

University Physics II · Final Examination

A cumulative final on calculus-based electricity, magnetism, optics and modern physics, weighted towards sources of magnetic fields, electromagnetic induction, inductance and AC circuits, Maxwell's equations and electromagnetic waves, geometric and physical optics, and the photon and matter-wave evidence of modern physics. The earlier units return through superposed Coulomb forces and stable equilibrium, electric potential, capacitor design, internal resistance, RC transients and charged-particle motion.

SUGGESTED TIME	TOTAL MARKS	QUESTIONS	MULTIPLE CHOICE
150 minutes	117	19	10

SECTION	UPII · Final · University Physics II — Final examination
IN THE FORMAT OF	A cumulative university final: conceptual multiple choice, structured problems, a data-analysis question and two extended derivations
DEMAND	Routine 6 · Demanding 11 · Discriminating 2 <i>Pitched at what a strong course grade requires rather than at what a homework set rehearses. Routine substitutions carry few marks; most sit on problems that have to be set up before they can be solved, on derivations from a named law, and on checking a result against its limits. Every paper still opens with entry points, so you can tell which marks you should be securing first.</i>
COURSE UNITS	UPII-01 · UPII-02 · UPII-03 · UPII-04 · UPII-05 · UPII-06 · UPII-07 · UPII-08 · UPII-09 · UPII-10 · UPII-11 · UPII-12 · UPII-13 · UPII-14 · UPII-15

INSTRUCTIONS

- Answer all questions in the spaces provided.
- Show your working. Method marks are awarded even where the final answer is wrong.
- Give your answers to a sensible number of significant figures, with a unit.
- The mark scheme for this paper follows the last question.

Written by GioPhysics from its own University Physics course maps. These are practice examinations in a university style: they are not the examinations of any university, contain no past-paper questions, and carry no academic credit. For a course you are taking, your instructor's syllabus and past papers remain the authority.

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University Physics II · Final Examination

Answer all questions. 117 marks in 150 minutes.

1 MULTIPLE CHOICE · ROUTINE · UPII-01.04 · UPII-01.05 · UPII-02.03 [1]
CORE COURSE

A thin, smooth insulating ring of radius R is held fixed, and gravity can be neglected. Point charges $+Q$ are fixed at the two ends of one diameter of the ring, and a small bead carrying charge $+q$, with $q > 0$, is threaded on the ring so that it can slide freely round it. The bead is placed at a point on the ring a quarter of a turn from each fixed charge. Which statement is correct?

- A The electric field of the fixed charges is zero at that point, so the bead would stay there even if the ring were removed.
- B The field there is not zero but points radially outwards, at right angles to the ring, and a bead nudged a short way along the ring is pushed back towards that point.
- C The field there is not zero but points radially outwards, and a bead nudged a short way along the ring is pushed on towards the nearer fixed charge.
- D The bead cannot be in equilibrium at that point, because both fixed charges repel it; only a negatively charged bead could rest there.

Answer

2 MULTIPLE CHOICE · ROUTINE · UPII-04.06 [1]
CORE COURSE

In a region of space the electric potential is $V(x, y) = \alpha x^2 y - \beta y$, with $\alpha = 20 \text{ V/m}^3$ and $\beta = 30 \text{ V/m}$. What is the electric field at the point $(x, y) = (1.0 \text{ m}, 2.0 \text{ m})$?

- A $(-80 \hat{i} + 10 \hat{j}) \text{ V/m}$
- B $(+80 \hat{i} - 10 \hat{j}) \text{ V/m}$
- C $(-40 \hat{i} + 10 \hat{j}) \text{ V/m}$
- D $(+10 \hat{i} - 80 \hat{j}) \text{ V/m}$

Answer

3 MULTIPLE CHOICE · ROUTINE · UPII-06.06 · UPII-06.08 [1]
CORE COURSE

A cordless drill runs from a battery pack. A high-resistance voltmeter across the pack's terminals reads 18.0 V while the drill is switched off and 16.8 V while the drill draws a steady current of 8.0 A . Model the pack as a constant emf in series with a constant internal resistance. At what rate is energy dissipated as heat inside the pack while the drill runs?

- A 144 W
- B 134 W
- C 9.6 W
- D $2.16 \times 10^3 \text{ W}$

Answer

4

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-08.02 · UPII-08.03

[1]

Electrons leave a small source inside a long solenoid, each moving at 30° to the solenoid's axis. Inside the solenoid the magnetic field is uniform and parallel to the axis, and there is no electric field, so each electron follows a helix whose axis is parallel to the field. What is the ratio of the pitch of the helix, the distance an electron moves parallel to the axis during one revolution, to the radius of the helix?

- A $2\pi/\sqrt{3}$
- B $\pi\sqrt{3}$
- C 2π
- D $2\pi\sqrt{3}$

Answer

5

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-09.05

[1]

A long straight hollow cylindrical conductor has inner radius 2.0 cm and outer radius 4.0 cm, and carries a steady current of 12 A distributed uniformly over the metal's cross-section. Take $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$. What is the magnitude of the magnetic field at a distance of 3.0 cm from the axis?

- A zero
- B $8.0 \times 10^{-5} \text{ T}$
- C $4.5 \times 10^{-5} \text{ T}$
- D $3.3 \times 10^{-5} \text{ T}$

Answer

6

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-10.03 · UPII-10.06

[1]

A strong cylindrical magnet is dropped down the centre of a long vertical copper pipe and soon falls at a steady speed far below that of free fall. Copper is not ferromagnetic. Which statement correctly explains the motion?

- A The magnet magnetises the copper, which then attracts it upwards in the way an iron pipe would.
- B Eddy currents flow only in the pipe just below the magnet, where the flux is increasing; above the magnet the flux is decreasing, so no current is induced there.
- C Eddy currents circulate in opposite senses in the pipe just below and just above the magnet; each exerts an upward force on the magnet, and the gravitational energy the magnet loses is dissipated in the pipe.
- D Eddy currents circulate in the same sense above and below the magnet, and the steady speed is reached once the pipe has warmed enough for its resistance to stop them growing.

Answer

7CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-11.06 · UPII-11.07

[1]

A series RLC circuit is driven by a sinusoidal source at its resonant frequency f_0 . The source frequency is then raised to $2f_0$ with the source amplitude unchanged. Which statement describes the circuit at $2f_0$? (X_L and X_C are the inductive and capacitive reactances.)

- A** $X_L = 2X_C$, and the current leads the source emf
- B** $X_L = 4X_C$, and the current lags the source emf
- C** $X_L = 4X_C$, and the current leads the source emf
- D** $X_L = X_C$ still, because L and C have not changed, so the impedance is still R

Answer

8CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-12.06 · UPII-12.07

[1]

The electric field of a plane electromagnetic wave in vacuum is $E = E_0 \sin(kz + \omega t) \hat{j}$, where E_0 , k and ω are positive constants and c is the speed of light. Which expression gives the magnetic field of the wave?

- A** $B = (E_0/c) \sin(kz + \omega t) \hat{i}$
- B** $B = -(E_0/c) \sin(kz + \omega t) \hat{i}$
- C** $B = cE_0 \sin(kz + \omega t) \hat{i}$
- D** $B = (E_0/c) \sin(kz + \omega t) \hat{k}$

Answer

9CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-13.02 · UPII-13.03

[1]

Light travelling in glass of refractive index 1.52 meets a flat boundary with water of refractive index 1.33. Above what angle of incidence, measured from the normal, is the light totally internally reflected?

- A** 41.1°
- B** 48.8°
- C** 61.0°
- D** No angle: total internal reflection requires the second medium to be air or vacuum

Answer

An electron is accelerated from rest through a potential difference of 150 V. Take $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$, $e = 1.60 \times 10^{-19} \text{ C}$ and $c = 3.00 \times 10^8 \text{ m/s}$. What is the electron's de Broglie wavelength?

- A 8.29 nm
- B 0.142 nm
- C 0.00243 nm
- D 0.100 nm

Answer

The storage capacitor of a laboratory pulse generator is a hollow metal globe of radius $b = 0.200$ m with a metal ball of radius $a = 0.120$ m supported by an insulating rod at its centre. Transformer oil, of dielectric constant $\kappa = 2.2$ and dielectric strength 1.2×10^7 V/m, fills the globe exactly up to the horizontal plane through its centre, so that the oil occupies the lower half of the gap between ball and globe; the upper half of the gap contains air, of dielectric constant 1.00 and dielectric strength 3.0×10^6 V/m. The ball is charged positive, to a potential difference of 100 kV relative to the globe, and the capacitor is then disconnected from the supply. Treat the wall of the globe as thin and neglect the supporting rod. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F/m.

(a) SHOW THAT

[4]

Starting from Gauss's law, show that if air alone filled the whole gap the capacitance would be $C_0 = 4\pi\epsilon_0 ab/(b - a)$. Hence, by explaining why the field in the half-filled gap is radial and has the same magnitude in the oil and in the air at the same distance from the centre, show that the half-filled capacitor has $C = 2\pi(1 + \kappa)\epsilon_0 ab/(b - a)$.

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(b) CALCULATE

[3]

Calculate the capacitance, the charge on the ball, and the magnitude of the electric field in the gap next to the surface of the ball, when the potential difference is 100 kV.

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(c) DETERMINE

[3]

Determine the free charge per unit area on the half of the ball's surface in contact with the oil and on the half in contact with the air, and check that together they carry the charge found in (b).

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(d) PREDICT

[2]

With the capacitor still disconnected, the oil is drained out through a valve in the bottom of the globe and air takes its place. Predict the potential difference that the charge on the ball would produce with air alone in the gap, and whether the air could withstand it.

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An ideal battery of emf $\varepsilon = 9.0 \text{ V}$, a switch and a resistor $R_1 = 30 \text{ k}\Omega$ are connected in series from the battery's positive terminal to a node P. From node P back to the battery's negative terminal run two branches in parallel: one contains a resistor $R_2 = 60 \text{ k}\Omega$, the other an initially uncharged capacitor $C = 220 \text{ }\mu\text{F}$. The switch is closed at $t = 0$.

(a) DETERMINE

[2]

Determine the current in R_1 immediately after the switch is closed, and a long time afterwards.

(b) DERIVE

[4]

Using Kirchhoff's loop and junction rules, derive the differential equation obeyed by the capacitor voltage $V(t)$, and hence obtain $V(t) = V_\infty(1 - e^{-(t/\tau)})$, giving V_∞ and τ in terms of ε , R_1 , R_2 and C .

(c) CALCULATE

[2]

Calculate τ , and the time at which the capacitor voltage reaches 4.5 V .

(d) EVALUATE

[2]

A student argues that because the capacitor charges through R_1 , the time constant must be $R_1C = 6.6 \text{ s}$. Evaluate this argument.

(e) CALCULATE

[1]

Calculate the energy stored in the capacitor a long time after the switch is closed.

A long straight insulated wire lies in the plane of a flat rectangular coil of $N = 200$ turns. The coil's two long sides, each of length $\ell = 0.200$ m, are parallel to the wire at distances $a = 0.020$ m and $b = 0.080$ m from it. The coil's ends are connected to an oscilloscope of very high input resistance. Take $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

(a) DERIVE

[3]

Starting from Ampère's law and the definition of magnetic flux, derive an expression for the magnetic flux through one turn of the coil when the wire carries a steady current I .

(b) CALCULATE

[2]

Calculate the mutual inductance of the wire and the coil.

(c) CALCULATE

[2]

The wire carries $I(t) = I_0 \sin(2\pi ft)$ with $I_0 = 15$ A and $f = 50$ Hz. Calculate the amplitude of the emf induced in the coil.

(d) EXPLAIN

[2]

The coil is now closed through a resistor. At an instant when the wire current is in its positive direction and increasing, explain the direction of the induced current in the long side nearer the wire, and explain whether the wire attracts or repels the coil.

(e) PREDICT

[2]

The coil is slid within its plane until the wire lies along the coil's centre line, parallel to and midway between its long sides. Predict the emf induced by the same alternating current, giving the reason.

A series circuit contains a resistor $R = 25\ \Omega$, an inductor $L = 80\ \text{mH}$ of negligible resistance and a capacitor $C = 50\ \mu\text{F}$. It is driven by a sinusoidal source of rms emf $120\ \text{V}$ whose frequency can be varied.

(a) **CALCULATE**

[2]

Calculate the resonant frequency, and the rms current and average power delivered at resonance.

(b) **DETERMINE**

[4]

The source frequency is set to $50\ \text{Hz}$. Determine the impedance, the rms current, the phase angle between the current and the source emf, and the average power delivered, stating whether the current leads or lags the emf.

(c) **SKETCH**

[3]

Sketch the phasor diagram for the circuit at $50\ \text{Hz}$, with the current phasor drawn horizontally, showing the rms voltage phasors across R , L and C and the source emf, each labelled with its magnitude.

(d) **EXPLAIN**

[2]

At resonance the rms voltage across the capacitor is $192\ \text{V}$, well above the $120\ \text{V}$ rms of the source. Explain how this is consistent with Kirchhoff's loop rule and with conservation of energy.

A compact telephoto camera lens is modelled as two thin lenses on a common axis: a converging lens of focal length $f_1 = +10.0$ cm, followed 6.0 cm behind it by a diverging lens of focal length $f_2 = -12.0$ cm. The image sensor sits behind the second lens. Use the real-is-positive convention, $1/u + 1/v = 1/f$ with magnification $m = -v/u$, in which a virtual object has a negative object distance u .

(a) DETERMINE

[3]

Light from a very distant object enters the first lens. Determine where the final image forms, and state whether it is real or virtual.

(b) SHOW THAT

[3]

The distant object subtends an angle of 0.50° at the lens. Show that the final image is 0.131 cm tall, and hence that the combination behaves as a single lens of focal length 15.0 cm.

(c) COMPARE

[2]

Compare this combination with a single converging lens that forms an image of the same size of the same distant object, in terms of the distance from the front of the lens to the sensor.

(d) PREDICT

[2]

The camera is refocused on an object 2.00 m in front of the first lens, with the lens separation unchanged. Predict whether the sensor must move towards or away from the second lens, and by how much.

A student uses a transmission grating ruled with 450 lines per millimetre, illuminated at normal incidence, to look for the deuterium line in the light of a discharge lamp that contains hydrogen and deuterium in comparable amounts. The red Balmer line of ordinary hydrogen has wavelength 656.28 nm; the corresponding line of deuterium has wavelength 656.10 nm. The lamp also emits the blue-green Balmer line of hydrogen, of wavelength 486.13 nm.

- (a) **CALCULATE** [2]

Calculate the grating spacing, and the highest order in which light of wavelength 656 nm can be observed.

- (b) **CALCULATE** [1]

Calculate the angle at which the hydrogen line appears in second order.

- (c) **SHOW THAT** [2]

By differentiating the grating equation, show that two lines separated in wavelength by a small amount $\Delta\lambda$ appear in order m with an angular separation $\Delta\theta = m\Delta\lambda/(d \cos \theta)$.

- (d) **CALCULATE** [1]

Calculate the angular separation of the hydrogen and deuterium lines in second order.

- (e) **DETERMINE** [2]

Determine the minimum number of grating lines that must be illuminated to resolve the two lines in second order, and the corresponding minimum width of the illuminated region of the grating.

- (f) **COMPARE** [2]

Compare working in third order with working in second order for this measurement.

In a photoelectric experiment, monochromatic light falls on a clean metal cathode in an evacuated tube. For each wavelength the reverse potential difference between anode and cathode is increased until the photocurrent just falls to zero, and this stopping potential V_s is recorded. Each stopping potential has an uncertainty of ± 0.03 V, and each wavelength is known to ± 1 nm. Take $e = 1.60 \times 10^{-19}$ C and $c = 3.00 \times 10^8$ m/s.

Stopping potential against wavelength for one cathode

Reading	Wavelength λ (nm)	Stopping potential V_s (V)
1	300	1.95
2	340	1.44
3	380	1.08
4	420	0.66
5	460	0.52
6	500	0.28

- (a) **STATE** [1]
State the photoelectric equation relating the stopping potential V_s to the frequency f of the light.

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- (b) **EVALUATE** [2]
Calculate the frequency of the light for each reading, and evaluate whether all six readings are consistent with a single straight line.

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- (c) **DETERMINE** [3]
Determine the Planck constant from the gradient of a graph of V_s against f .

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- (d) **DETERMINE** [2]
Determine the work function of the cathode, in electronvolts, and the threshold wavelength beyond which no photoelectrons are emitted.

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- (e) **ESTIMATE** [2]
Estimate the uncertainty in your value of the Planck constant, stating the assumptions you make.

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(f) JUSTIFY

[2]

Justify the claim that these data cannot be explained by a classical wave model, in which the energy an electron absorbs grows with the intensity of the light and the time it is exposed.

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18
CORE
COURSE

FREE RESPONSE · EXTENDED DERIVATION · DISCRIMINATING · UPII-12.04 · UPII-12.05 · UPII-12.07

[15]

A parallel-plate capacitor has circular plates of radius $R = 5.0$ cm separated by $d = 2.0$ mm in vacuum. It is being charged by a steady current $I = 0.40$ A carried by long straight leads that meet the centres of the plates along their common axis. Neglect fringing, so that the electric field is uniform between the plates and zero outside the gap. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F/m and $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

(a) EXPLAIN

[2]

Ampère's law without Maxwell's correction gives contradictory results for the magnetic field at a point beside the gap, depending on which surface is chosen to span a circular path around the axis. Explain the contradiction, and how the displacement current resolves it.

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(b) DERIVE

[4]

Using the Ampère–Maxwell law, derive the magnitude of the magnetic field between the plates at a distance $r < R$ from the axis.

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(c) DETERMINE

[2]

Determine the magnitude of the magnetic field in the midplane of the gap at a distance $r > R$ from the axis, and state how it compares with the field at the same distance from one of the leads.

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(d) SKETCH

[2]

Sketch a graph of the magnitude of B against r in the midplane of the gap, from $r = 0$ to $r = 3R$, marking the value at $r = R$.

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(e) **SHOW THAT** [4]

At the edge of the gap, $r = R$, show that the Poynting vector $S = E \times B/\mu_0$ points radially inwards, and that the total power flowing in through the cylindrical surface of radius R and height d equals the rate of increase of the energy stored in the capacitor, $d/dt(q^2/2C)$.

(f) **PREDICT** [1]

The capacitor is later discharged through the same leads. Predict the direction of the Poynting vector at the edge of the gap during the discharge.

19
CORE
COURSE

FREE RESPONSE · MODEL, SOLVE AND CHECK THE LIMITS · DISCRIMINATING · UPII-09.04 · UPII-10.07 · UPII-12.03 [15]

A long solenoid of radius $a = 4.0$ cm has $n = 8000$ turns per metre and carries a current $I_0 = 20$ A. Coaxial with it, outside the winding, a thin insulating ring of radius $b = 6.0$ cm and mass $m = 0.50$ g carries a charge $Q = 0.50$ μC spread uniformly around it. The ring can turn freely about the common axis, and the apparatus is in vacuum. The solenoid current is reduced steadily to zero over $T = 0.50$ s. Treat the solenoid as ideal, with no magnetic field outside its winding, and neglect the magnetic field of the ring itself. Take $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

(a) **DESCRIBE** [2]

Describe the electric field induced while the current is falling: its direction, the shape of its field lines, and why it cannot be described by an electric potential.

(b) **DERIVE** [4]

Using Faraday's law in the form $\oint E \cdot dl = -d\Phi_B/dt$, derive the magnitude of the induced electric field at distance r from the axis, for $r < a$ and for $r > a$, in terms of the rate of change dB/dt of the field inside the solenoid.

(c) **CALCULATE** [2]

Calculate the magnetic field inside the solenoid before the current starts to fall, and the magnitude of the induced electric field at the ring while the current is falling.

(d) **DERIVE** [3]

Using the rotational form of Newton's second law, $\tau = dL/dt$, derive the angular momentum the ring has acquired once the current has fallen to zero, and show that it depends neither on T nor on b , provided $b > a$.

(e) **CALCULATE** [2]

Calculate the angular speed of the ring once the current has fallen to zero.

(f) **EVALUATE** [2]

A student claims that the ring starts to turn because the solenoid's magnetic field pushes on its charges. Evaluate this claim.

University Physics II · Final Examination — mark scheme

One line per mark, as an examiner would award them. A note under a part is the error that part is set to catch.

1

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-01.04 · UPII-01.05 · UPII-02.03

A thin, smooth insulating ring of radius R is held fixed, and gravity can be neglected. Point charges $+Q$ are fixed at the two ends of one diameter of the ring, and a small bead carrying charge $+q$, with $q > 0$, is threaded on the ring so that it can slide freely round it. The bead is placed at a point on the ring a quarter of a turn from each fixed charge. Which statement is correct?

KEY B The field there is not zero but points radially outwards, at right angles to the ring, and a bead nudged a short way along the ring is pushed back towards that point.

Each fixed charge is a distance $R\sqrt{2}$ from the quarter point and produces a field of magnitude $kQ/(2R^2)$ there, where $k = 1/(4\pi\epsilon_0)$, directed away from itself. By symmetry the components along the ring cancel and the components along the radius add, giving $\sqrt{2}kQ/(2R^2)$ radially outwards; the only point where the field of the two fixed charges vanishes is the centre of the ring. A mistakes an equilibrium for a null point: the bead is at rest only because the ring pushes inwards on it, balancing the outward electric force, and without the ring it would be driven away from the centre. C forgets what a nudge along the ring does to the two distances: the bead moves closer to one fixed charge and further from the other, so the push along the ring from the nearer charge grows while the opposing push from the further charge shrinks, and the bead is driven back; the quarter point is a minimum of potential energy along the ring. D confuses equilibrium with the absence of force: both repulsions act, but their components along the ring are equal and opposite and the ring supplies the rest. A negative bead would also be in equilibrium at that point, but a nudge would then be amplified rather than undone, so its equilibrium would be unstable.

[1]

U1

2

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-04.06

In a region of space the electric potential is $V(x, y) = \alpha x^2 y - \beta y$, with $\alpha = 20 \text{ V/m}^3$ and $\beta = 30 \text{ V/m}$. What is the electric field at the point $(x, y) = (1.0 \text{ m}, 2.0 \text{ m})$?

KEY A $(-80 \hat{i} + 10 \hat{j}) \text{ V/m}$

The field is minus the gradient of the potential: $E_x = -\partial V/\partial x = -2\alpha xy = -80 \text{ V/m}$ and $E_y = -\partial V/\partial y = -(\alpha x^2 - \beta) = +10 \text{ V/m}$. B gives both components the sign of the gradient itself, the field of a candidate who forgets that E points down the potential slope. C drops the factor of 2 from differentiating x^2 , a slip that leaves E_y untouched and so looks half right. D attaches each derivative to the wrong component, taking E_x from $\partial V/\partial y$ and E_y from $\partial V/\partial x$; the numbers are the right ones, which is exactly what makes it convincing.

[1]

U2

3CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-06.06 · UPII-06.08

[1]

A cordless drill runs from a battery pack. A high-resistance voltmeter across the pack's terminals reads 18.0 V while the drill is switched off and 16.8 V while the drill draws a steady current of 8.0 A. Model the pack as a constant emf in series with a constant internal resistance. At what rate is energy dissipated as heat inside the pack while the drill runs?

KEY C 9.6 W

The terminal voltage falls by 1.2 V when 8.0 A flows, and that fall is the potential difference across the internal resistance, so $r = (1.2 \text{ V})/(8.0 \text{ A}) = 0.15 \Omega$ and the heating rate inside the pack is $I^2 r = (8.0 \text{ A})^2 (0.15 \Omega) = 9.6 \text{ W}$, the same as $(1.2 \text{ V})(8.0 \text{ A})$. A is $\mathcal{E} I = (18.0 \text{ V})(8.0 \text{ A})$, the whole rate at which the emf does work, which counts the energy delivered to the drill as well as the heat left in the pack. B is the terminal voltage times the current, $(16.8 \text{ V})(8.0 \text{ A}) = 134 \text{ W}$: this is the power delivered to the drill, the energy that leaves the pack rather than the energy that stays in it. D is $\mathcal{E}^2/r$, the heating rate if the terminals were short-circuited; it puts the whole emf across the internal resistance, which happens only when the external resistance is zero, not while the drill is running.

U3

4CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-08.02 · UPII-08.03

[1]

Electrons leave a small source inside a long solenoid, each moving at 30° to the solenoid's axis. Inside the solenoid the magnetic field is uniform and parallel to the axis, and there is no electric field, so each electron follows a helix whose axis is parallel to the field. What is the ratio of the pitch of the helix, the distance an electron moves parallel to the axis during one revolution, to the radius of the helix?

KEY D $2\pi\sqrt{3}$

Resolve the velocity along and across the field. The perpendicular component $v \sin 30^\circ$ drives the circular motion, with radius $r = mv \sin 30^\circ / (eB)$ and period $T = 2\pi m / (eB)$, which does not depend on the speed. The magnetic force has no component along the field, so the parallel component $v \cos 30^\circ$ stays constant and carries the electron a pitch $p = (v \cos 30^\circ)T = 2\pi m v \cos 30^\circ / (eB)$ in one revolution. The ratio is $p/r = 2\pi \cot 30^\circ = 2\pi\sqrt{3} \approx 10.9$, whatever the speed, the field strength or the electron's charge and mass. A comes from interchanging the components, putting $\sin 30^\circ$ into the pitch and $\cos 30^\circ$ into the radius. B compares the pitch with the diameter rather than the radius, or counts half a period as one revolution. C assumes the electron advances one circumference per revolution, which is true only when the two components are equal, at 45° to the field.

U2

5CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-09.05

[1]

A long straight hollow cylindrical conductor has inner radius 2.0 cm and outer radius 4.0 cm, and carries a steady current of 12 A distributed uniformly over the metal's cross-section. Take $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$. What is the magnitude of the magnetic field at a distance of 3.0 cm from the axis?

KEY D $3.3 \times 10^{-5} \text{ T}$

By symmetry B is azimuthal and uniform on a circle of radius 3.0 cm about the axis, so Ampère's law gives $B(2\pi r) = \mu_0 I_{\text{enc}}$. The enclosed current is the fraction of the metal inside that circle, $I_{\text{enc}} = 12 \text{ A} \times (3.0^2 - 2.0^2)/(4.0^2 - 2.0^2) = 5.0 \text{ A}$, so $B = \mu_0(5.0 \text{ A})/(2\pi \times 0.030 \text{ m}) = 3.3 \times 10^{-5} \text{ T}$. B uses the whole 12 A, as if the point lay outside the conductor. C takes the enclosed fraction as r^2/b^2 , the rule for a solid wire, and so counts current in the hollow core where there is none. A carries over the electrostatic result that the field inside a conductor vanishes; that is true of E in equilibrium, not of the magnetic field of a current flowing through the metal.

U3

6

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-10.03 · UPII-10.06

[1]

A strong cylindrical magnet is dropped down the centre of a long vertical copper pipe and soon falls at a steady speed far below that of free fall. Copper is not ferromagnetic. Which statement correctly explains the motion?

KEY C Eddy currents circulate in opposite senses in the pipe just below and just above the magnet; each exerts an upward force on the magnet, and the gravitational energy the magnet loses is dissipated in the pipe.

Below the magnet the flux through each ring of the pipe is increasing, so the induced current opposes the increase and repels the magnet; above it the flux is decreasing, so the induced current acts to maintain the flux and attracts the magnet back. The two rings carry currents in opposite senses, yet both forces on the magnet point up, and at the steady speed their sum equals the weight while the lost gravitational energy becomes internal energy of the copper. A fails for copper, whose magnetisation is negligible, and would fail even for iron: a long uniform ferromagnetic pipe pulls the magnet sideways towards its wall, not upwards along the axis. A copper pipe with a slot cut along its length also slows the magnet far less, because the slot interrupts the circulating currents, which a magnetisation mechanism cannot explain. B applies Lenz's law only to increasing flux, but a decreasing flux induces a current just as surely. D gets the sense of the currents wrong and invents a thermal cause for the steady speed, which is really set by a drag force that grows in proportion to the speed.

U1

7

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-11.06 · UPII-11.07

[1]

A series RLC circuit is driven by a sinusoidal source at its resonant frequency f_0 . The source frequency is then raised to $2f_0$ with the source amplitude unchanged. Which statement describes the circuit at $2f_0$? (X_L and X_C are the inductive and capacitive reactances.)

KEY B $X_L = 4X_C$, and the current lags the source emf

At f_0 the two reactances are equal. Doubling the frequency doubles $X_L = 2\pi fL$ and halves $X_C = 1/(2\pi fC)$, so $X_L = 4X_C$; the circuit is inductive, and in an inductive circuit the current lags the emf. A doubles X_L but forgets that X_C falls at the same time. C has the ratio right and the phase backwards: a leading current belongs to a capacitive circuit, below resonance. D treats resonance as a property of the components alone, when it is a property of those components at one particular frequency.

U1

8

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-12.06 · UPII-12.07

[1]

The electric field of a plane electromagnetic wave in vacuum is $E = E_0 \sin(kz + \omega t) \hat{j}$, where E_0 , k and ω are positive constants and c is the speed of light. Which expression gives the magnetic field of the wave?

KEY A $B = (E_0/c) \sin(kz + \omega t) \hat{i}$

The phase $kz + \omega t$ stays constant if z decreases as t increases, so this wave travels towards $-z$. The energy flux $S = E \times B/\mu_0$ must then point along $-\hat{k}$; with E along $\hat{j}$ that needs B along $+\hat{i}$, because $\hat{j} \times \hat{i} = -\hat{k}$, and the amplitudes are related by $B_0 = E_0/c$. Faraday's law confirms it: $-\partial E_y/\partial z = -kE_0 \cos(kz + \omega t)$ equals $-\partial B_x/\partial t$ only for the field in A. B gives the magnetic field the orientation, $-\hat{i}$, that belongs to a wave travelling towards $+z$; with this E it would make S point along $+\hat{k}$, against the direction the wave actually travels, and it is chosen by a candidate who assumes a wave always moves forwards without reading the sign in the phase. C inverts the amplitude relation, giving a magnetic field about 10^{17} times too large. D puts B along the direction of propagation, but a field that varies only with z can have no varying z -component without violating Gauss's law for magnetism: electromagnetic waves in vacuum are transverse.

U2

9

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-13.02 · UPII-13.03

[1]

Light travelling in glass of refractive index 1.52 meets a flat boundary with water of refractive index 1.33. Above what angle of incidence, measured from the normal, is the light totally internally reflected?

KEY C 61.0°

Total internal reflection needs only that the second medium have the lower refractive index. At the critical angle the refracted ray grazes the boundary, so $1.52 \sin \theta_c = 1.33 \sin 90^\circ$ and $\theta_c = \sin^{-1}(1.33/1.52) = 61.0^\circ$. A is $\sin^{-1}(1/1.52)$, the critical angle for glass to air, the value a candidate remembers rather than works out. B is $\sin^{-1}(1/1.33)$, the critical angle for water to air, reached by dropping the glass out of the problem. D mistakes the everyday example for the condition: every boundary from a higher to a lower index has a critical angle, and it is larger the closer the two indices are.

U3

10

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-15.05

[1]

An electron is accelerated from rest through a potential difference of 150 V. Take $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$, $e = 1.60 \times 10^{-19} \text{ C}$ and $c = 3.00 \times 10^8 \text{ m/s}$. What is the electron's de Broglie wavelength?

KEY D 0.100 nm

The kinetic energy is $eV = 2.40 \times 10^{-17} \text{ J}$, tiny beside $m_e c^2$, so the classical momentum $p = \sqrt{2m_e eV} = 6.61 \times 10^{-24} \text{ kg}\cdot\text{m/s}$ is adequate and $\lambda = h/p = 1.00 \times 10^{-10} \text{ m} = 0.100 \text{ nm}$. A is $hc/(eV)$, the wavelength of a 150 eV photon, which applies $E = pc$ to a particle with mass. B drops the factor of 2 inside the square root, which is what happens when the kinetic energy is written as mv^2 instead of $\frac{1}{2}mv^2$. C is $h/(m_e c)$, the Compton wavelength, which replaces the electron's actual speed with the speed of light.

U3

11

CORE
COURSE

STRUCTURED · DEMANDING · UPII-03.04 · UPII-04.08 · UPII-05.05 · UPII-05.07 · UPII-05.08

[12]

The storage capacitor of a laboratory pulse generator is a hollow metal globe of radius $b = 0.200 \text{ m}$ with a metal ball of radius $a = 0.120 \text{ m}$ supported by an insulating rod at its centre. Transformer oil, of dielectric constant $\kappa = 2.2$ and dielectric strength $1.2 \times 10^7 \text{ V/m}$, fills the globe exactly up to the horizontal plane through its centre, so that the oil occupies the lower half of the gap between ball and globe; the upper half of the gap contains air, of dielectric constant 1.00 and dielectric strength $3.0 \times 10^6 \text{ V/m}$. The ball is charged positive, to a potential difference of 100 kV relative to the globe, and the capacitor is then disconnected from the supply. Treat the wall of the globe as thin and neglect the supporting rod. Take $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.

(a) SHOW THAT · U2

[4]

Starting from Gauss's law, show that if air alone filled the whole gap the capacitance would be $C_0 = 4\pi\epsilon_0 ab/(b - a)$. Hence, by explaining why the field in the half-filled gap is radial and has the same magnitude in the oil and in the air at the same distance from the centre, show that the half-filled capacitor has $C = 2\pi(1 + \kappa)\epsilon_0 ab/(b - a)$.

- 1 Gaussian sphere of radius r , $a < r < b$, concentric with the ball: by symmetry E is radial and uniform over it, so $4\pi r^2 E = Q/\epsilon_0$ and $E = Q/(4\pi\epsilon_0 r^2)$
- 2 $V = \int E \, dr$ from $r = a$ to $r = b = (Q/(4\pi\epsilon_0))(1/a - 1/b)$, so $C_0 = Q/V = 4\pi\epsilon_0 ab/(b - a)$
- 3 A radial field $E = A/r^2$ in each half lies along the flat oil-air boundary with no component across it; because the ball and the globe are equipotentials, $\int E \, dr$ from a to b is the same V along every radius, which forces the same A in the oil and in the air and keeps the tangential field continuous at the boundary, so the two halves share one potential difference and act as capacitors in parallel
- 4 The air half has capacitance $C_0/2$ and the oil half $\kappa C_0/2$, so $C = (1 + \kappa)C_0/2 = 2\pi(1 + \kappa)\epsilon_0 ab/(b - a)$

Candidates who stack the oil and the air as capacitors in series have put the boundary in the wrong place: layers in series need a boundary across the field lines, whereas this boundary runs along them, so the halves share the potential difference and their capacitances add. The step from C_0 to C earns nothing without the reason the field is the same in both media.

(b) CALCULATE · U3

[3]

Calculate the capacitance, the charge on the ball, and the magnitude of the electric field in the gap next to the surface of the ball, when the potential difference is 100 kV.

1 $C = 2\pi(3.2)(8.85 \times 10^{-12} \text{ F/m})(0.120 \text{ m})(0.200 \text{ m})/(0.080 \text{ m}) = 5.34 \times 10^{-11} \text{ F} (53.4 \text{ pF})$

2 $Q = CV = (5.34 \times 10^{-11} \text{ F})(1.00 \times 10^5 \text{ V}) = 5.34 \times 10^{-6} \text{ C}$

3 $E(a) = Vb/(a(b - a)) = (1.00 \times 10^5 \text{ V})(0.200 \text{ m})/((0.120 \text{ m})(0.080 \text{ m})) = 2.08 \times 10^6 \text{ V/m}$, the same in the oil and in the air, and below the $3.0 \times 10^6 \text{ V/m}$ that air withstands

Dividing the field in the oil by κ , to $9.5 \times 10^5 \text{ V/m}$, treats the oil as if it faced a fixed charge of its own; here the conductors fix one potential difference along every radius, so the field is the same in both media and it is the charge on the ball that differs between the halves. Using $V/(b - a) = 1.25 \times 10^6 \text{ V/m}$, as if the field were uniform, understates the field at the ball by 40%.

(c) DETERMINE · U2

[3]

Determine the free charge per unit area on the half of the ball's surface in contact with the oil and on the half in contact with the air, and check that together they carry the charge found in (b).

1 Just outside a conductor the free surface charge density is $\sigma = \kappa\epsilon_0 E$, with $\kappa = 1.00$ for the air, and $E(a)$ is the same on both halves

2 $\sigma_{\text{air}} = (8.85 \times 10^{-12} \text{ F/m})(2.08 \times 10^6 \text{ V/m}) = 1.84 \times 10^{-5} \text{ C/m}^2$, and $\sigma_{\text{oil}} = \kappa\sigma_{\text{air}} = 4.06 \times 10^{-5} \text{ C/m}^2$

3 Each half has area $2\pi a^2 = 9.05 \times 10^{-2} \text{ m}^2$, so $Q = 2\pi a^2(\sigma_{\text{oil}} + \sigma_{\text{air}}) = 5.34 \times 10^{-6} \text{ C}$, agreeing with (b); the oil-covered half carries $\kappa/(1 + \kappa) = 69\%$ of it

Candidates who assume a conductor always spreads its charge evenly put $2.67 \times 10^{-6} \text{ C}$ on each half. A conductor must be an equipotential, not uniformly charged: the bound charge on the oil next to the ball cancels part of the free charge there, so the oil-covered half draws κ times as much free charge per unit area to keep the field the same on both sides.

(d) PREDICT · U1

[2]

With the capacitor still disconnected, the oil is drained out through a valve in the bottom of the globe and air takes its place. Predict the potential difference that the charge on the ball would produce with air alone in the gap, and whether the air could withstand it.

1 The charge stays at $5.34 \times 10^{-6} \text{ C}$ while the capacitance falls to $C_0 = 2C/(1 + \kappa) = 33.4 \text{ pF}$, so the potential difference would rise by the factor $(1 + \kappa)/2 = 1.6$, to $1.60 \times 10^5 \text{ V}$

2 The field at the surface of the ball would rise by the same factor, to $3.33 \times 10^6 \text{ V/m}$, above the $3.0 \times 10^6 \text{ V/m}$ that air withstands, so the air cannot withstand it: breakdown starts next to the ball while the last of the oil is still draining, before the potential difference reaches $1.60 \times 10^5 \text{ V}$

Candidates who keep the potential difference at 100 kV, as if the supply were still connected, predict no change in the field and a safe gap. With the charge fixed, removing the dielectric lowers the capacitance and raises both the potential difference and the field; and the oil's high dielectric strength never protected the capacitor, because the air half always carried the same field.

An ideal battery of emf $\epsilon = 9.0$ V, a switch and a resistor $R_1 = 30$ k Ω are connected in series from the battery's positive terminal to a node P. From node P back to the battery's negative terminal run two branches in parallel: one contains a resistor $R_2 = 60$ k Ω , the other an initially uncharged capacitor $C = 220$ μ F. The switch is closed at $t = 0$.

(a) DETERMINE · U1

[2]

Determine the current in R_1 immediately after the switch is closed, and a long time afterwards.

- 1 At $t = 0$ the uncharged capacitor has no potential difference across it, so P is at the potential of the negative terminal and $I_1 = 9.0$ V/30 k $\Omega = 3.0 \times 10^{-4}$ A (0.30 mA)
- 2 A long time later the capacitor carries no current, so R_1 and R_2 are in series and $I_1 = 9.0$ V/90 k $\Omega = 1.0 \times 10^{-4}$ A (0.10 mA)

Candidates who treat the capacitor as an open circuit at $t = 0$ give 0.10 mA for both times. The uncharged capacitor behaves as a wire at the first instant and as a break only once charged.

(b) DERIVE · U2

[4]

Using Kirchhoff's loop and junction rules, derive the differential equation obeyed by the capacitor voltage $V(t)$, and hence obtain $V(t) = V_\infty(1 - e^{-(t/\tau)})$, giving V_∞ and τ in terms of ϵ , R_1 , R_2 and C .

- 1 Loop rule round the battery, R_1 and the capacitor: $\epsilon - I_1 R_1 - V = 0$, so $I_1 = (\epsilon - V)/R_1$
- 2 Junction rule at P: $I_1 = V/R_2 + C \, dV/dt$
- 3 Combining: $C \, dV/dt = \epsilon/R_1 - V(1/R_1 + 1/R_2)$, which rearranges to $\tau \, dV/dt = V_\infty - V$ with $V_\infty = \epsilon R_2/(R_1 + R_2)$ and $\tau = C R_1 R_2/(R_1 + R_2)$
- 4 Separates the variables and integrates with $V(0) = 0$: $\ln((V_\infty - V)/V_\infty) = -t/\tau$, so $V(t) = V_\infty(1 - e^{-(t/\tau)})$

The junction rule is the step most often missing: a single loop equation with $I_1 = C \, dV/dt$ ignores the current R_2 draws and reproduces the student's argument in part (d).

(c) CALCULATE · U3

[2]

Calculate τ , and the time at which the capacitor voltage reaches 4.5 V.

- 1 $\tau = (220 \times 10^{-6} \text{ F})(20 \text{ k}\Omega) = 4.4$ s, with $V_\infty = 6.0$ V
- 2 $4.5 \text{ V} = (6.0 \text{ V})(1 - e^{-(t/\tau)})$ gives $e^{-(t/\tau)} = 0.25$, so $t = \tau \ln 4 = 6.1$ s

Using 9.0 V as the final voltage gives $t = \tau \ln 2 = 3.0$ s; the threshold is three-quarters of the way to 6.0 V, not half of the way to 9.0 V.

(d) EVALUATE · U1

[2]

A student argues that because the capacitor charges through R_1 , the time constant must be $R_1 C = 6.6$ s. Evaluate this argument.

- 1 The argument fails because R_2 diverts part of the current arriving at P: seen from the capacitor, the network is equivalent to a source $\epsilon R_2/(R_1 + R_2) = 6.0$ V behind R_1 and R_2 in parallel, 20 k Ω
- 2 So $\tau = 4.4$ s rather than 6.6 s, and the student's value would place the 4.5 V crossing at 9.1 s instead of 6.1 s, an error of 50% in a timing design

The part rewards a judgement backed by the equivalent resistance, not a restatement of part (b). An answer that says only ' R_2 also matters' earns the first mark at most.

(e) CALCULATE · U3

[1]

Calculate the energy stored in the capacitor a long time after the switch is closed.

- 1 $U = \frac{1}{2} C V_\infty^2 = \frac{1}{2} (220 \times 10^{-6} \text{ F})(6.0 \text{ V})^2 = 3.96 \times 10^{-3}$ J (4.0 mJ)

Using the battery emf gives 8.9 mJ; the capacitor ends at the divider voltage across R_2 .

A long straight insulated wire lies in the plane of a flat rectangular coil of $N = 200$ turns. The coil's two long sides, each of length $\ell = 0.200$ m, are parallel to the wire at distances $a = 0.020$ m and $b = 0.080$ m from it. The coil's ends are connected to an oscilloscope of very high input resistance. Take $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$.

(a) DERIVE · U2

[3]

Starting from Ampère's law and the definition of magnetic flux, derive an expression for the magnetic flux through one turn of the coil when the wire carries a steady current I .

- 1 Ampère's law round a circle of radius r centred on the wire gives $B(2\pi r) = \mu_0 I$, so $B = \mu_0 I / (2\pi r)$, directed perpendicular to the plane of the coil
- 2 Because B varies across the coil, take a strip of width dr and length ℓ parallel to the wire, so $d\Phi = B\ell dr$
- 3 $\Phi = (\mu_0 I \ell / 2\pi) \int dr/r$ from $r = a$ to $r = b = (\mu_0 I \ell / 2\pi) \ln(b/a)$

Multiplying the field at the coil's centre line by the coil's area gives a plausible answer that is about 13% too small. The integral is required because the field falls off across the coil.

(b) CALCULATE · U3

[2]

Calculate the mutual inductance of the wire and the coil.

- 1 $M = N\Phi/I = N\mu_0 \ell \ln(b/a)/(2\pi)$
- 2 $M = 200 \times (2.0 \times 10^{-7} \text{ T}\cdot\text{m/A})(0.200 \text{ m}) \ln 4 = 1.11 \times 10^{-5} \text{ H} (11.1 \mu\text{H})$

Forgetting the factor N gives $5.5 \times 10^{-8} \text{ H}$. The flux linkage, not the flux through one turn, defines the mutual inductance.

(c) CALCULATE · U3

[2]

The wire carries $I(t) = I_0 \sin(2\pi ft)$ with $I_0 = 15$ A and $f = 50$ Hz. Calculate the amplitude of the emf induced in the coil.

- 1 $\mathcal{E} = -M dI/dt = -2\pi f M I_0 \cos(2\pi ft)$, so the amplitude is $2\pi f M I_0$
- 2 $\mathcal{E}_0 = 2\pi(50 \text{ Hz})(1.11 \times 10^{-5} \text{ H})(15 \text{ A}) = 5.2 \times 10^{-2} \text{ V} (52 \text{ mV})$

Quoting the rms value, 37 mV, answers a different question; the amplitude is asked for. Using f in place of $\omega = 2\pi f$ gives 8.3 mV.

(d) EXPLAIN · U1

[2]

The coil is now closed through a resistor. At an instant when the wire current is in its positive direction and increasing, explain the direction of the induced current in the long side nearer the wire, and explain whether the wire attracts or repels the coil.

- 1 The flux of the wire's field through the coil is increasing, so by Lenz's law the induced current makes a field opposing it inside the coil, which requires the near side to carry current antiparallel to the wire's current
- 2 Antiparallel currents repel and parallel currents attract; the near side, antiparallel and closer, feels the stronger force, so the net force pushes the coil away from the wire

Candidates often get the direction of the current right and stop. The far side carries current parallel to the wire and is attracted, so the net force has to be argued from the $1/r$ dependence of the field.

(e) PREDICT · U1

[2]

The coil is slid within its plane until the wire lies along the coil's centre line, parallel to and midway between its long sides. Predict the emf induced by the same alternating current, giving the reason.

- 1 The wire's field points into the plane on one side of the wire and out of it on the other, with equal magnitudes at equal distances
- 2 The two halves of the coil are mirror images about the wire, so the net flux is zero for any current: $M = 0$ and no emf is induced

The coil is now threaded by the strongest fields in the arrangement, and candidates who reason from field strength near the wire predict a larger emf. Induction depends on net flux linkage, which the symmetry makes zero.

A series circuit contains a resistor $R = 25\ \Omega$, an inductor $L = 80\text{ mH}$ of negligible resistance and a capacitor $C = 50\ \mu\text{F}$. It is driven by a sinusoidal source of rms emf 120 V whose frequency can be varied.

(a) CALCULATE · U3

[2]

Calculate the resonant frequency, and the rms current and average power delivered at resonance.

1 $f_0 = 1/(2\pi\sqrt{LC}) = 1/(2\pi\sqrt{(0.080\text{ H})(50 \times 10^{-6}\text{ F})}) = 79.6\text{ Hz}$

2 At resonance $Z = R$, so $I_{\text{rms}} = 120\text{ V}/25\ \Omega = 4.80\text{ A}$ and $P = I_{\text{rms}}^2 R = 576\text{ W}$

Quoting $\omega_0 = 500\text{ rad/s}$ as the frequency in hertz is the slip this part catches; the factor 2π separates angular frequency from frequency.

(b) DETERMINE · U3

[4]

The source frequency is set to 50 Hz . Determine the impedance, the rms current, the phase angle between the current and the source emf, and the average power delivered, stating whether the current leads or lags the emf.

1 $X_L = 2\pi(50\text{ Hz})(0.080\text{ H}) = 25.1\ \Omega$ and $X_C = 1/(2\pi(50\text{ Hz})(50 \times 10^{-6}\text{ F})) = 63.7\ \Omega$

2 $Z = \sqrt{(25\ \Omega)^2 + (25.1\ \Omega - 63.7\ \Omega)^2} = 45.9\ \Omega$, so $I_{\text{rms}} = 120\text{ V}/45.9\ \Omega = 2.61\text{ A}$

3 $\tan \phi = (X_L - X_C)/R = -1.54$, so the phase angle is 57.0° , with the current leading the emf because below resonance the circuit is capacitive

4 $P = I_{\text{rms}}^2 R = (2.61\text{ A})^2(25\ \Omega) = 171\text{ W}$, equivalently $V_{\text{rms}} I_{\text{rms}} \cos \phi$ with power factor 0.544

Computing the power as $V_{\text{rms}} \times I_{\text{rms}} = 314\text{ W}$ ignores the power factor; only the resistor dissipates energy on average, and the check $I_{\text{rms}}^2 R$ exposes the error at once.

(c) SKETCH · U1

[3]

Sketch the phasor diagram for the circuit at 50 Hz , with the current phasor drawn horizontally, showing the rms voltage phasors across R , L and C and the source emf, each labelled with its magnitude.

1 $V_R = 65.3\text{ V}$ drawn along the current phasor

2 $V_L = 65.7\text{ V}$ drawn 90° ahead of the current and $V_C = 166\text{ V}$ drawn 90° behind it, visibly longer than V_L , for phasors rotating anticlockwise

3 The source emf, 120 V , drawn as the vector sum of the three, lying 57° behind the current phasor so that the current leads

A diagram in which V_L and V_C are drawn in the same direction, or in which the emf lies ahead of the current, contradicts part (b); the sketch is marked for consistency with the candidate's own phase angle.

(d) EXPLAIN · U1

[2]

At resonance the rms voltage across the capacitor is 192 V , well above the 120 V rms of the source. Explain how this is consistent with Kirchhoff's loop rule and with conservation of energy.

1 The loop rule holds for instantaneous voltages, or for phasors added with their phases: at resonance V_L and V_C are equal in size and 180° apart, so they cancel and the whole source voltage appears across R

2 No energy is created: the inductor and capacitor exchange stored energy back and forth each cycle with zero average power, and the only average power drawn from the source is $I^2 R$ dissipated in the resistor

Adding the rms voltages as plain numbers at resonance, $120\text{ V} + 192\text{ V} + 192\text{ V} = 504\text{ V}$, appears to exceed the 120 V source more than fourfold; this part is set to break that habit.

A compact telephoto camera lens is modelled as two thin lenses on a common axis: a converging lens of focal length $f_1 = +10.0$ cm, followed 6.0 cm behind it by a diverging lens of focal length $f_2 = -12.0$ cm. The image sensor sits behind the second lens. Use the real-is-positive convention, $1/u + 1/v = 1/f$ with magnification $m = -v/u$, in which a virtual object has a negative object distance u .

(a) DETERMINE · U3

[3]

Light from a very distant object enters the first lens. Determine where the final image forms, and state whether it is real or virtual.

- 1 The first lens alone would form its image at its focal point, 10.0 cm behind it, which is 4.0 cm behind the second lens
- 2 The second lens intercepts rays that are still converging, so that point is a virtual object for it, with $u_2 = -4.0$ cm
- 3 $1/v_2 = 1/f_2 - 1/u_2 = -1/12.0 + 1/4.0$, so $v_2 = +6.0$ cm: a real image 6.0 cm behind the second lens, 12.0 cm behind the first

Taking $u_2 = +4.0$ cm, as if the first image were a real object in front of the second lens, gives $v_2 = -3.0$ cm, a virtual image that no sensor could record. Declaring the sign of the virtual object is the step the part is set on.

(b) SHOW THAT · U3

[3]

The distant object subtends an angle of 0.50° at the lens. Show that the final image is 0.131 cm tall, and hence that the combination behaves as a single lens of focal length 15.0 cm.

- 1 The first image has height $h_1 = f_1\theta = (10.0 \text{ cm})(8.73 \times 10^{-3} \text{ rad}) = 0.0873 \text{ cm}$
- 2 The second lens magnifies it by $m_2 = -v_2/u_2 = -(6.0 \text{ cm})/(-4.0 \text{ cm}) = +1.50$, so the final image is $1.50 \times 0.0873 \text{ cm} = 0.131 \text{ cm}$ tall
- 3 A single lens images a distant object at height $f\theta$, so $f_{\text{eff}} = (0.131 \text{ cm})/(8.73 \times 10^{-3} \text{ rad}) = 15.0 \text{ cm}$

Candidates who leave θ in degrees obtain a final image 7.5 cm tall, larger than any camera sensor, and should reject it on sight; the small-angle relation $h = f\theta$ needs radians.

(c) COMPARE · U1

[2]

Compare this combination with a single converging lens that forms an image of the same size of the same distant object, in terms of the distance from the front of the lens to the sensor.

- 1 A single lens giving the same image size must have $f = 15.0$ cm, so its sensor sits 15.0 cm behind the lens
- 2 The combination places the sensor only 12.0 cm behind its front lens, 3.0 cm (20%) shorter for the same image size, because the diverging lens shifts the effective lens position 3.0 cm in front of the first lens

The telephoto idea is that effective focal length and physical length are different quantities. Candidates who add the focal lengths or the separations have not used the image-size result of part (b).

(d) PREDICT · U3

[2]

The camera is refocused on an object 2.00 m in front of the first lens, with the lens separation unchanged. Predict whether the sensor must move towards or away from the second lens, and by how much.

- 1 First lens: $1/v_1 = 1/10.0 - 1/200$, so $v_1 = 10.53$ cm and the virtual object for the second lens is at $u_2 = -(10.53 - 6.0) \text{ cm} = -4.53 \text{ cm}$
- 2 $1/v_2 = -1/12.0 + 1/4.53$ gives $v_2 = 7.27$ cm, so the sensor must move 1.27 cm away from the second lens

The shift is larger than the 1.22 cm a single 15.0 cm lens would need. Candidates who treat 2.00 m as effectively infinite should notice that it is only about 13 focal lengths.

A student uses a transmission grating ruled with 450 lines per millimetre, illuminated at normal incidence, to look for the deuterium line in the light of a discharge lamp that contains hydrogen and deuterium in comparable amounts. The red Balmer line of ordinary hydrogen has wavelength 656.28 nm; the corresponding line of deuterium has wavelength 656.10 nm. The lamp also emits the blue-green Balmer line of hydrogen, of wavelength 486.13 nm.

(a) CALCULATE · U3

[2]

Calculate the grating spacing, and the highest order in which light of wavelength 656 nm can be observed.

1 $d = (1.00 \times 10^{-3} \text{ m})/450 = 2.22 \times 10^{-6} \text{ m}$

2 $d \sin \theta = m\lambda$ with $\sin \theta < 1$ requires $m < d/\lambda = 3.39$, so the highest order is $m = 3$

Rounding $d/\lambda = 3.39$ to the nearest integer rather than down is harmless here but not in general; the order must be the largest integer with $\sin \theta$ below 1.

(b) CALCULATE · U3

[1]

Calculate the angle at which the hydrogen line appears in second order.

1 $\sin \theta = 2(656.28 \times 10^{-9} \text{ m})/(2.222 \times 10^{-6} \text{ m}) = 0.591$, so $\theta = 36.2^\circ$

Using the small-angle form $\theta \approx m\lambda/d$ gives 33.8° ; at an angle this large the approximation fails.

(c) SHOW THAT · U2

[2]

By differentiating the grating equation, show that two lines separated in wavelength by a small amount $\Delta\lambda$ appear in order m with an angular separation $\Delta\theta = m\Delta\lambda/(d \cos \theta)$.

1 Differentiates $d \sin \theta = m\lambda$ with d and m fixed: $d \cos \theta d\theta = m d\lambda$

2 For small separations replaces the differentials by differences, $\Delta\theta = m\Delta\lambda/(d \cos \theta)$, with $\Delta\theta$ in radians

Candidates who differentiate $\sin \theta$ as $\cos \theta$ but forget that the result is in radians later quote a separation in degrees that is 57 times too small.

(d) CALCULATE · U3

[1]

Calculate the angular separation of the hydrogen and deuterium lines in second order.

1 $\Delta\theta = 2(0.18 \times 10^{-9} \text{ m})/((2.222 \times 10^{-6} \text{ m}) \cos 36.2^\circ) = 2.0 \times 10^{-4} \text{ rad } (0.0115^\circ)$

Subtracting two angles each computed to 0.01° loses the answer entirely; the differential form is the reliable route for a separation this small.

(e) DETERMINE · U3

[2]

Determine the minimum number of grating lines that must be illuminated to resolve the two lines in second order, and the corresponding minimum width of the illuminated region of the grating.

1 Resolving power $\lambda/\Delta\lambda = mN$, so $N \geq (656.19 \text{ nm})/(2 \times 0.18 \text{ nm}) = 1.82 \times 10^3$ lines (1823)

2 Width $\geq 1823/(450 \text{ lines per mm}) = 4.1 \text{ mm}$

Resolution is set by the number of lines illuminated, not by the angular separation alone. A grating lit by a 2 mm beam separates the line centres in angle but cannot resolve them, however carefully it is read.

(f) COMPARE · U1

[2]

Compare working in third order with working in second order for this measurement.

- 1 Third order spreads the lines further: $m/\cos \theta$ rises from 2.48 at 36.2° to 6.47 at 62.4° , so $\Delta\theta = 5.2 \times 10^{-4}$ rad, about 2.6 times larger, and only 1216 lines (2.7 mm of grating) are needed to resolve the pair
- 2 Against that, gives a supported cost of third order: the pair lies at 62.4° , near the limit of usable angles, and higher orders are usually fainter; or the third-order spectrum overlaps the fourth, so the 486.13 nm line appears in fourth order at 61.0° , only 1.3° from the red pair, and each line must be identified with care

A comparison that lists only the advantage of higher order earns one mark. The fourth-order image of the 486.13 nm line, 1.3° from the red pair, is the practical trap, and a candidate who checks where other lamp lines fall has done what a spectroscopist would.

17
CORE
COURSE

DATA ANALYSIS · DEMANDING · UPII-15.04

[12]

In a photoelectric experiment, monochromatic light falls on a clean metal cathode in an evacuated tube. For each wavelength the reverse potential difference between anode and cathode is increased until the photocurrent just falls to zero, and this stopping potential V_s is recorded. Each stopping potential has an uncertainty of ± 0.03 V, and each wavelength is known to ± 1 nm. Take $e = 1.60 \times 10^{-19}$ C and $c = 3.00 \times 10^8$ m/s.

(a) STATE · U2

[1]

State the photoelectric equation relating the stopping potential V_s to the frequency f of the light.

- 1 $eV_s = hf - \phi$, where ϕ is the work function of the cathode, so V_s against f is a straight line of gradient h/e and intercept $-\phi/e$

Writing $V_s = hf - \phi$ without the charge e mixes volts with joules; the equation must balance in units before a gradient can be interpreted.

(b) EVALUATE · U4

[2]

Calculate the frequency of the light for each reading, and evaluate whether all six readings are consistent with a single straight line.

- 1 $f = c/\lambda$ gives 10.00, 8.82, 7.89, 7.14, 6.52 and 6.00×10^{14} Hz for readings 1 to 6
- 2 Readings 1, 2, 3, 5 and 6 lie within ± 0.03 V of a straight line, but reading 4 lies about 0.10 V below it, more than three times its uncertainty, so it is anomalous and is excluded from the fit

A line forced through all six points gives $h \approx 6.70 \times 10^{-34}$ J·s and a shifted intercept. The anomalous reading is only found by comparing each residual with the stated uncertainty, which is the skill being tested.

(c) DETERMINE · U4

[3]

Determine the Planck constant from the gradient of a graph of V_s against f .

- 1 Plots V_s against f , or fits a line, using readings 1, 2, 3, 5 and 6
- 2 Gradient = $(1.95 \text{ V} - 0.28 \text{ V}) / ((10.00 - 6.00) \times 10^{14} \text{ Hz}) = 4.2 \times 10^{-15} \text{ V·s}$, or $4.13 \times 10^{-15} \text{ V·s}$ by least squares (accept 4.0 to $4.3 \times 10^{-15} \text{ V·s}$)
- 3 $h = e \times \text{gradient} = 6.6 \times 10^{-34} \text{ J·s}$ from the least-squares gradient, or $6.7 \times 10^{-34} \text{ J·s}$ from the two end points (accept 6.4 to $6.9 \times 10^{-34} \text{ J·s}$)

Plotting against wavelength rather than frequency produces a curve, and a gradient read from it has no meaning. Candidates must linearise first.

- (d) DETERMINE · U4 [2]
- Determine the work function of the cathode, in electronvolts, and the threshold wavelength beyond which no photoelectrons are emitted.
- 1 Extrapolating to $f = 0$ gives an intercept of about -2.19 V, so $\phi = 2.2$ eV (accept 2.1 to 2.3 eV)
 - 2 $f_0 = \phi/h = 5.3 \times 10^{14}$ Hz, so $\lambda_0 = c/f_0 = 5.7 \times 10^2$ nm (accept 540 to 590 nm, or a value consistent with the candidate's own gradient and intercept)
- Reading the intercept from a graph whose frequency axis starts at 6×10^{14} Hz rather than at zero gives a wrong work function; the intercept must be calculated from the line's equation or read from a true origin.*
- (e) ESTIMATE · U4 [2]
- Estimate the uncertainty in your value of the Planck constant, stating the assumptions you make.
- 1 Takes the ± 0.03 V uncertainty as the dominant one and draws the steepest and shallowest lines through the error bars of the extreme readings at 300 nm and 500 nm, with gradients of about 4.33 and 4.03×10^{-15} V·s
 - 2 Half the spread gives $h = (6.6 \pm 0.2) \times 10^{-34}$ J·s, an uncertainty of about 4%; the ± 1 nm wavelength uncertainty shifts each frequency by at most 0.33% and the gradient by about 1%, small beside the 4% from the voltages
- An estimate without its assumption, which uncertainty dominates, earns nothing for the second mark. Candidates who quote the least-squares standard error alone, about 0.07×10^{-34} J·s, should say that it reflects the scatter and not the ± 0.03 V on each reading.*
- (f) JUSTIFY · U1 [2]
- Justify the claim that these data cannot be explained by a classical wave model, in which the energy an electron absorbs grows with the intensity of the light and the time it is exposed.
- 1 In the wave model the maximum energy of an ejected electron should depend on intensity and exposure time, not on frequency, and light of any frequency should eventually eject electrons
 - 2 The data show the maximum kinetic energy eV_s rising linearly with frequency, and the line extrapolates to a threshold near 5.3×10^{14} Hz (about 570 nm) below which no electrons would be emitted: a frequency dependence and a threshold the wave model does not predict, but which follow if each electron absorbs a single quantum of energy hf
- Answers that describe photons without pointing to a feature of this table, the linear dependence on f or the threshold it extrapolates to, have asserted the claim rather than justified it.*

18
CORE
COURSE

FREE RESPONSE · EXTENDED DERIVATION · DISCRIMINATING · UPII-12.04 · UPII-12.05 · UPII-12.07

[15]

A parallel-plate capacitor has circular plates of radius $R = 5.0$ cm separated by $d = 2.0$ mm in vacuum. It is being charged by a steady current $I = 0.40$ A carried by long straight leads that meet the centres of the plates along their common axis. Neglect fringing, so that the electric field is uniform between the plates and zero outside the gap. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F/m and $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

- (a) **EXPLAIN** · U1 [2]
- Ampère's law without Maxwell's correction gives contradictory results for the magnetic field at a point beside the gap, depending on which surface is chosen to span a circular path around the axis. Explain the contradiction, and how the displacement current resolves it.
- 1 A flat disc spanning a circle round the axis in the midplane of the gap is crossed by no conduction current, but a surface with the same boundary that bulges out to cut one of the leads is pierced by the current I , so $\oint \mathbf{B} \cdot d\mathbf{l}$ would have to be both zero and $\mu_0 I$
 - 2 Maxwell added the displacement current $\epsilon_0 d\Phi_E/dt$; the electric flux through the gap is q/ϵ_0 , so $\epsilon_0 d\Phi_E/dt = dq/dt = I$, and $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_C + \epsilon_0 d\Phi_E/dt) = \mu_0 I$ for both surfaces
- Candidates who say only that 'a changing electric field makes a magnetic field' have named the correction without showing that it removes the contradiction; the second mark needs $\epsilon_0 d\Phi_E/dt$ equal to I .*
- (b) **DERIVE** · U2 [4]
- Using the Ampère–Maxwell law, derive the magnitude of the magnetic field between the plates at a distance $r < R$ from the axis.
- 1 By symmetry $\mathbf{B}$ is azimuthal and uniform in magnitude on a circle of radius r centred on the axis in a plane between the plates, so $\oint \mathbf{B} \cdot d\mathbf{l} = 2\pi r B$
 - 2 No conduction current crosses the disc bounded by that circle; the field $E = q/(\epsilon_0 \pi R^2)$ is uniform, so $\Phi_E = \pi r^2 E = q r^2/(\epsilon_0 R^2)$
 - 3 The displacement current through the disc is $\epsilon_0 d\Phi_E/dt = I r^2/R^2$
 - 4 $2\pi r B = \mu_0 I r^2/R^2$, so $B = \mu_0 I r/(2\pi R^2)$
- The commonest error is to take the whole displacement current I through a circle of radius $r < R$, which gives $\mu_0 I/(2\pi r)$ and a field that diverges on the axis. Only the fraction r^2/R^2 of the changing flux is enclosed.*
- (c) **DETERMINE** · U2 [2]
- Determine the magnitude of the magnetic field in the midplane of the gap at a distance $r > R$ from the axis, and state how it compares with the field at the same distance from one of the leads.
- 1 A disc of radius $r > R$ encloses all the changing electric flux, so the displacement current through it is I and $2\pi r B = \mu_0 I$, giving $B = \mu_0 I/(2\pi r)$
 - 2 This is identical to the field at distance r from the long lead: outside the gap the magnetic field cannot distinguish conduction current from displacement current
- Candidates who expect the field beside the gap to be weaker, because no charge flows there, are reasoning from conduction current alone; the result is the physical content of Maxwell's correction.*
- (d) **SKETCH** · U1 [2]
- Sketch a graph of the magnitude of B against r in the midplane of the gap, from $r = 0$ to $r = 3R$, marking the value at $r = R$.
- 1 B rises linearly from zero on the axis to a maximum at $r = R$, marked as $\mu_0 I/(2\pi R) = 1.6 \times 10^{-6} \text{ T}$
 - 2 Beyond $r = R$, B falls as $1/r$, continuous at $r = R$, reaching one-third of the maximum ($5.3 \times 10^{-7} \text{ T}$) at $r = 3R$
- A sketch with a jump at $r = R$, or with the peak anywhere but at the edge, is inconsistent with parts (b) and (c) and is not credited for shape.*

(e) SHOW THAT · U2

[4]

At the edge of the gap, $r = R$, show that the Poynting vector $S = E \times B/\mu_0$ points radially inwards, and that the total power flowing in through the cylindrical surface of radius R and height d equals the rate of increase of the energy stored in the capacitor, $d/dt(q^2/2C)$.

- 1 E is directed along the axis from the positive plate, the same sense as the current in the leads, and B at $r = R$ circulates in the sense given by the right-hand rule about that current; with the axis as $\hat{z}$, $E \times B$ lies along $\hat{z} \times \hat{\phi} = -\hat{r}$, radially inwards
- 2 $|S| = EB/\mu_0 = (q/(\epsilon_0\pi R^2))(I/(2\pi R)) = qI/(2\pi^2\epsilon_0 R^3)$
- 3 Power entering through the side of the gap $= |S| \times 2\pi R d = qId/(\pi\epsilon_0 R^2)$
- 4 With $C = \epsilon_0\pi R^2/d$, $d/dt(q^2/2C) = (q/C)(dq/dt) = qId/(\pi\epsilon_0 R^2)$, equal to the power entering: the energy reaches the capacitor through the sides of the gap, not along the wires

The inward direction is the discriminating step. A candidate who takes B in the wrong sense finds energy leaving a capacitor that is being charged, and should treat that contradiction as a signal to recheck, not as a result.

(f) PREDICT · U1

[1]

The capacitor is later discharged through the same leads. Predict the direction of the Poynting vector at the edge of the gap during the discharge.

- 1 E keeps its direction while the current, and so B , reverses, so S points radially outwards: energy leaves the gap through its sides

Candidates who reverse both E and B predict inward flow again; the field between the plates depends on the charge, which keeps its sign while it decreases.

19 CORE COURSE

FREE RESPONSE · MODEL, SOLVE AND CHECK THE LIMITS · DISCRIMINATING · UPII-09.04 · UPII-10.07 · UPII-12.03

[15]

A long solenoid of radius $a = 4.0$ cm has $n = 8000$ turns per metre and carries a current $I_0 = 20$ A. Coaxial with it, outside the winding, a thin insulating ring of radius $b = 6.0$ cm and mass $m = 0.50$ g carries a charge $Q = 0.50$ μC spread uniformly around it. The ring can turn freely about the common axis, and the apparatus is in vacuum. The solenoid current is reduced steadily to zero over $T = 0.50$ s. Treat the solenoid as ideal, with no magnetic field outside its winding, and neglect the magnetic field of the ring itself. Take $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

(a) DESCRIBE · U1

[2]

Describe the electric field induced while the current is falling: its direction, the shape of its field lines, and why it cannot be described by an electric potential.

- 1 By axial symmetry the induced field is azimuthal, with closed circular field lines centred on the axis and a magnitude that depends only on r ; its sense is the one that would drive a current opposing the fall in flux, the same sense as the solenoid current
- 2 Its circulation round a closed field line, $\oint E \cdot dl = -d\Phi_B/dt$, is not zero, whereas any field derived from a potential has zero circulation round every closed path, so no potential function exists

Describing the field as radial, by analogy with the field of a charge, is the misconception this part is set to catch; an induced field has no sources or sinks, only circulation.

(b) DERIVE · U2

[4]

Using Faraday's law in the form $\oint E \cdot dl = -d\Phi_B/dt$, derive the magnitude of the induced electric field at distance r from the axis, for $r < a$ and for $r > a$, in terms of the rate of change dB/dt of the field inside the solenoid.

- 1 Chooses a circular path of radius r centred on the axis, on which E is tangential and uniform, so $\oint E \cdot dl = 2\pi r E$
- 2 For $r < a$ the flux enclosed is $\pi r^2 B$, so $2\pi r E = \pi r^2 |dB/dt|$ and $E = (r/2)|dB/dt|$
- 3 For $r > a$ only the solenoid's cross-section carries flux, $\pi a^2 B$, so $2\pi r E = \pi a^2 |dB/dt|$
- 4 $E = (a^2/(2r))|dB/dt|$, which agrees with the inside result at $r = a$

Using the area of the path instead of the area that carries flux for $r > a$ gives a field that grows without limit outside the solenoid. The continuity check at $r = a$ catches it immediately, which is why the prompt asks for both regions.

- (c) **CALCULATE** · U3 [2]
- Calculate the magnetic field inside the solenoid before the current starts to fall, and the magnitude of the induced electric field at the ring while the current is falling.
- 1 $B_0 = \mu_0 n I_0 = (4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(8000 \text{ m}^{-1})(20 \text{ A}) = 0.201 \text{ T}$, so $|dB/dt| = 0.201 \text{ T}/0.50 \text{ s} = 0.402 \text{ T/s}$
 - 2 At $r = b$: $E = (a^2/(2b))|dB/dt| = ((0.040 \text{ m})^2/(2 \times 0.060 \text{ m}))(0.402 \text{ T/s}) = 5.4 \times 10^{-3} \text{ V/m}$
Substituting $r = b$ into the inside expression $(r/2)|dB/dt|$ gives $1.2 \times 10^{-2} \text{ V/m}$, more than twice too large, and is the error of a candidate who has not decided which region the ring is in.
- (d) **DERIVE** · U2 [3]
- Using the rotational form of Newton's second law, $\tau = dL/dt$, derive the angular momentum the ring has acquired once the current has fallen to zero, and show that it depends neither on T nor on b , provided $b > a$.
- 1 Every element of the ring's charge feels a tangential force at radius b , so the torque about the axis is $\tau = QEb = (Qa^2/2)|dB/dt|$
 - 2 Integrating over the whole fall in field, $L = \int \tau dt = (Qa^2/2) \int |dB/dt| dt = Qa^2 B_0/2$
 - 3 The ring radius b has cancelled and only the total change in B appears, so neither b nor the duration T of the decrease affects L
- The cancellation of b is the result to notice: a larger ring feels a weaker field but with a longer lever arm. Candidates who stop at τ and multiply by T without integrating obtain the right number here only because the fall is uniform, and have not shown the independence of T .*
- (e) **CALCULATE** · U3 [2]
- Calculate the angular speed of the ring once the current has fallen to zero.
- 1 $L = (0.50 \times 10^{-6} \text{ C})(0.040 \text{ m})^2(0.201 \text{ T})/2 = 8.0 \times 10^{-11} \text{ kg}\cdot\text{m}^2/\text{s}$
 - 2 Moment of inertia $mb^2 = (0.50 \times 10^{-3} \text{ kg})(0.060 \text{ m})^2 = 1.8 \times 10^{-6} \text{ kg}\cdot\text{m}^2$, so $\omega = L/(mb^2) = 4.5 \times 10^{-5} \text{ rad/s}$
Using $Qb^2 B_0/2$, the ring's own area rather than the solenoid's, gives an angular speed 2.25 times too large; the flux lives only inside the winding.
- (f) **EVALUATE** · U1 [2]
- A student claims that the ring starts to turn because the solenoid's magnetic field pushes on its charges. Evaluate this claim.
- 1 The claim fails: a magnetic force $qv \times B$ needs moving charges and the ring starts from rest, and in any case the ideal solenoid's field is zero at $r = b$, so no magnetic force acts on the ring at any time
 - 2 The torque comes from the induced, non-conservative electric field, which exists at the ring even though B is zero there: what matters is the changing flux enclosed by the ring's path, not the field at the ring's own position
- This is the discriminating idea of the question. Candidates who hold that a field must be present where a force acts are obliged by the ideal solenoid to accept that a changing magnetic flux elsewhere produces an electric field here.*

What the command words are asking for

Examiners mark to the command word as well as to the physics. An answer that explains where the question said state has spent time earning one mark; one that states a result where the question said derive has earned none, however correct the result.

State	Give the result, definition or law concisely. No justification is required.
Describe	Give the relevant features of a situation or process in words, precisely enough that someone else could reproduce it.
Explain	Give the physical reason a result or behaviour occurs, connecting it explicitly to the principle that governs it.
Calculate	Obtain a numerical value, showing the relation used, the substitution and the units of the result.
Determine	Obtain a result by any valid route the problem allows — analytic, graphical or numerical — with enough working to be followed.
Derive	Start from a named fundamental law or definition and reach the required result by explicit, justified steps.
Show that	Establish a result the question gives you. Every step must be shown, because the stated answer earns nothing on its own.
Estimate	Reach an order-of-magnitude value from stated, reasonable assumptions, and say what those assumptions are.
Sketch	Draw the shape of a graph or field with axes labelled and its key features marked — intercepts, asymptotes, turning points, limits. Exact scale is not required.
Justify	Support a claim or a choice with evidence, and with the reasoning that connects that evidence to the claim.
Evaluate	Weigh a model, argument or result against criteria, including its assumptions and domain of validity, and reach a stated judgement.
Compare	Identify the similarities and differences between two situations, models or results, relating each difference to its cause.
Predict	Say what will happen when a condition changes, reasoning from a principle rather than guessing, and give the reason.

WHAT IS BEING TESTED

University assessment objectives

Each course has the two papers a university course is examined by: a midterm on the first half of its units, and a cumulative final weighted towards the second half. Every paper combines short conceptual multiple-choice items with structured problems, a data or estimation question, and at least one extended derivation. Constants are given where a question needs them; a calculator and your own formula sheet are assumed.

U1 33 MARKS · 28%	Conceptual understanding Identify the governing principle and the conditions under which it holds, and reason qualitatively about what a result must look like before calculating it.
U2 37 MARKS · 32%	Mathematical modelling and derivation Translate a physical situation into mathematics: choose coordinates and approximations, set up the governing equations, solve them symbolically, and derive stated results from named laws.

U3

38 MARKS · 32%

Quantitative problem solving

Carry a multi-step calculation to a numerical result with correct units and sensible precision, and check it against dimensions, limiting cases and orders of magnitude.

U4

9 MARKS · 8%

Data, estimation and experimental reasoning

Analyse measurements and their uncertainties, linearise and fit data, estimate orders of magnitude from stated assumptions, and judge whether evidence supports a model.

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