

University Physics II · Midterm Examination

Electrostatics through magnetic forces: charge and Coulomb's law, electric fields and Gauss's law, electric potential, capacitance and dielectrics, DC and multi-loop circuits with RC transients, and the Lorentz force on moving charges. Most marks sit on setting up a field, circuit or force model before calculating, and on testing every result against symmetry, conservation and limiting cases.

SUGGESTED TIME	TOTAL MARKS	QUESTIONS	MULTIPLE CHOICE
90 minutes	72	13	8

SECTION

UPII · Midterm · University Physics II — Midterm examination

IN THE FORMAT OF

A university midterm: conceptual multiple choice, structured problems, a data-analysis question and an extended derivation

DEMAND

Routine 6 · Demanding 6 · Discriminating 1

Pitched at what a strong course grade requires rather than at what a homework set rehearses. Routine substitutions carry few marks; most sit on problems that have to be set up before they can be solved, on derivations from a named law, and on checking a result against its limits. Every paper still opens with entry points, so you can tell which marks you should be securing first.

COURSE UNITS

UPII-01 · UPII-02 · UPII-03 · UPII-04 · UPII-05 · UPII-06 · UPII-07 · UPII-08

INSTRUCTIONS

- Answer all questions in the spaces provided.
- Show your working. Method marks are awarded even where the final answer is wrong.
- Give your answers to a sensible number of significant figures, with a unit.
- The mark scheme for this paper follows the last question.

Written by GioPhysics from its own University Physics course maps. These are practice examinations in a university style: they are not the examinations of any university, contain no past-paper questions, and carry no academic credit. For a course you are taking, your instructor's syllabus and past papers remain the authority.

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University Physics II · Midterm Examination

Answer all questions. 72 marks in 90 minutes.

1 MULTIPLE CHOICE · ROUTINE · UPII-01.01 · UPII-01.02 · UPII-01.03 [1]
CORE COURSE

A neutral solid metal sphere stands on an insulating support. A glass rod carrying positive charge is held close to the left-hand side of the sphere without touching it. While the rod is held in place, a wire connected to earth briefly touches the right-hand side of the sphere, the side farther from the rod, and is then removed. Finally the rod is taken away. What is the final charge on the sphere?

- A Positive, the same sign as the rod, spread uniformly over the surface
- B Zero, because the earth connection was broken before the rod was removed
- C Negative, concentrated on the left-hand side where the rod had been
- D Negative, spread uniformly over the surface

Answer

2 MULTIPLE CHOICE · ROUTINE · UPII-02.02 · UPII-02.03 [1]
CORE COURSE

A point charge $+9Q$ is fixed at $x = 0$ and a point charge $-4Q$ is fixed at $x = L$ on the x -axis, where $Q > 0$. Apart from points infinitely far away, where is the electric field zero?

- A $x = 3L$
- B $x = 1.8L$
- C $x = 0.6L$
- D Nowhere: the field is non-zero at every point a finite distance from the charges

Answer

3 MULTIPLE CHOICE · DEMANDING · UPII-03.01 · UPII-03.02 · UPII-03.08 [1]
CORE COURSE

A very long, straight, thin line of charge with uniform linear charge density λ passes perpendicularly through the centres of two opposite faces of an imaginary cube of edge length a . What is the electric flux through one of the four faces that the line does not pass through?

- A $\lambda a / (6\epsilon_0)$
- B $\lambda a / (4\epsilon_0)$
- C $\lambda a / (\pi\epsilon_0)$
- D $\lambda a / \epsilon_0$

Answer

4CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-02.03 · UPII-04.04

[1]

Four point charges sit at the corners of a square of side a centred on the origin in the xy -plane: $+q$ at $(-a/2, a/2)$ and at $(a/2, a/2)$, and $-q$ at $(-a/2, -a/2)$ and at $(a/2, -a/2)$, with $q > 0$. Take the electric potential to be zero infinitely far away and $k = 1/(4\pi\epsilon_0)$. Which statement describes the electric potential V and the electric field E at the origin?

- A $V = 0$ and $E = 0$
- B $V > 0$ and $E = 0$
- C $V = 0$, and E has magnitude $4\sqrt{2}kq/a^2$ in the $+y$ direction
- D $V = 0$, and E has magnitude $4\sqrt{2}kq/a^2$ in the $-y$ direction

Answer **5**CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-05.02

[1]

Two initially uncharged capacitors, of capacitance $2.0\ \mu\text{F}$ and $6.0\ \mu\text{F}$, are connected in series across a $12\ \text{V}$ battery and allowed to charge fully. What is the potential difference across the $6.0\ \mu\text{F}$ capacitor?

- A $3.0\ \text{V}$
- B $6.0\ \text{V}$
- C $9.0\ \text{V}$
- D $12\ \text{V}$

Answer **6**CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-06.03 · UPII-06.05

[1]

Wire X and wire Y are made of the same metal, are at the same temperature, and are joined end to end so that they carry the same steady current. The diameter of X is twice the diameter of Y. What are the ratio of the drift speed in X to that in Y, v_X/v_Y , and the ratio of the electric field inside X to that inside Y, E_X/E_Y ?

- A $v_X/v_Y = 1/2$ and $E_X/E_Y = 1/2$
- B $v_X/v_Y = 1/4$ and $E_X/E_Y = 1/4$
- C $v_X/v_Y = 1/4$ and $E_X/E_Y = 1$
- D $v_X/v_Y = 1$ and $E_X/E_Y = 1/4$

Answer

7CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-07.06 · UPII-07.07

[1]

A charged capacitor discharges through a fixed resistor. The potential difference across the capacitor falls to half its initial value in 3.0 s. How long after the discharge begins has the energy stored in the capacitor fallen to one-eighth of its initial value?

- A 1.5 s
- B 3.0 s
- C 4.5 s
- D 9.0 s

Answer

8CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-08.03 · UPII-08.05

[1]

In a mass spectrometer, ions start from rest, are accelerated through the same potential difference, and then enter a uniform magnetic field perpendicular to their velocity, in which they move on circular arcs. Ion X is ${}^4\text{He}^+$ (charge $+e$, mass $4u$) and ion Y is ${}^{16}\text{O}^{2+}$ (charge $+2e$, mass $16u$), where u is the atomic mass unit. What is the ratio of the radius of Y's path to the radius of X's path?

- A $1/2$
- B 1
- C $\sqrt{2}$
- D 2

Answer

Three small charged beads are fixed on a horizontal insulating board, which defines the xy -plane. Bead 1 carries $+4.0$ nC at the origin, bead 2 carries -2.0 nC at $(0.30$ m, $0)$ and bead 3 carries $+3.0$ nC at $(0, 0.40$ m). Point P is at $(0.30$ m, 0.40 m). Bead 2 was initially neutral and gained its charge by being rubbed with a cloth that was also initially neutral, and no charge escaped to the surroundings. A fourth bead carrying $+1.0$ nC is available for part (d), and in part (e) bead 2 is replaced by a bead carrying $+2.0$ nC at the same position. Treat all beads as point charges and ignore the surroundings. Take $k = 1/(4\pi\epsilon_0) = 8.99 \times 10^9$ N m² C⁻² and $e = 1.60 \times 10^{-19}$ C.

- (a) STATE [1]
State the charge on the cloth after the rubbing.

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- (b) CALCULATE [1]
Calculate the number of electrons transferred during the rubbing, and state whether they moved from the cloth to the bead or from the bead to the cloth.

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- (c) CALCULATE [4]
Calculate the magnitude and direction of the net electrostatic force on bead 1.

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- (d) DETERMINE [2]
Determine the electric potential at P, and the work an external agent must do to bring the $+1.0$ nC bead slowly from a very large distance to rest at P.

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- (e) COMPARE [2]
Bead 2 is replaced by the $+2.0$ nC bead. Compare the net force on bead 1 in the original and the new arrangements, and state one measurement at P that would distinguish the two arrangements.

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A length of high-voltage coaxial cable has a solid inner conductor of radius $a = 1.0$ mm and a thin outer conducting sheath of radius $b = 4.0$ mm on the same axis. The space between them is filled with polyethylene of dielectric constant $\kappa = 2.3$, which breaks down when the electric field in it reaches $E_{bd} = 2.0 \times 10^7$ V m⁻¹. The inner conductor carries free charge $+\lambda$ per unit length and the sheath carries $-\lambda$ per unit length. The cable is long enough for end effects to be neglected. A redesign with a thinner inner conductor, of radius 0.50 mm and the same sheath, is under consideration. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F m⁻¹.

(a) **DERIVE**

[3]

Using Gauss's law, derive an expression for the magnitude of the electric field in the polyethylene at a distance r from the axis, where $a < r < b$. Justify your choice of Gaussian surface.

(b) **SKETCH**

[2]

Sketch a graph of the magnitude of the electric field against r from $r = 0$ to $r = 2b$, marking on the r -axis the radii at which the graph changes form.

(c) **SHOW THAT**

[2]

Show that the capacitance per unit length of the cable is $C/L = 2\pi\kappa\epsilon_0/\ln(b/a)$.

(d) **DETERMINE**

[3]

Determine the largest potential difference that can be applied between the conductors of the present design without breakdown of the polyethylene.

(e) **EVALUATE**

[3]

The proposed redesign thins the inner conductor to 0.50 mm on the grounds that a wider gap needs a larger potential difference to break down. Evaluate this proposal, and determine the inner radius that gives the highest voltage rating for $b = 4.0$ mm.

A 12.0 V supply of internal resistance $r_1 = 0.50 \, \Omega$ and a rechargeable 9.0 V cell of internal resistance $r_2 = 1.5 \, \Omega$ are connected in parallel across a load of resistance $R_L = 4.0 \, \Omega$. Both positive terminals are joined to node X, both negative terminals are joined to node Y, and the load connects X to Y. The connecting wires have negligible resistance, and the emfs and internal resistances stay constant. In parts (d) and (e) an initially uncharged 0.47 F capacitor, with a manufacturing tolerance of $\pm 20\%$, is also connected directly between X and Y, and the design specification requires the load voltage to stay above 5.0 V for at least 1.0 s after both sources are disconnected.

(a) DETERMINE

[4]

Using Kirchhoff's rules, determine the current in the supply, in the cell and in the load, stating the direction of each, and the potential difference V_{XY} between X and Y.

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(b) SHOW THAT

[2]

Show that the power delivered by the emf of the 12.0 V supply, 41.1 W, equals the total rate of energy transfer in the two internal resistances, the load and the cell.

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(c) PREDICT

[2]

Without the capacitor, the load resistance is gradually reduced from 4.0 Ω . Predict the load resistance at which the current in the 9.0 V cell becomes zero, and what the cell does when the load resistance is smaller still.

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(d) CALCULATE

[3]

With the load at 4.0 Ω , the 0.47 F capacitor is connected between X and Y and the circuit is left for a long time. At $t = 0$ both the supply and the cell are disconnected, leaving only the capacitor and the load. Calculate the time at which the potential difference across the load has fallen to 5.0 V.

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(e) EVALUATE

[2]

Evaluate whether every capacitor within the $\pm 20\%$ tolerance meets the specification that the load voltage stays above 5.0 V for at least 1.0 s after disconnection.

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A thin rectangular strip of doped semiconductor lies in the xy -plane. It is 20 mm long in the x -direction, $w = 5.0$ mm wide in the y -direction and $t = 0.50$ mm thick in the z -direction. A steady current $I = 4.0$ mA flows through it in the $+x$ direction. A uniform magnetic field of adjustable magnitude is applied along the z -axis, and a high-resistance voltmeter connected to contacts on the two long edges records the Hall voltage $V_H = V(y = w) - V(y = 0)$. A positive value of B means the field points in the $+z$ direction; the final reading was taken with the field reversed. Take $e = 1.60 \times 10^{-19}$ C. For part (e), copper has density 8.96×10^3 kg m $^{-3}$ and molar mass 63.5 g mol $^{-1}$, and $N_A = 6.02 \times 10^{23}$ mol $^{-1}$.

Hall voltage readings at $I = 4.0$ mA (the last row was taken with the field reversed)

B / T	V_H / mV
0.00	0.42
0.10	2.95
0.20	5.40
0.30	7.95
0.40	10.39
0.50	12.92
0.60	15.40
−0.50	−12.08

(a) **JUSTIFY**

[2]

Justify, from the sign of the readings for positive B , whether the charge carriers in the strip are positive or negative.

(b) **EXPLAIN**

[2]

The reading at $B = 0$ is not zero. Explain how this offset can arise, and use the readings at $B = +0.50$ T and $B = -0.50$ T to obtain the Hall voltage at 0.50 T with the offset removed.

(c) **DETERMINE**

[3]

Using the seven readings with $B \geq 0$, determine the gradient of a graph of V_H against B .

- (d) **CALCULATE** [2]
Calculate the number density of charge carriers in the semiconductor.

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- (e) **ESTIMATE** [2]
Estimate the Hall voltage that the same current and a field of 0.50 T would produce in a copper strip of the same dimensions, stating your assumption about the number of conduction electrons, and say what your estimate implies for the choice of material in a Hall probe.

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13
CORE
COURSE

FREE RESPONSE · EXTENDED DERIVATION · DISCRIMINATING · UPII-01.06 · UPII-02.05 · UPII-03.02 · UPII-04.06 [17]
· UPII-04.07

A thin flat annulus, shaped like a washer, of inner radius a and outer radius b lies in the xy -plane centred on the origin and carries a uniform surface charge density $\sigma > 0$. The z -axis passes through the centre of the hole, perpendicular to the annulus. Take the electric potential to be zero infinitely far away and neglect gravity. For part (e), the particle is an electron of charge magnitude $q = 1.60 \times 10^{-19}$ C and mass $m = 9.11 \times 10^{-31}$ kg, with $a = 2.0$ cm, $b = 6.0$ cm, $\sigma = 1.0 \times 10^{-9}$ C m $^{-2}$ and $\epsilon_0 = 8.85 \times 10^{-12}$ F m $^{-1}$.

- (a) **DERIVE** [4]
Starting from the potential of a point charge, derive an expression for the electric potential $V(z)$ at a point on the z -axis.

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- (b) **DETERMINE** [2]
Determine the z -component of the electric field on the axis from your answer to (a), and explain why the potential on the axis alone is enough to give the complete field there.

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- (c) **SHOW THAT** [3]
Show that for $|z| \gg b$ the field on the axis reduces to that of a point charge $Q = \pi\sigma(b^2 - a^2)$, and state the field obtained in the limit $a \rightarrow 0$, $b \rightarrow \infty$, naming the charge distribution this limit describes.

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(d) **DERIVE** [3]

A particle of charge $-q$ and mass m is released from rest on the axis at $z = z_0$, where $0 < z_0 \ll a$. Using Newton's second law, derive the equation of motion for small z , show that the motion along the axis is simple harmonic, and derive its angular frequency ω .

(e) **CALCULATE** [2]

Calculate the period of the oscillation for the electron described in the stem.

(f) **PREDICT** [3]

In practice the electron cannot be released exactly on the axis. By applying Gauss's law to a small closed cylinder of radius ρ and length $2h$, centred on the origin with its axis along z and with ρ and h both much smaller than a , predict the radial component of the electric field at a small distance ρ from the axis near the centre, and hence predict whether the oscillation in (d) survives a small sideways displacement.

University Physics II · Midterm Examination — mark scheme

One line per mark, as an examiner would award them. A note under a part is the error that part is set to catch.

1 MULTIPLE CHOICE · ROUTINE · UPII-01.01 · UPII-01.02 · UPII-01.03 **[1]**

CORE
COURSE

A neutral solid metal sphere stands on an insulating support. A glass rod carrying positive charge is held close to the left-hand side of the sphere without touching it. While the rod is held in place, a wire connected to earth briefly touches the right-hand side of the sphere, the side farther from the rod, and is then removed. Finally the rod is taken away. What is the final charge on the sphere?

KEY D Negative, spread uniformly over the surface

A is tempting because charging by contact does leave an object with the same sign as its source, but the rod never touches the sphere. While the wire touches the sphere, the sphere and the earth form one conductor, and electrons are drawn up the wire towards the positive rod, so the sphere ends with an excess of electrons and a charge opposite in sign to the rod's. B would be right only if the last two steps were reversed: removing the rod first lets the extra electrons flow back to earth. Here the wire is removed while the rod still holds them on the sphere, so they are trapped. C has the right sign but keeps the distribution the rod induced. Once the rod has gone nothing holds the excess charge on one side, and on an isolated conducting sphere it spreads uniformly over the outer surface. The point where the earth wire touches is irrelevant: touching the far side, where induced positive charge sat, still lets electrons flow onto the sphere.

U1

2 MULTIPLE CHOICE · ROUTINE · UPII-02.02 · UPII-02.03 **[1]**

CORE
COURSE

A point charge $+9Q$ is fixed at $x = 0$ and a point charge $-4Q$ is fixed at $x = L$ on the x -axis, where $Q > 0$. Apart from points infinitely far away, where is the electric field zero?

KEY A $x = 3L$

C is the null point for two like charges, where the fields oppose each other between the charges. Here the charges have opposite signs, so between them both fields point in the $+x$ direction and add; any cancellation must lie outside the pair, on the side of the smaller-magnitude charge, where its greater nearness can compensate for its smaller size. Setting $9Q/x^2 = 4Q/(x - L)^2$ for $x > L$ gives $3(x - L) = 2x$, so $x = 3L$. B comes from writing the same balance with $1/r$ instead of $1/r^2$, $9/x = 4/(x - L)$: the right region with the wrong law. D is tempting for anyone who pictures field lines running from the positive to the negative charge and concludes that opposite charges never cancel. Beyond $x = L$ the two fields do point in opposite directions; just beyond L the $-4Q$ field dominates, while far away the $+9Q$ field dominates because $9Q > 4Q$, so the two must be equal somewhere between.

U3

3CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-03.01 · UPII-03.02 · UPII-03.08

[1]

A very long, straight, thin line of charge with uniform linear charge density λ passes perpendicularly through the centres of two opposite faces of an imaginary cube of edge length a . What is the electric flux through one of the four faces that the line does not pass through?

KEY B $\lambda a/(4\epsilon_0)$

D is the flux through the whole closed cube, which by Gauss's law is the enclosed charge λa divided by ϵ_0 ; the question asks for one face. A shares that total equally among all six faces, which ignores the direction of the field: the field of a long line is radial and perpendicular to the line, so it is parallel to the two faces the line pierces and carries no flux through them. All of $\lambda a/\epsilon_0$ therefore passes through the other four faces, and the fourfold rotational symmetry about the line makes their shares equal. C is the most careful-looking wrong answer. It takes the field at the centre of a face, $\lambda/(2\pi\epsilon_0(a/2)) = \lambda/(\pi\epsilon_0 a)$, and multiplies it by the area a^2 , as if that field were uniform and normal over the whole face. Away from the centre the field is both weaker and tilted from the normal, so this overestimates. The cube is not a surface on which Gauss's law gives the field, yet it still gives the flux exactly.

U1

4CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-02.03 · UPII-04.04

[1]

Four point charges sit at the corners of a square of side a centred on the origin in the xy -plane: $+q$ at $(-a/2, a/2)$ and at $(a/2, a/2)$, and $-q$ at $(-a/2, -a/2)$ and at $(a/2, -a/2)$, with $q > 0$. Take the electric potential to be zero infinitely far away and $k = 1/(4\pi\epsilon_0)$. Which statement describes the electric potential V and the electric field E at the origin?

KEY D $V = 0$, and E has magnitude $4\sqrt{2}kq/a^2$ in the $-y$ direction

A treats the field as if it were the potential. Potential is a scalar, and the four contributions $\pm kq/(a/\sqrt{2})$ cancel in pairs, so $V = 0$; the field is a vector, and here its contributions do not cancel. Each positive charge pushes a test charge at the origin away from itself and each negative charge pulls it towards itself, so all four fields have a component in the $-y$ direction while their x -components cancel. Each field has magnitude $kq/(a^2/2) = 2kq/a^2$ and a y -component of magnitude $2kq/(\sqrt{2}a^2)$, and four of these give $4\sqrt{2}kq/a^2$. A would be correct for the alternating arrangement with like charges on opposite corners, which is why it is tempting. C has the right magnitude but reverses the direction: the field at the centre points from the positive pair towards the negative pair. B comes from adding kq/r for every charge as if potential could not be negative, and then assuming a symmetric arrangement must cancel the field.

U1

5CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-05.02

[1]

Two initially uncharged capacitors, of capacitance $2.0 \mu\text{F}$ and $6.0 \mu\text{F}$, are connected in series across a 12 V battery and allowed to charge fully. What is the potential difference across the $6.0 \mu\text{F}$ capacitor?

KEY A 3.0 V

Because both capacitors start uncharged and the two plates joined between them form an isolated conductor, the capacitors carry equal charge. The series combination is $(2.0 \times 6.0)/(2.0 + 6.0) \mu\text{F} = 1.5 \mu\text{F}$, so the charge is $18 \mu\text{C}$ and the potential difference across the $6.0 \mu\text{F}$ capacitor is $18 \mu\text{C}/6.0 \mu\text{F} = 3.0 \text{ V}$. C is the potential difference across the $2.0 \mu\text{F}$ capacitor, and it is chosen by anyone who divides the voltage in proportion to capacitance, as series resistors divide it in proportion to resistance; at equal charge $V = Q/C$, so the larger capacitor takes the smaller share. B splits the 12 V equally, which is true only for equal capacitances. D is the answer for a parallel connection, in which each capacitor sits directly across the battery.

U3

6

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPII-06.03 · UPII-06.05

[1]

Wire X and wire Y are made of the same metal, are at the same temperature, and are joined end to end so that they carry the same steady current. The diameter of X is twice the diameter of Y. What are the ratio of the drift speed in X to that in Y, v_X/v_Y , and the ratio of the electric field inside X to that inside Y, E_X/E_Y ?

KEY **B** $v_X/v_Y = 1/4$ and $E_X/E_Y = 1/4$

The current $I = nqAv$ is the same in both wires and so is the carrier density n , because the metal is the same; doubling the diameter quadruples the cross-sectional area A , so the drift speed in X is a quarter of that in Y. The field follows from the microscopic form of Ohm's law, $E = \rho j = \rho I/A$: the same resistivity and current through four times the area give a quarter of the field. A scales both quantities with the diameter rather than with the area. C gets the drift speed right but assumes the field is the same throughout a connected circuit, as if the battery alone set it; the field in each segment is whatever is needed to drive the common current through that segment's resistance per unit length. D assumes carriers in the same metal must drift at the same speed, when the drift speed is set by the current density and not by the material alone.

U1

7

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-07.06 · UPII-07.07

[1]

A charged capacitor discharges through a fixed resistor. The potential difference across the capacitor falls to half its initial value in 3.0 s. How long after the discharge begins has the energy stored in the capacitor fallen to one-eighth of its initial value?

KEY **C** 4.5 s

The stored energy is $U = \frac{1}{2}CV^2$, so it decays as $e^{-(2t/RC)}$, twice as fast as the voltage: it halves every 1.5 s, and one-eighth is three energy halvings, 4.5 s. Equivalently, $U = U_0/8$ requires $V = V_0/\sqrt{8}$, and $RC \ln \sqrt{8} = 1.5 \times RC \ln 2 = 1.5 \times 3.0$ s. D applies the voltage's halving time of 3.0 s to the energy three times over, forgetting that energy goes as the square of the voltage. B is the time at which the energy has fallen to one-quarter, because the voltage has then halved; it answers a different fraction. A is the energy's halving time itself, one of the three steps required, and is chosen by candidates who correctly see that energy decays twice as fast but stop there.

U3

8

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPII-08.03 · UPII-08.05

[1]

In a mass spectrometer, ions start from rest, are accelerated through the same potential difference, and then enter a uniform magnetic field perpendicular to their velocity, in which they move on circular arcs. Ion X is ${}^4\text{He}^+$ (charge +e, mass 4u) and ion Y is ${}^{16}\text{O}^{2+}$ (charge +2e, mass 16u), where u is the atomic mass unit. What is the ratio of the radius of Y's path to the radius of X's path?

KEY **C** $\sqrt{2}$

After acceleration through a potential difference V each ion has kinetic energy qV , so its momentum is $\sqrt{2mqV}$ and its radius is $r = mv/(qB) = \sqrt{(2mV/q)}/B$. The radius therefore depends on $\sqrt{(m/q)}$: $\sqrt{(16/2)}$ against $\sqrt{(4/1)}$, a ratio of $\sqrt{2}$. D is the ratio of m/q , which is the answer if both ions enter the field with the same speed, as they would after a velocity selector; it is the right formula applied to the wrong entry condition. B is the answer for equal kinetic energies, when $r \propto \sqrt{m/q}$, but Y's double charge gives it twice the kinetic energy from the same potential difference. A is the answer for equal momenta, when $r \propto 1/q$. Each wrong option is the correct result for a different real arrangement, which is why the entry condition has to be read before any formula is chosen.

U2

9

CORE
COURSE

STRUCTURED · ROUTINE · UPII-01.01 · UPII-01.03 · UPII-01.04 · UPII-01.05 · UPII-04.01 · UPII-04.03 · UPII-04.04

[10]

Three small charged beads are fixed on a horizontal insulating board, which defines the xy-plane. Bead 1 carries +4.0 nC at the origin, bead 2 carries -2.0 nC at (0.30 m, 0) and bead 3 carries +3.0 nC at (0, 0.40 m). Point P is at (0.30 m, 0.40 m). Bead 2 was initially neutral and gained its charge by being rubbed with a cloth that was also initially neutral, and no charge escaped to the surroundings. A fourth bead carrying +1.0 nC is available for part (d), and in part (e) bead 2 is replaced by a bead carrying +2.0 nC at the same position. Treat all beads as point charges and ignore the surroundings. Take $k = 1/(4\pi\epsilon_0) = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ and $e = 1.60 \times 10^{-19} \text{ C}$.

- (a) **STATE** · U1 [1]
State the charge on the cloth after the rubbing.
- 1 +2.0 nC: the cloth loses exactly the electrons the bead gains, so the charge of cloth and bead together stays zero
An answer of zero or −2.0 nC treats rubbing as creating charge rather than transferring it.
- (b) **CALCULATE** · U3 [1]
Calculate the number of electrons transferred during the rubbing, and state whether they moved from the cloth to the bead or from the bead to the cloth.
- 1 $N = (2.0 \times 10^{-9} \text{ C}) / (1.60 \times 10^{-19} \text{ C}) = 1.25 \times 10^{10}$ electrons, moving from the cloth to the bead
Candidates who say positive charge moved to the cloth have missed the model used at this level: charging by friction transfers electrons, while the positive charge stays bound in the nuclei.
- (c) **CALCULATE** · U3 [4]
Calculate the magnitude and direction of the net electrostatic force on bead 1.
- 1 Force from bead 2: $(8.99 \times 10^9)(4.0 \times 10^{-9})(2.0 \times 10^{-9}) / (0.30)^2 = 7.99 \times 10^{-7} \text{ N}$, attractive, so along +x towards bead 2
- 2 Force from bead 3: $(8.99 \times 10^9)(4.0 \times 10^{-9})(3.0 \times 10^{-9}) / (0.40)^2 = 6.74 \times 10^{-7} \text{ N}$, repulsive, so along −y away from bead 3
- 3 Combines the perpendicular components: $|F| = \sqrt{(7.99 \times 10^{-7})^2 + (6.74 \times 10^{-7})^2} = 1.05 \times 10^{-6} \text{ N}$
- 4 Direction: $\tan^{-1}(6.74/7.99) = 40^\circ$ from the +x axis, rotated towards the −y axis (components $+7.99 \times 10^{-7} \text{ N}$ along x and $-6.74 \times 10^{-7} \text{ N}$ along y)
Adding the two magnitudes to get $1.47 \times 10^{-6} \text{ N}$, or fixing directions from the algebraic sign of q_1q_2 instead of from attraction and repulsion, are the errors this part is set to catch.
- (d) **DETERMINE** · U3 [2]
Determine the electric potential at P, and the work an external agent must do to bring the +1.0 nC bead slowly from a very large distance to rest at P.
- 1 Distances from P are 0.50 m, 0.40 m and 0.30 m, so $V_P = (8.99 \times 10^9)[(4.0/0.50) - (2.0/0.40) + (3.0/0.30)] \times 10^{-9} \text{ V} = 117 \text{ V}$
- 2 $W = qV_P = (1.0 \times 10^{-9} \text{ C})(117 \text{ V}) = 1.17 \times 10^{-7} \text{ J}$, positive work done by the agent
Potential is a signed scalar sum, so the −2.0 nC bead subtracts. Adding magnitudes gives 207 V; resolving potentials into components as if they were vectors earns nothing.
- (e) **COMPARE** · U1 [2]
Bead 2 is replaced by the +2.0 nC bead. Compare the net force on bead 1 in the original and the new arrangements, and state one measurement at P that would distinguish the two arrangements.
- 1 The force from bead 2 reverses to −x with the same size, so the net force on bead 1 keeps its magnitude of $1.05 \times 10^{-6} \text{ N}$ but now points at 40° from the −x axis towards −y: same magnitude, different direction
- 2 The potential at P distinguishes them: 117 V originally against 207 V in the new arrangement (equivalently $1.17 \times 10^{-7} \text{ J}$ against $2.07 \times 10^{-7} \text{ J}$ of work to bring the +1.0 nC bead to P); accept instead the electric field at P, about 386 N C^{-1} almost exactly along +x originally against 448 N C^{-1} at 31° above the +x axis in the new arrangement
The part checks that candidates see a force magnitude at one point does not fix a charge configuration; a second, independent measurement is needed to separate them.

A length of high-voltage coaxial cable has a solid inner conductor of radius $a = 1.0$ mm and a thin outer conducting sheath of radius $b = 4.0$ mm on the same axis. The space between them is filled with polyethylene of dielectric constant $\kappa = 2.3$, which breaks down when the electric field in it reaches $E_{\text{bd}} = 2.0 \times 10^7$ V m⁻¹. The inner conductor carries free charge $+\lambda$ per unit length and the sheath carries $-\lambda$ per unit length. The cable is long enough for end effects to be neglected. A redesign with a thinner inner conductor, of radius 0.50 mm and the same sheath, is under consideration. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F m⁻¹.

(a) DERIVE · U2

[3]

Using Gauss's law, derive an expression for the magnitude of the electric field in the polyethylene at a distance r from the axis, where $a < r < b$. Justify your choice of Gaussian surface.

- 1 The charge distribution is unchanged by translation along the axis and by rotation about it, so E is radial and depends only on r ; a coaxial cylinder of radius r and length ℓ therefore has E normal to and constant over its curved surface, and zero flux through its flat ends
- 2 Gauss's law in the dielectric in terms of free charge, $\oint \mathbf{D} \cdot d\mathbf{A} = q_{\text{free}}$ with $\mathbf{D} = \kappa \epsilon_0 \mathbf{E}$ (or with the bound charge included, which reduces the enclosed charge by the factor κ): $\kappa \epsilon_0 E(2\pi r \ell) = \lambda \ell$
- 3 $E = \lambda/(2\pi \kappa \epsilon_0 r)$, directed radially outward

Two errors are targeted: choosing a spherical surface because Gauss's law 'uses spheres', and dropping κ by applying the vacuum form of the law to the free charge alone.

(b) SKETCH · U1

[2]

Sketch a graph of the magnitude of the electric field against r from $r = 0$ to $r = 2b$, marking on the r -axis the radii at which the graph changes form.

- 1 $E = 0$ for $r < a$ (inside the conducting inner conductor) and $E = 0$ for $r > b$ (a Gaussian cylinder there encloses zero net charge), with a and b marked on the r -axis
- 2 At $r = a$ the field jumps from zero to its maximum, then falls along a $1/r$ curve (concave up, not a straight line) to one-quarter of that maximum at $r = b$, where it drops to zero

Curves falling as $1/r^2$, or falling continuously to zero instead of jumping at the conductor surfaces, describe a point charge or a smeared distribution rather than this cable.

(c) SHOW THAT · U2

[2]

Show that the capacitance per unit length of the cable is $C/L = 2\pi \kappa \epsilon_0 / \ln(b/a)$.

- 1 Potential difference between the conductors: $V = \int_a^b E \, dr = (\lambda/(2\pi \kappa \epsilon_0)) \int_a^b dr/r = (\lambda/(2\pi \kappa \epsilon_0)) \ln(b/a)$
- 2 Capacitance per unit length is charge per unit length over potential difference: $C/L = \lambda/V = 2\pi \kappa \epsilon_0 / \ln(b/a)$

Writing $V = E(b - a)$, the parallel-plate relation, is the error caught here; it does not hold for a field that varies as $1/r$.

(d) DETERMINE · U3

[3]

Determine the largest potential difference that can be applied between the conductors of the present design without breakdown of the polyethylene.

- 1 The field varies as $1/r$, so it is greatest at $r = a$, the surface of the inner conductor, and breakdown begins there
- 2 Eliminating λ between the results of (a) and (c): $E(a) = V/(a \ln(b/a))$, so $V_{\text{max}} = E_{\text{bd}} a \ln(b/a) = (2.0 \times 10^7 \text{ V m}^{-1})(1.0 \times 10^{-3} \text{ m}) \ln 4$
- 3 $V_{\text{max}} = 2.8 \times 10^4$ V (27.7 kV)

Multiplying the breakdown field by the gap, $(2.0 \times 10^7)(3.0 \times 10^{-3}) = 6.0 \times 10^4$ V, assumes a uniform field and more than doubles the true rating.

(e) EVALUATE · U3

[3]

The proposed redesign thins the inner conductor to 0.50 mm on the grounds that a wider gap needs a larger potential difference to break down. Evaluate this proposal, and determine the inner radius that gives the highest voltage rating for $b = 4.0$ mm.

- 1 With $a = 0.50$ mm, $V_{\max} = (2.0 \times 10^7)(0.50 \times 10^{-3}) \ln 8 = 2.1 \times 10^4$ V, lower than 2.8×10^4 V: thinning the conductor raises the field at its surface faster than $\ln(b/a)$ grows, so the proposal lowers the rating
- 2 Maximising $V_{\max} = E_{\text{bd}} a \ln(b/a)$ with respect to a : $dV_{\max}/da = E_{\text{bd}}[\ln(b/a) - 1] = 0$, so $a = b/e = 1.47$ mm
- 3 The optimum rating is $E_{\text{bd}} b/e = 2.9 \times 10^4$ V, about 6% above the present design; the judgement is to reject the proposal and make the inner conductor slightly thicker, not thinner

The intuition that a wider gap withstands more is right for parallel plates, where the field is uniform, and wrong here, where the field concentrates on the thin conductor. The top answers find the optimum rather than only rejecting the proposal.

11
CORE
COURSE

STRUCTURED · DEMANDING · UPII-06.06 · UPII-06.08 · UPII-07.01 · UPII-07.02 · UPII-07.03 · UPII-07.06 · UPII-07.08

[13]

A 12.0 V supply of internal resistance $r_1 = 0.50 \Omega$ and a rechargeable 9.0 V cell of internal resistance $r_2 = 1.5 \Omega$ are connected in parallel across a load of resistance $R_L = 4.0 \Omega$. Both positive terminals are joined to node X, both negative terminals are joined to node Y, and the load connects X to Y. The connecting wires have negligible resistance, and the emfs and internal resistances stay constant. In parts (d) and (e) an initially uncharged 0.47 F capacitor, with a manufacturing tolerance of $\pm 20\%$, is also connected directly between X and Y, and the design specification requires the load voltage to stay above 5.0 V for at least 1.0 s after both sources are disconnected.

(a) DETERMINE · U3

[4]

Using Kirchhoff's rules, determine the current in the supply, in the cell and in the load, stating the direction of each, and the potential difference V_{XY} between X and Y.

- 1 Defines I_1 and I_2 as the currents leaving the positive terminals of the supply and the cell and I_L as the load current from X to Y, and applies the junction rule at X: $I_1 + I_2 = I_L$
- 2 Loop rule for each branch: $12.0 - 0.50I_1 = V_{XY}$, $9.0 - 1.5I_2 = V_{XY}$ and $V_{XY} = 4.0I_L$ (or an equivalent pair of independent loop equations)
- 3 Solving $(12.0 - V_{XY})/0.50 + (9.0 - V_{XY})/1.5 = V_{XY}/4.0$ gives $V_{XY} = 10.3$ V (10.29 V)
- 4 $I_1 = 3.43$ A out of the supply's positive terminal; $I_2 = -0.857$ A, that is 0.857 A into the cell's positive terminal; $I_L = 2.57$ A from X to Y through the load

Candidates who assume every source drives current out of its positive terminal, and then quietly drop the minus sign on I_2 , discard the physics of the question: the negative sign is the result.

(b) SHOW THAT · U3

[2]

Show that the power delivered by the emf of the 12.0 V supply, 41.1 W, equals the total rate of energy transfer in the two internal resistances, the load and the cell.

- 1 Dissipation: $I_1^2 r_1 = (3.43)^2(0.50) = 5.88$ W, $I_2^2 r_2 = (0.857)^2(1.5) = 1.10$ W and $I_L^2 R_L = (2.57)^2(4.0) = 26.4$ W, totalling 33.4 W
- 2 The cell stores $\epsilon_2 |I_2| = (9.0 \text{ V})(0.857 \text{ A}) = 7.71$ W as chemical energy because current is driven into its positive terminal, and $33.4 \text{ W} + 7.71 \text{ W} = 41.1 \text{ W} = (12.0 \text{ V})(3.429 \text{ A})$

Counting only the $I^2 R$ terms leaves 7.7 W unaccounted for; the audit balances only when the cell is recognised as acting as a load.

(c) PREDICT · U1

[2]

Without the capacitor, the load resistance is gradually reduced from $4.0\ \Omega$. Predict the load resistance at which the current in the $9.0\ \text{V}$ cell becomes zero, and what the cell does when the load resistance is smaller still.

- 1 Zero current in the cell means no potential drop across r_2 , so $V_{XY} = 9.0\ \text{V}$; then $I_1 = (12.0 - 9.0)/0.50 = 6.0\ \text{A}$, all of which passes through the load, so $R_L = 9.0\ \text{V}/6.0\ \text{A} = 1.5\ \Omega$
- 2 For $R_L < 1.5\ \Omega$, V_{XY} falls below $9.0\ \text{V}$, so current leaves the cell's positive terminal: the cell discharges and shares the load current with the supply (at $R_L = 1.0\ \Omega$ it delivers $0.55\ \text{A}$)

The part rewards reasoning from the condition $V_{XY} = \varepsilon_2$ rather than re-solving the whole network for trial resistances.

(d) CALCULATE · U3

[3]

With the load at $4.0\ \Omega$, the $0.47\ \text{F}$ capacitor is connected between X and Y and the circuit is left for a long time. At $t = 0$ both the supply and the cell are disconnected, leaving only the capacitor and the load. Calculate the time at which the potential difference across the load has fallen to $5.0\ \text{V}$.

- 1 In the steady state no current flows into the capacitor, so the currents found in (a) are unchanged and the capacitor is charged to $V_0 = V_{XY} = 10.3\ \text{V}$
- 2 After disconnection it discharges through the load alone: $V = V_0 e^{-(t/RC)}$ with $RC = (4.0\ \Omega)(0.47\ \text{F}) = 1.88\ \text{s}$
- 3 $t = RC \ln(V_0/V) = (1.88\ \text{s}) \ln(10.29/5.0) = 1.36\ \text{s}$

Two errors are targeted: starting the discharge from the $12.0\ \text{V}$ emf instead of the terminal voltage, and including internal resistances in the time constant after the sources have been removed.

(e) EVALUATE · U4

[2]

Evaluate whether every capacitor within the $\pm 20\%$ tolerance meets the specification that the load voltage stays above $5.0\ \text{V}$ for at least $1.0\ \text{s}$ after disconnection.

- 1 The worst case is the smallest capacitance, $0.80 \times 0.47\ \text{F} = 0.376\ \text{F}$, giving $RC = (4.0\ \Omega)(0.376\ \text{F}) = 1.504\ \text{s}$; the starting voltage of $10.3\ \text{V}$ does not depend on C
- 2 $t = (1.504\ \text{s}) \ln(10.29/5.0) = 1.08\ \text{s}$, which exceeds $1.0\ \text{s}$, so every capacitor in tolerance meets the specification, but with a margin of only about $0.08\ \text{s}$ (about 8%), so a lower load resistance or a lower starting voltage could cause failure

Checking the nominal value ($1.36\ \text{s}$) and declaring a comfortable pass is the error; a design judgement has to be made at the edge of the tolerance band.

12
CORE
COURSE

DATA ANALYSIS · DEMANDING · UPII-06.03 · UPII-08.02 · UPII-08.08

[11]

A thin rectangular strip of doped semiconductor lies in the xy -plane. It is $20\ \text{mm}$ long in the x -direction, $w = 5.0\ \text{mm}$ wide in the y -direction and $t = 0.50\ \text{mm}$ thick in the z -direction. A steady current $I = 4.0\ \text{mA}$ flows through it in the $+x$ direction. A uniform magnetic field of adjustable magnitude is applied along the z -axis, and a high-resistance voltmeter connected to contacts on the two long edges records the Hall voltage $V_H = V(y = w) - V(y = 0)$. A positive value of B means the field points in the $+z$ direction; the final reading was taken with the field reversed. Take $e = 1.60 \times 10^{-19}\ \text{C}$. For part (e), copper has density $8.96 \times 10^3\ \text{kg m}^{-3}$ and molar mass $63.5\ \text{g mol}^{-1}$, and $N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$.

(a) JUSTIFY · U1

[2]

Justify, from the sign of the readings for positive B, whether the charge carriers in the strip are positive or negative.

- 1 For current in +x, positive carriers move in +x and negative carriers in -x; in both cases $F = q\mathbf{v} \times \mathbf{B}$ with B along +z points in -y ($q(\mathbf{v} \hat{x}) \times (B \hat{z}) = -qvB \hat{y}$ for positive carriers, $(-e)(-\mathbf{v} \hat{x}) \times (B \hat{z}) = -e\mathbf{v}B \hat{y}$ for electrons), so carriers of either sign accumulate on the y = 0 edge
- 2 $V_H = V(y = w) - V(y = 0)$ is positive, so the y = 0 edge is at the lower potential and the charge accumulated there is negative: the carriers are electrons and the material is n-type

Applying the right-hand rule to the conventional current alone gives the same deflection for both signs and never reaches the answer; the polarity is informative precisely because the deflection direction does not depend on the carrier sign.

(b) EXPLAIN · U4

[2]

The reading at B = 0 is not zero. Explain how this offset can arise, and use the readings at B = +0.50 T and B = -0.50 T to obtain the Hall voltage at 0.50 T with the offset removed.

- 1 The two edge contacts are not exactly opposite each other, so they also pick up part of the resistive potential drop along the strip; this offset keeps its sign when B is reversed, whereas the Hall voltage reverses sign
- 2 Half the difference of the two readings removes the offset: $V_H = [12.92 - (-12.08)]/2 = 12.50$ mV (half their sum, 0.42 mV, recovers the offset and agrees with the zero-field reading)

Subtracting the zero-field reading from the +0.50 T reading gives the same 12.50 mV here only because the offset stayed constant through the run; reversing the field measures the offset at the same field magnitude, so it also cancels any part of the contact voltage that changes with the size of B but not with its direction. The part checks that the physical origin of the offset is understood.

(c) DETERMINE · U4

[3]

Using the seven readings with B ≥ 0, determine the gradient of a graph of V_H against B.

- 1 Uses all seven readings through a single best-fit straight line of V_H against B, drawn on a graph or found by least squares, rather than one reading or two adjacent readings
- 2 Reads the gradient from two widely separated points on that line, for example about (0.00 T, 0.43 mV) and (0.60 T, 15.40 mV), giving $(15.40 - 0.43) \text{ mV} / (0.60 - 0.00) \text{ T}$, or takes the least-squares slope directly
- 3 Gradient = $25.0 \text{ mV T}^{-1} = 2.50 \times 10^{-2} \text{ V T}^{-1}$ (accept 24.6 to 25.4 mV T⁻¹)

Dividing a single reading by its field, $12.92/0.50 = 25.8 \text{ mV T}^{-1}$, builds the offset into the result; the gradient of a line is immune to a constant offset, which is why it is used.

(d) CALCULATE · U3

[2]

Calculate the number density of charge carriers in the semiconductor.

- 1 Balancing the magnetic and Hall-field forces on the carriers gives $V_H = IB/(n e t)$, so the gradient equals $I/(n e t)$ and $n = I/(e t \times \text{gradient})$
- 2 $n = (4.0 \times 10^{-3}) / [(1.60 \times 10^{-19})(0.50 \times 10^{-3})(2.50 \times 10^{-2})] = 2.0 \times 10^{21} \text{ m}^{-3}$

Using the width w instead of the thickness t, the dimension along B, gives a value ten times too small, $2.0 \times 10^{20} \text{ m}^{-3}$.

(e) ESTIMATE · U4

[2]

Estimate the Hall voltage that the same current and a field of 0.50 T would produce in a copper strip of the same dimensions, stating your assumption about the number of conduction electrons, and say what your estimate implies for the choice of material in a Hall probe.

- 1 Assuming one conduction electron per copper atom: $n \approx (8.96 \times 10^3 \text{ kg m}^{-3} / 0.0635 \text{ kg mol}^{-1})(6.02 \times 10^{23} \text{ mol}^{-1}) \approx 8.5 \times 10^{28} \text{ m}^{-3}$
- 2 $V_H = IB/(n e t) \approx (4.0 \times 10^{-3})(0.50)/[(8.5 \times 10^{28})(1.60 \times 10^{-19})(0.50 \times 10^{-3})] \approx 3 \times 10^{-10} \text{ V}$, about 4×10^7 times smaller than the semiconductor's 12.5 mV and far below its 0.42 mV offset, so Hall probes are made from low-carrier-density semiconductors

Candidates who expect the better conductor to give the larger signal have the dependence upside down: $V_H \propto 1/n$.

13
CORE
COURSE

FREE RESPONSE · EXTENDED DERIVATION · DISCRIMINATING · UPII-01.06 · UPII-02.05 · UPII-03.02 · UPII-04.06 · UPII-04.07

[17]

A thin flat annulus, shaped like a washer, of inner radius a and outer radius b lies in the xy -plane centred on the origin and carries a uniform surface charge density $\sigma > 0$. The z -axis passes through the centre of the hole, perpendicular to the annulus. Take the electric potential to be zero infinitely far away and neglect gravity. For part (e), the particle is an electron of charge magnitude $q = 1.60 \times 10^{-19} \text{ C}$ and mass $m = 9.11 \times 10^{-31} \text{ kg}$, with $a = 2.0 \text{ cm}$, $b = 6.0 \text{ cm}$, $\sigma = 1.0 \times 10^{-9} \text{ C m}^{-2}$ and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$.

(a) DERIVE · U2

[4]

Starting from the potential of a point charge, derive an expression for the electric potential $V(z)$ at a point on the z -axis.

- 1 Divides the annulus into thin rings of radius r and width dr , each carrying $dq = \sigma(2\pi r dr)$, all of it at the same distance $\sqrt{r^2 + z^2}$ from the field point
- 2 $dV = dq/(4\pi\epsilon_0\sqrt{r^2 + z^2}) = (\sigma/(2\epsilon_0)) r dr/\sqrt{r^2 + z^2}$, added as scalars with no components to resolve
- 3 Integrates from $r = a$ to $r = b$: $\int_a^b r dr/\sqrt{r^2 + z^2} = \sqrt{b^2 + z^2} - \sqrt{a^2 + z^2}$
- 4 $V(z) = (\sigma/(2\epsilon_0))[\sqrt{b^2 + z^2} - \sqrt{a^2 + z^2}]$

Integrating the field instead takes on a vector integral the problem does not need; using r instead of $\sqrt{r^2 + z^2}$ for the distance produces a potential that does not depend on z .

(b) DETERMINE · U2

[2]

Determine the z -component of the electric field on the axis from your answer to (a), and explain why the potential on the axis alone is enough to give the complete field there.

- 1 $E_z = -dV/dz = (\sigma z/(2\epsilon_0))[1/\sqrt{a^2 + z^2} - 1/\sqrt{b^2 + z^2}]$, positive for $z > 0$, so the field on the axis points away from the annulus
- 2 V known only along the axis gives only $\partial V/\partial z$, but rotational symmetry about the z -axis forbids any transverse component on the axis, so $E_x = E_y = 0$ there and E_z is the whole field

Dropping the minus sign in $E = -dV/dz$ gives a field pointing towards a positive distribution; the symmetry argument is needed because a potential known on a line cannot by itself give derivatives across that line.

(c) **SHOW THAT** · U2 [3]

Show that for $|z| \gg b$ the field on the axis reduces to that of a point charge $Q = \pi\sigma(b^2 - a^2)$, and state the field obtained in the limit $a \rightarrow 0$, $b \rightarrow \infty$, naming the charge distribution this limit describes.

- 1 For $|z| \gg b$, $1/\sqrt{a^2 + z^2} \approx (1/|z|)(1 - a^2/(2z^2))$ and likewise for b , so the bracket in E_z is approximately $(b^2 - a^2)/(2|z|^3)$
- 2 $E_z \approx \sigma(b^2 - a^2)/(4\epsilon_0 z^2)$ for $z > 0$, which equals $Q/(4\pi\epsilon_0 z^2)$ with $Q = \pi\sigma(b^2 - a^2)$, the total charge of the annulus
- 3 With $a \rightarrow 0$ and $b \rightarrow \infty$ the bracket tends to $1/|z|$, so $E_z = \sigma/(2\epsilon_0)$ directed away from the plane on each side and independent of distance: the field of an infinite uniformly charged plane

The expansion must be carried to second order; keeping only $1/|z|$ from each term makes the two cancel and suggests, wrongly, that the far field vanishes.

(d) **DERIVE** · U2 [3]

A particle of charge $-q$ and mass m is released from rest on the axis at $z = z_0$, where $0 < z_0 \ll a$. Using Newton's second law, derive the equation of motion for small z , show that the motion along the axis is simple harmonic, and derive its angular frequency ω .

- 1 For $|z| \ll a$, $1/\sqrt{a^2 + z^2} \approx 1/a$ and $1/\sqrt{b^2 + z^2} \approx 1/b$, so $E_z \approx (\sigma z/(2\epsilon_0))(1/a - 1/b) = \sigma(b - a)z/(2\epsilon_0 ab)$
- 2 Newton's second law with force $-qE_z$: $m d^2z/dt^2 = -[q\sigma(b - a)/(2\epsilon_0 ab)]z$, a restoring force proportional to displacement, so the motion is simple harmonic about the centre
- 3 $\omega = \sqrt{[q\sigma(b - a)/(2\epsilon_0 mab)]}$

Setting $z = 0$ in the prefactor as well as inside the bracket removes the force entirely; the approximation must keep the leading power of z .

(e) **CALCULATE** · U3 [2]

Calculate the period of the oscillation for the electron described in the stem.

- 1 $\omega^2 = (1.60 \times 10^{-19})(1.0 \times 10^{-9})(0.040)/[2(8.85 \times 10^{-12})(9.11 \times 10^{-31})(0.020)(0.060)] = 3.3 \times 10^{14} \text{ s}^{-2}$, so $\omega = 1.82 \times 10^7 \text{ rad s}^{-1}$
- 2 $T = 2\pi/\omega = 3.5 \times 10^{-7} \text{ s}$ ($3.45 \times 10^{-7} \text{ s}$)

Leaving a and b in centimetres makes ω^2 a hundred times too small and the period ten times too long, since ω^2 scales as $1/\text{length}$; a dimensional check catches it.

(f) **PREDICT** · U1 [3]

In practice the electron cannot be released exactly on the axis. By applying Gauss's law to a small closed cylinder of radius p and length $2h$, centred on the origin with its axis along z and with p and h both much smaller than a , predict the radial component of the electric field at a small distance p from the axis near the centre, and hence predict whether the oscillation in (d) survives a small sideways displacement.

- 1 The cylinder lies inside the hole and encloses no charge; with $E_z \approx Kz$, where $K = \sigma(b - a)/(2\epsilon_0 ab)$, the outward flux through the two end caps is $2(Kh)(\pi p^2)$
- 2 Zero total flux requires an equal inward flux through the curved surface: $E_p(2\pi p)(2h) = -2\pi Khp^2$, so $E_p = -Kp/2$, directed towards the axis
- 3 The force on the electron, $-qE_p$, points away from the axis and grows with p , so the centre is unstable sideways: a small transverse displacement grows, the electron drifts off the axis towards the annulus and the axial oscillation is not sustained, as Earnshaw's theorem requires of any electrostatic equilibrium

This part separates the top grade. The one-dimensional analysis in (d) looks complete, and only a candidate who uses Gauss's law as a constraint on the field, rather than as a way to compute it, sees that stability along the axis forces instability across it.

What the command words are asking for

Examiners mark to the command word as well as to the physics. An answer that explains where the question said state has spent time earning one mark; one that states a result where the question said derive has earned none, however correct the result.

State	Give the result, definition or law concisely. No justification is required.
Describe	Give the relevant features of a situation or process in words, precisely enough that someone else could reproduce it.
Explain	Give the physical reason a result or behaviour occurs, connecting it explicitly to the principle that governs it.
Calculate	Obtain a numerical value, showing the relation used, the substitution and the units of the result.
Determine	Obtain a result by any valid route the problem allows — analytic, graphical or numerical — with enough working to be followed.
Derive	Start from a named fundamental law or definition and reach the required result by explicit, justified steps.
Show that	Establish a result the question gives you. Every step must be shown, because the stated answer earns nothing on its own.
Estimate	Reach an order-of-magnitude value from stated, reasonable assumptions, and say what those assumptions are.
Sketch	Draw the shape of a graph or field with axes labelled and its key features marked — intercepts, asymptotes, turning points, limits. Exact scale is not required.
Justify	Support a claim or a choice with evidence, and with the reasoning that connects that evidence to the claim.
Evaluate	Weigh a model, argument or result against criteria, including its assumptions and domain of validity, and reach a stated judgement.
Compare	Identify the similarities and differences between two situations, models or results, relating each difference to its cause.
Predict	Say what will happen when a condition changes, reasoning from a principle rather than guessing, and give the reason.

WHAT IS BEING TESTED

University assessment objectives

Each course has the two papers a university course is examined by: a midterm on the first half of its units, and a cumulative final weighted towards the second half. Every paper combines short conceptual multiple-choice items with structured problems, a data or estimation question, and at least one extended derivation. Constants are given where a question needs them; a calculator and your own formula sheet are assumed.

U1 16 MARKS · 22%	Conceptual understanding Identify the governing principle and the conditions under which it holds, and reason qualitatively about what a result must look like before calculating it.
U2 18 MARKS · 25%	Mathematical modelling and derivation Translate a physical situation into mathematics: choose coordinates and approximations, set up the governing equations, solve them symbolically, and derive stated results from named laws.

U3

29 MARKS · 40%

Quantitative problem solving

Carry a multi-step calculation to a numerical result with correct units and sensible precision, and check it against dimensions, limiting cases and orders of magnitude.

U4

9 MARKS · 13%

Data, estimation and experimental reasoning

Analyse measurements and their uncertainties, linearise and fit data, estimate orders of magnitude from stated assumptions, and judge whether evidence supports a model.

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