

University Physics III · Final Examination

The whole of University Physics III, weighted towards its second half: oscillations, waves, optics and kinetic theory, then thermodynamic cycles and entropy, special relativity, the Schrödinger equation, the hydrogen atom and nuclear energetics. Most marks are for building a model from a named law, auditing it against conservation laws and the second law, and checking where its approximations break.

SUGGESTED TIME	TOTAL MARKS	QUESTIONS	MULTIPLE CHOICE
150 minutes	118	19	10

SECTION	UPIII · Final · University Physics III — Final examination
IN THE FORMAT OF	A cumulative university final: conceptual multiple choice, structured problems, a data-analysis question and two extended derivations
DEMAND	Routine 7 · Demanding 10 · Discriminating 2 <i>Pitched at what a strong course grade requires rather than at what a homework set rehearses. Routine substitutions carry few marks; most sit on problems that have to be set up before they can be solved, on derivations from a named law, and on checking a result against its limits. Every paper still opens with entry points, so you can tell which marks you should be securing first.</i>
COURSE UNITS	UPIII-01 · UPIII-02 · UPIII-03 · UPIII-04 · UPIII-05 · UPIII-06 · UPIII-07 · UPIII-08 · UPIII-09 · UPIII-10 · UPIII-11

INSTRUCTIONS

- Answer all questions in the spaces provided.
- Show your working. Method marks are awarded even where the final answer is wrong.
- Give your answers to a sensible number of significant figures, with a unit.
- The mark scheme for this paper follows the last question.

Written by GioPhysics from its own University Physics course maps. These are practice examinations in a university style: they are not the examinations of any university, contain no past-paper questions, and carry no academic credit. For a course you are taking, your instructor's syllabus and past papers remain the authority.

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University Physics III · Final Examination

Answer all questions. 118 marks in 150 minutes.

1 MULTIPLE CHOICE · ROUTINE · UPIII-01.07 [1]
CORE COURSE

A particle of mass 0.150 kg moves along the positive x-axis in the potential $U(x) = A/x^2 - B/x$, where $A = 0.360 \text{ J}\cdot\text{m}^2$ and $B = 1.20 \text{ J}\cdot\text{m}$. What is the angular frequency of small oscillations about its stable equilibrium position?

- A 0.969 rad/s
- B 6.09 rad/s
- C 10.5 rad/s
- D 13.6 rad/s

Answer

2 MULTIPLE CHOICE · DEMANDING · UPIII-02.02 [1]
CORE COURSE

The one-dimensional wave equation for waves of speed v is $\partial^2 y / \partial x^2 = (1/v^2) \partial^2 y / \partial t^2$. In each function below, A , k and b are positive constants and $\omega = kv$. Which function is NOT a solution of this equation?

- A $y = A \cos(kx) \sin(\omega t)$
- B $y = A/[1 + (x + vt)^2/b^2]$
- C $y = A \sin(kx - \omega t) + A \sin(kx + \omega t)$
- D $y = A \exp[-(x^2 + v^2 t^2)/b^2]$

Answer

3 MULTIPLE CHOICE · DEMANDING · UPIII-04.09 [1]
CORE COURSE

A Galilean beam expander widens a collimated laser beam of diameter 2.00 mm. The beam passes first through a thin diverging lens of focal length -25.0 mm and then through a thin converging lens of focal length $+100 \text{ mm}$ on the same axis. Which lens separation makes the beam emerge collimated, and what is its diameter then?

- A Separation 125 mm; diameter 8.00 mm
- B Separation 75.0 mm; diameter 0.500 mm
- C Separation 75.0 mm; diameter 8.00 mm
- D Separation 75.0 mm; diameter 2.00 mm

Answer

4

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-05.09

[1]

Sunlight is reflected from the calm surface of a lake, of refractive index 1.33, towards an angler whose sunglasses have polarising lenses with vertical transmission axes. At what angle below the horizontal must the angler look at the water for the reflected glare to be completely polarised, so that the sunglasses block it entirely?

- A 53.1°
- B 36.9°
- C 41.2°
- D 48.8°

Answer

5

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-06.05

[1]

A sample of argon (molar mass 39.9 g/mol) is at 320 K. At what temperature would a sample of helium (molar mass 4.00 g/mol) have the same root-mean-square atomic speed? Treat both as ideal monatomic gases.

- A 32.1 K
- B 101 K
- C 320 K
- D 3190 K

Answer

6

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPIII-07.08

[1]

A rigid, thermally insulated container is divided by a partition into two equal halves. One half holds 1.00 mol of an ideal gas and the other is evacuated. The partition is punctured and the gas fills the whole container. Take $R = 8.314 \text{ J/(mol}\cdot\text{K)}$. Which statement is correct?

- A No heat enters the gas, so its entropy change is zero.
- B The gas does work as it expands, so its temperature falls.
- C The internal energy of the gas is unchanged, and its entropy rises by 5.76 J/K.
- D The entropy of the gas rises by 5.76 J/K, but the entropy of the universe is unchanged because the container is insulated.

Answer

7CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-08.03

[1]

A bunch of protons travels at constant speed $0.800c$ along a straight section of a linear accelerator, between two beam-position monitors that are fixed in the laboratory 36.0 m apart. Take $c = 3.00 \times 10^8\text{ m/s}$. Which pair gives the distance between the monitors and the time the bunch takes to travel from one to the other, both as measured in the rest frame of the bunch?

- A 21.6 m and 90.0 ns
- B 36.0 m and 90.0 ns
- C 21.6 m and 150 ns
- D 60.0 m and 250 ns

Answer

8CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-09.02

[1]

Light of wavelength 400 nm ejects photoelectrons from a metal surface whose work function is 2.15 eV . The wavelength is then changed to 200 nm while the intensity is kept the same. Take $hc = 1240\text{ eV}\cdot\text{nm}$. What is the stopping potential with the 200 nm light?

- A 0.95 V
- B 1.90 V
- C 4.05 V
- D 6.20 V

Answer

9CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-10.04

[1]

An electron in a hydrogen atom has orbital angular momentum quantum number $l = 3$. What is the smallest angle that its orbital angular momentum vector can make with the z -axis?

- A 0°
- B 30.0°
- C 54.7°
- D 60.0°

Answer

Rest energies: proton 938.3 MeV, neutron 939.6 MeV, electron 0.511 MeV, muon 105.7 MeV, charged pion 139.6 MeV; neutrino masses are negligible. Which of the following processes is allowed by every conservation law?

- A $n \rightarrow p + e^- + \nu_e$
- B $p \rightarrow n + e^+ + \nu_e$, for a free proton at rest
- C $\mu^- \rightarrow e^- + \gamma$
- D $\pi^+ \rightarrow \mu^+ + \nu_\mu$

Answer

An aerial-mapping camera photographs a field from a height of 120 m. Its lens is a thin converging lens of focal length 8.80 mm and aperture diameter 3.14 mm, and its image sensor has square pixels 2.40 μm wide. Take the wavelength of the light as 550 nm in air. To reduce reflections, the lens glass, of refractive index 1.52, carries a single coating of magnesium fluoride, of refractive index 1.38, designed for 550 nm at normal incidence.

(a) CALCULATE

[2]

Calculate how far behind the lens the sensor must be placed to focus the field, and how far this is from the lens's focal plane.

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(b) CALCULATE

[2]

Calculate the width of the strip of ground that is imaged onto one pixel.

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(c) JUSTIFY

[3]

Justify whether diffraction at the lens aperture or the size of the pixels sets the smallest ground detail this camera can resolve at 550 nm.

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(d) DETERMINE

[2]

Determine the minimum thickness of the coating that minimises the reflection of 550 nm light at normal incidence.

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(e) EXPLAIN

[2]

When white light is reflected from the coated lens, the reflection looks purple. Explain why.

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The pendulum of a longcase clock is a light rigid rod carrying a solid brass sphere of mass 2.30 kg and radius $r = 0.0400$ m, whose centre is $L = 0.990$ m below the pivot. The moment of inertia of a solid sphere about an axis through its centre is $(2/5)mr^2$. Take $g = 9.81$ m/s². The clock's escapement gives the pendulum a small push once each swing to keep its amplitude at 4.00° .

(a) DERIVE

[3]

Derive, from the rotational form of Newton's second law, an expression for the period of small oscillations of this pendulum in terms of L , r and g , stating the approximation you make.

(b) CALCULATE

[2]

Calculate the period, and the fraction by which it exceeds the period of a simple pendulum of length 0.990 m.

(c) DETERMINE

[3]

With the escapement disconnected, the amplitude falls from 4.00° to 2.00° in 12.0 minutes. Determine the average power the escapement must supply to keep the amplitude at 4.00° , stating the assumption about the damping that your method requires.

(d) CALCULATE

[2]

For a finite amplitude θ_0 the period is $T \approx T_0(1 + \theta_0^2/16)$, with θ_0 in radians. The clock is adjusted to keep perfect time at an amplitude of 4.00° , but friction in its gear train lets the amplitude wander between 3.50° and 4.50° . Calculate the largest time the clock gains or loses in one day as a result.

(e) EVALUATE

[2]

The clock must keep time to within 1 s per day. Evaluate which idealisation of the simple-pendulum model, the point-mass bob or the small-angle limit, limits this clock's timekeeping.

A pair of extraction fans of the same model stand 6.0 m apart on a flat roof. Each has 8 blades and radiates a steady tone at its blade-passing frequency, the number of blades multiplied by the fan's rotation rate in revolutions per second, and the controller of fan A shows a rotation rate of 24.50 revolutions per second. With both fans running, a technician's sound level meter, filtered to reject everything except a narrow band around this tone and quick enough to follow any change in the tone's amplitude, records a level that rises and falls smoothly and regularly between a maximum of 79.5 dB and a minimum of 71.9 dB, with successive maxima 5.00 s apart. The meter responds to the pressure at its microphone and reports the sound intensity level of a single plane wave with the same pressure amplitude. Treat the fans as small sources in open air and ignore reflections. Take the reference intensity as $1.00 \times 10^{-12} \text{ W/m}^2$, the density of air as 1.20 kg/m^3 , the speed of sound as 343 m/s and atmospheric pressure as $1.01 \times 10^5 \text{ Pa}$.

(a) CALCULATE

[2]

Calculate the pressure amplitude of the sound at the meter when the level is at a maximum and when it is at a minimum.

(b) EXPLAIN

[2]

The sound from the two fans reaches the meter travelling in different directions. Explain why the excess pressures of the two waves at the meter can be added as numbers, whereas the displacements of the air in the two waves cannot, and why the superposition principle holds for these waves.

(c) DETERMINE

[4]

Determine the sound intensity level that the louder fan alone would produce at the meter, the level that the quieter fan alone would produce, and the difference between the two fans' rotation rates.

(d) DESCRIBE

[2]

The technician adjusts the controller of fan B until the level at the meter stops rising and falling. Describe how the steady level now depends on the fans and on where the meter is placed, giving the range of steady levels the meter could read at its present position, and the change in the difference between its distances from the two fans that takes the level from its highest to its lowest value, ignoring any change in each fan's own amplitude.

In the laboratory frame S , three events are recorded on the x -axis. Event A: a detector at $x = 0$ fires at $t = 0$. Event B: a detector at $x = 900$ m fires at $t = 2.00 \mu\text{s}$. Event C: the same detector at $x = 900$ m fires again at $t = 4.00 \mu\text{s}$. Frame S' moves at $+0.800c$ along the x -axis relative to S , and its origin coincides with that of S at event A. Take $c = 3.00 \times 10^8$ m/s.

(a) CALCULATE

[2]

Calculate the invariant interval $s^2 = (c\Delta t)^2 - (\Delta x)^2$ between events A and B, and classify the separation.

(b) DETERMINE

[3]

Determine the time interval $\Delta t'$ and the spatial separation $\Delta x'$ between A and B measured in S' , and state which event occurs first in S' .

(c) JUSTIFY

[2]

Justify the claim that event A cannot have caused event B, giving two separate arguments.

(d) DETERMINE

[2]

Determine the velocity, relative to S , of the inertial frame in which events A and B are simultaneous.

(e) PREDICT

[2]

Predict the order of events A and C in S' and in every other inertial frame, giving the reason.

A radioisotope power source for a deep-space probe contains 3.50 kg of plutonium-238 (^{238}Pu , $Z = 94$), which decays by alpha emission to uranium-234 with a half-life of 87.7 years. Atomic masses: ^{238}Pu 238.049 560 u, ^{234}U 234.040 952 u, ^4He 4.002 603 u. Take $1 \text{ u} = 931.494 \text{ MeV}/c^2$, $1 \text{ MeV} = 1.602 \text{ 18} \times 10^{-13} \text{ J}$, $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$, $1 \text{ year} = 3.156 \times 10^7 \text{ s}$, and the molar mass of ^{238}Pu as 238.05 g/mol.

- (a) STATE [1]

State the decay equation, showing the nucleon number and proton number of every particle.

- (b) EXPLAIN [2]

Explain why the Q value of this decay can be calculated directly from atomic masses, even though the decay is a nuclear process.

- (c) CALCULATE [2]

Calculate the Q value of the decay in MeV.

- (d) SHOW THAT [2]

Show that, for a parent nucleus at rest, the alpha particle carries about 98% of the Q value as kinetic energy.

- (e) CALCULATE [3]

Calculate the initial thermal power of the source, assuming that all of the decay energy is absorbed within it.

- (f) DETERMINE [2]

Determine how long after the source is made its thermal power falls to 1500 W.

An engine takes 0.500 mol of an ideal diatomic gas ($C_V = 5R/2$, $\gamma = 1.40$) around a three-stage cycle. $A \rightarrow B$: reversible isothermal compression at 300 K, in contact with a cold reservoir at 300 K, until the volume is one quarter of its value at A. $B \rightarrow C$: heating at constant volume, by contact with a hot reservoir at 600 K, until the gas reaches temperature T_C , when contact is broken. $C \rightarrow A$: reversible adiabatic expansion back to state A. Take $R = 8.314 \text{ J/(mol}\cdot\text{K)}$.

(a) SHOW THAT

[2]

Show that $T_C = 522 \text{ K}$.

(b) CALCULATE

[4]

Calculate the heat Q absorbed by the gas and the work W done by the gas in each of the three stages, and the net work done by the gas per cycle.

(c) COMPARE

[2]

Compare the thermal efficiency of this cycle with the Carnot efficiency of an engine operating between the same two reservoirs.

(d) DETERMINE

[3]

Determine the total entropy change of the gas and the two reservoirs per cycle, and identify the stage in which entropy is generated.

(e) SHOW THAT

[2]

Show that the work this cycle loses, compared with a Carnot engine that absorbs the same heat Q_{BC} from the 600 K reservoir, equals $T_{\text{cold}} \times S_{\text{gen}}$, where $T_{\text{cold}} = 300 \text{ K}$ and S_{gen} is the entropy generated per cycle.

In a low-energy electron diffraction experiment, electrons accelerated from rest through a potential difference V strike a clean copper crystal surface along its normal. The rows of surface atoms, a distance a apart, act as a reflection grating: first-order diffracted beams leave the surface at an angle ϕ to the normal, where $a \sin \phi = \lambda$ and λ is the de Broglie wavelength of the electrons. The angle ϕ is measured at several accelerating potential differences, and the energy scale of the electron gun is calibrated to $\pm 2\%$. Take $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$, $m_e = 9.109 \times 10^{-31} \text{ kg}$ and $e = 1.602 \times 10^{-19} \text{ C}$. In bulk copper the atoms of a plane of this orientation form a square array of side 0.2556 nm .

Angle of a first-order diffracted beam from the copper surface at several accelerating potential differences

accelerating potential difference V / V	first-order beam angle $\phi / ^\circ$
50	42.9
75	33.4
100	28.9
125	25.2
150	23.2
175	21.1
200	19.9

(a) **SHOW THAT**

[2]

Show that the model predicts a straight line through the origin when $\sin \phi$ is plotted against $1/\sqrt{V}$, and give its gradient in terms of h , m_e , e and a .

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(b) **DETERMINE**

[3]

Using all of the readings, determine the gradient of the graph of $\sin \phi$ against $1/\sqrt{V}$.

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(c) **CALCULATE**

[3]

Calculate the row spacing a , with its uncertainty arising from the calibration of the energy scale.

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(d) EVALUATE

[2]

A calculation predicts that the outermost layer of copper atoms relaxes towards the layer beneath it, reducing the spacing between those two layers by about 2%. Evaluate whether these data support a surface whose atoms keep their bulk arrangement within the layer, and whether this experiment could detect the predicted relaxation.

In muonic hydrogen a negative muon, of mass 1.8835×10^{-28} kg, takes the place of the electron and is bound to a proton of mass 1.6726×10^{-27} kg; its energy levels are measured to find the size of the proton. The relative motion obeys the hydrogen-atom Schrödinger equation with the reduced mass $\mu = m_\mu m_p / (m_\mu + m_p)$ in place of the electron mass. For a state with no angular dependence ($l = 0$) the equation reads $-(\hbar^2/2\mu)(1/r^2) d/dr(r^2 d\psi/dr) - (e^2/(4\pi\epsilon_0 r))\psi = E\psi$. Take $\hbar = 1.0546 \times 10^{-34}$ J·s, $e = 1.602 \times 10^{-19}$ C, $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, the proton radius as 0.84 fm and the Bohr radius of ordinary hydrogen as 5.29×10^{-11} m. You may use $\int_0^\infty r^n e^{-\beta r} dr = n!/\beta^{n+1}$.

(a) DERIVE

[5]

By substituting the trial solution $\psi(r) = Ce^{(-r/a)}$ into the Schrödinger equation, derive expressions for the length a and the energy E , and evaluate both for muonic hydrogen.

(b) SHOW THAT

[2]

Show that the normalised wavefunction has $C = 1/\sqrt{(\pi a^3)}$.

(c) DETERMINE

[2]

Determine the most probable distance of the muon from the proton, in metres.

(d) SKETCH

[2]

Sketch the radial probability density $P(r) = 4\pi r^2 |\psi(r)|^2$ against r/a , marking the most probable radius found in (c) and the mean radius $\langle r \rangle = 1.5a$.

(e) ESTIMATE

[3]

Estimate the probability that the muon is found inside the proton, stating the assumption you make, and compare it with the corresponding probability for the electron in ordinary hydrogen.

The neutron's mass is determined most precisely from the gamma ray emitted when a slow neutron is captured by a proton at rest: $n + p \rightarrow d + \gamma$. A crystal-diffraction spectrometer measures the photon energy as $E_\gamma = 2.223\,25$ MeV, with an uncertainty of ± 0.4 eV. Take the rest energies $m_{\text{p}}c^2 = 938.2721$ MeV and $m_{\text{d}}c^2 = 1875.6129$ MeV, and treat the neutron's kinetic energy before capture as negligible. Write $M = m_{\text{n}} + m_{\text{p}}$ for the total rest mass before capture, and $B = (M - m_{\text{d}})c^2$ for the binding energy of the deuteron.

(a) **DERIVE** [4]

Derive, from conservation of energy and momentum, an exact expression for E_γ in terms of M , m_{d} and c .

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(b) **SHOW THAT** [3]

Show that the result of (a) can be written exactly as $E_\gamma = B - B^2/(2Mc^2)$, and explain the physical meaning of the term $B^2/(2Mc^2)$.

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(c) **DETERMINE** [3]

Determine the rest energy of the neutron, $m_{\text{n}}c^2$, from the measured photon energy.

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(d) **EVALUATE** [2]

Evaluate whether the recoil term must be included when the neutron's rest energy is quoted from this measurement.

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(e) **COMPARE** [3]

In the reverse process, photodisintegration $\gamma + d \rightarrow n + p$, a photon strikes a deuteron at rest. Compare the minimum photon energy for this process with the photon energy emitted in capture, and explain the difference.

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University Physics III · Final Examination — mark scheme

One line per mark, as an examiner would award them. A note under a part is the error that part is set to catch.

1 MULTIPLE CHOICE · ROUTINE · UPIII-01.07 **[1]**
CORE COURSE

A particle of mass 0.150 kg moves along the positive x-axis in the potential $U(x) = A/x^2 - B/x$, where $A = 0.360 \text{ J}\cdot\text{m}^2$ and $B = 1.20 \text{ J}\cdot\text{m}$. What is the angular frequency of small oscillations about its stable equilibrium position?

KEY B 6.09 rad/s

Equilibrium is where $dU/dx = -2A/x^3 + B/x^2 = 0$, at $x_0 = 2A/B = 0.600 \text{ m}$, and the effective spring constant is the curvature there: $U''(x_0) = 6A/x_0^4 - 2B/x_0^3 = 16.67 - 11.11 = 5.56 \text{ J/m}^2$, so $\omega = \sqrt{U''/m} = 6.09 \text{ rad/s}$. A is the same result converted to a frequency, $f = \omega/2\pi = 0.969 \text{ Hz}$, and then labelled in rad/s, a units slip that costs the whole mark. C keeps only the curvature of the repulsive term, $6A/x_0^4$, as if the attractive term were a constant offset; but $-B/x$ has a curvature of its own, $-2B/x^3$, and it softens the well. D differentiates the attractive term with the wrong sign, adding 11.11 J/m^2 instead of subtracting it. A candidate who remembers that the attraction is what creates the minimum, and so must reduce the stiffness below that of the repulsion alone, rejects both C and D at once.

U2

2 MULTIPLE CHOICE · DEMANDING · UPIII-02.02 **[1]**
CORE COURSE

The one-dimensional wave equation for waves of speed v is $\partial^2 y / \partial x^2 = (1/v^2) \partial^2 y / \partial t^2$. In each function below, A , k and b are positive constants and $\omega = kv$. Which function is NOT a solution of this equation?

KEY D $y = A \exp[-(x^2 + v^2 t^2)/b^2]$

D contains vt , but only inside $x^2 + v^2 t^2$, which is not a function of $x - vt$ or of $x + vt$. Differentiating, $\partial^2 y / \partial x^2 = (4x^2/b^4 - 2/b^2)y$ while $(1/v^2)\partial^2 y / \partial t^2 = (4v^2 t^2/b^4 - 2/b^2)y$, and the two agree only on the lines $x = \pm vt$, so D fails. A is tempting to reject because it is a product that does not travel, but it is a standing wave: $\partial^2 y / \partial x^2 = -k^2 y$ and $(1/v^2)\partial^2 y / \partial t^2 = -(\omega^2/v^2)y = -k^2 y$, so it satisfies the equation. B is not sinusoidal, and a candidate who has met only harmonic waves may reject it, but any twice-differentiable function of $x + vt$ is a solution; this one is a pulse moving in the $-x$ direction. C is the sum of two travelling-wave solutions, and because the wave equation is linear any sum of solutions is a solution; it equals $2A \sin(kx) \cos(\omega t)$, another standing wave. The item rewards testing each function against the equation rather than judging it by its appearance.

U2

3 MULTIPLE CHOICE · DEMANDING · UPIII-04.09 **[1]**
CORE COURSE

A Galilean beam expander widens a collimated laser beam of diameter 2.00 mm. The beam passes first through a thin diverging lens of focal length -25.0 mm and then through a thin converging lens of focal length $+100 \text{ mm}$ on the same axis. Which lens separation makes the beam emerge collimated, and what is its diameter then?

KEY C Separation 75.0 mm; diameter 8.00 mm

The diverging lens spreads the collimated beam so that it appears to come from that lens's focal point, a virtual point source 25.0 mm in front of it. The converging lens collimates light from a point source only if the source sits at its own focal point, 100 mm in front of it, so the lenses must be $100 \text{ mm} - 25.0 \text{ mm} = 75.0 \text{ mm}$ apart; similar triangles from the virtual source then give a diameter of $2.00 \text{ mm} \times 100 \text{ mm} / 25.0 \text{ mm} = 8.00 \text{ mm}$. A adds the magnitudes of the focal lengths, which is the separation of a Keplerian expander made from two converging lenses; with a diverging first lens the virtual source would then be 150 mm from the converging lens, beyond its focal point, and the beam would leave converging. B has the right separation but inverts the ratio of the focal lengths, as though the light ran backwards through the system. D assumes that a beam which is collimated going in and coming out keeps its width, forgetting that the geometry which fixes the separation also scales the beam by $f_2/|f_1| = 4$.

U3

4

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-05.09

[1]

Sunlight is reflected from the calm surface of a lake, of refractive index 1.33, towards an angler whose sunglasses have polarising lenses with vertical transmission axes. At what angle below the horizontal must the angler look at the water for the reflected glare to be completely polarised, so that the sunglasses block it entirely?

KEY **B 36.9°**

Reflected light is completely polarised, parallel to the surface, when the angle of incidence is Brewster's angle, $\tan \theta_B = 1.33$, so $\theta_B = 53.1^\circ$ measured from the normal to the water, which is vertical. The reflected ray therefore rises at $90^\circ - 53.1^\circ = 36.9^\circ$ above the horizontal, and the angler looks down along it at 36.9° below the horizontal; the horizontally polarised glare is then perpendicular to the vertical transmission axes and is blocked. A is Brewster's angle itself, reported without converting from the normal to the horizontal. D is the critical angle, $\arcsin(1/1.33) = 48.8^\circ$, which governs light trying to leave the water rather than light reflected from above it, and C is its complement, 41.2° . A check on the key: at Brewster's angle the reflected and refracted rays are perpendicular, and $53.1^\circ + \arcsin(\sin 53.1^\circ/1.33) = 53.1^\circ + 36.9^\circ = 90^\circ$.

U3

5

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-06.05

[1]

A sample of argon (molar mass 39.9 g/mol) is at 320 K. At what temperature would a sample of helium (molar mass 4.00 g/mol) have the same root-mean-square atomic speed? Treat both as ideal monatomic gases.

KEY **A 32.1 K**

Since $v_{rms} = \sqrt{3RT/M}$, equal speeds require equal T/M : $T_{He} = 320\text{ K} \times 4.00/39.9 = 32.1\text{ K}$. C is the answer of a candidate who remembers that temperature fixes the mean translational kinetic energy per atom and slides from equal kinetic energy to equal speed; at a common temperature the lighter helium atoms move faster, so helium must be colder to match argon. D inverts the mass ratio, heating the lighter gas when it must be cooled. B scales the temperature by the square root of the mass ratio, which would be right only if v_{rms} were proportional to $T/\sqrt{M}$ rather than to $\sqrt{T/M}$.

U3

6

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPIII-07.08

[1]

A rigid, thermally insulated container is divided by a partition into two equal halves. One half holds 1.00 mol of an ideal gas and the other is evacuated. The partition is punctured and the gas fills the whole container. Take $R = 8.314\text{ J/(mol}\cdot\text{K)}$. Which statement is correct?

KEY **C The internal energy of the gas is unchanged, and its entropy rises by 5.76 J/K.**

In a free expansion the gas pushes against nothing, so $W = 0$; with $Q = 0$ as well, $\Delta U = 0$ and an ideal gas keeps its temperature. Entropy is a state function, so ΔS can be found along any reversible path between the same end states, here an isothermal doubling of volume: $\Delta S = nR \ln 2 = 5.76\text{ J/K}$. A applies $\Delta S = \int dQ/T$ to the actual process, but that integral gives the entropy change only along a reversible path, and a free expansion is irreversible. B imports the result for a reversible adiabatic expansion, in which the gas does work on a piston; expanding into a vacuum, there is nothing to do work on. D gets the gas right and then concludes that insulation keeps the entropy of the universe constant. Insulation means only that the surroundings' entropy is unchanged, so the universe gains the full 5.76 J/K, which is exactly what makes the process irreversible.

U1

7

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-08.03

[1]

A bunch of protons travels at constant speed $0.800c$ along a straight section of a linear accelerator, between two beam-position monitors that are fixed in the laboratory 36.0 m apart. Take $c = 3.00 \times 10^8\text{ m/s}$. Which pair gives the distance between the monitors and the time the bunch takes to travel from one to the other, both as measured in the rest frame of the bunch?

KEY **A 21.6 m and 90.0 ns**

The monitors are at rest in the laboratory, so 36.0 m is the proper length, and the bunch measures it contracted by $\gamma = 1/\sqrt{1 - 0.800^2} = 5/3$ to 21.6 m . In the laboratory the trip takes $36.0\text{ m}/(2.40 \times 10^8\text{ m/s}) = 150\text{ ns}$, timed by two separated clocks; a clock riding with the bunch is present at both events, so it records the proper time, $150\text{ ns}/\gamma = 90.0\text{ ns}$. The pair is self-consistent: in the bunch frame the second monitor approaches at $2.40 \times 10^8\text{ m/s}$ and covers 21.6 m in 90.0 ns . B pairs the proper length with the proper time, but 36.0 m at $0.800c$ takes 150 ns , so the two cannot hold in one frame. C contracts the length but keeps the laboratory time. D is self-consistent, since 60.0 m at $0.800c$ takes 250 ns , and is the answer of a candidate who multiplied by γ throughout. The remedy is to name the proper quantity first: a proper length is the longest length any inertial frame measures, and a proper time the shortest time between the two events.

U1

8

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-09.02

[1]

Light of wavelength 400 nm ejects photoelectrons from a metal surface whose work function is 2.15 eV . The wavelength is then changed to 200 nm while the intensity is kept the same. Take $hc = 1240\text{ eV}\cdot\text{nm}$. What is the stopping potential with the 200 nm light?

KEY **C 4.05 V**

Each photon now carries $hc/\lambda = 1240\text{ eV}\cdot\text{nm}/200\text{ nm} = 6.20\text{ eV}$, and the most energetic photoelectron leaves with $6.20\text{ eV} - 2.15\text{ eV} = 4.05\text{ eV}$, so the stopping potential is 4.05 V . A is the stopping potential for the 400 nm light, from $3.10\text{ eV} - 2.15\text{ eV} = 0.95\text{ eV}$, chosen by a candidate who reasons that an unchanged intensity leaves the electron energies unchanged; intensity sets how many photoelectrons are emitted, not their maximum energy. B doubles the old stopping potential because the photon energy doubled, forgetting that the work function is subtracted once from each photon's energy rather than scaling with it. D is the photon energy itself, with the work function left out altogether.

U3

9

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-10.04

[1]

An electron in a hydrogen atom has orbital angular momentum quantum number $l = 3$. What is the smallest angle that its orbital angular momentum vector can make with the z -axis?

KEY **B 30.0°**

The magnitude is $|L| = \sqrt{l(l+1)}\hbar = \sqrt{12}\hbar$ and the largest z -component is $L_z = l\hbar = 3\hbar$, so $\cos\theta = 3/\sqrt{12}$ and $\theta = 30.0^\circ$. A is the classical picture, and the answer of a candidate who takes $|L| = l\hbar$; because $\sqrt{l(l+1)}$ always exceeds l , the vector can never lie along the axis, which is the uncertainty principle forbidding L_x and L_y from both vanishing when L_z has a definite value. D takes the inverse sine of the same ratio instead of the inverse cosine. C is the angle for a spin- $1/2$ vector, from $\cos\theta = (1/2)/(\sqrt{3}/2)$, a memorised number that belongs to $s = 1/2$ rather than to $l = 3$.

U1

Rest energies: proton 938.3 MeV, neutron 939.6 MeV, electron 0.511 MeV, muon 105.7 MeV, charged pion 139.6 MeV; neutrino masses are negligible. Which of the following processes is allowed by every conservation law?

KEY D $\pi^+ \rightarrow \mu^+ + \nu_\mu$

D conserves charge (+1 $\rightarrow$ +1), baryon number (0 $\rightarrow$ 0) and muon lepton number (0 $\rightarrow$ -1 + 1), and the pion's 139.6 MeV rest energy exceeds the muon's 105.7 MeV, so energy allows it too; it is the pion's dominant decay. A has the right particles apart from the neutrino: lepton number goes from 0 to +2, and the real decay emits an electron antineutrino. B is what happens inside proton-rich nuclei, which is why it tempts, but a free proton at rest has 938.3 MeV, less than the 940.1 MeV of the products, so energy conservation forbids it; inside a nucleus the difference in binding energy pays. C conserves charge, energy and total lepton number, but it turns a muon-type lepton into an electron-type lepton, violating muon and electron lepton numbers separately, and it has never been observed.

U1

An aerial-mapping camera photographs a field from a height of 120 m. Its lens is a thin converging lens of focal length 8.80 mm and aperture diameter 3.14 mm, and its image sensor has square pixels 2.40 μm wide. Take the wavelength of the light as 550 nm in air. To reduce reflections, the lens glass, of refractive index 1.52, carries a single coating of magnesium fluoride, of refractive index 1.38, designed for 550 nm at normal incidence.

(a) **CALCULATE** · U3

[2]

Calculate how far behind the lens the sensor must be placed to focus the field, and how far this is from the lens's focal plane.

- $1/s' = 1/f - 1/s = 1/(8.80 \times 10^{-3} \text{ m}) - 1/(120 \text{ m})$, so $s' = 8.80065 \times 10^{-3} \text{ m}$ behind the lens
- $s' - f = 6.5 \times 10^{-7} \text{ m} = 0.65 \mu\text{m}$, about a quarter of a pixel width, so the field is effectively at infinity and the sensor sits in the focal plane

Mixing millimetres with metres, or writing $1/s' = 1/f + 1/s$, puts the image well away from the focal plane; the tiny value of $s' - f$ is itself the check that the units and the sign are right.

(b) **CALCULATE** · U3

[2]

Calculate the width of the strip of ground that is imaged onto one pixel.

- The magnification has magnitude $s'/s = 8.8006 \times 10^{-3} \text{ m}/120 \text{ m} = 7.334 \times 10^{-5}$
- Ground width = $2.40 \times 10^{-6} \text{ m}/(7.334 \times 10^{-5}) = 3.27 \times 10^{-2} \text{ m} = 32.7 \text{ mm}$

Multiplying the pixel width by the magnification instead of dividing gives a ground width of order 10^{-10} m ; the image is smaller than the object, so the strip of ground must be far wider than the pixel.

(c) **JUSTIFY** · U3

[3]

Justify whether diffraction at the lens aperture or the size of the pixels sets the smallest ground detail this camera can resolve at 550 nm.

- Rayleigh criterion for a circular aperture: $\theta_{\min} = 1.22\lambda/D = 1.22 \times 550 \times 10^{-9} \text{ m}/(3.14 \times 10^{-3} \text{ m}) = 2.14 \times 10^{-4} \text{ rad}$
- On the ground this is $120 \text{ m} \times 2.14 \times 10^{-4} \text{ rad} = 25.6 \text{ mm}$; equivalently $f\theta_{\min} = 1.88 \mu\text{m}$ on the sensor
- One pixel covers 32.7 mm of ground (2.40 μm on the sensor), coarser than the 25.6 mm diffraction limit, so the pixels limit the resolution: two features need at least two pixels, about 65 mm, to be separated

Comparing an angle with a pixel width directly is the error; both limits must be put on the ground, or both on the sensor, before they can be compared.

(d) DETERMINE · U2 [2]

Determine the minimum thickness of the coating that minimises the reflection of 550 nm light at normal incidence.

1 Both reflections, air to coating (1.00 to 1.38) and coating to glass (1.38 to 1.52), go from lower to higher index, so both waves are shifted by π and cancellation needs a path difference $2n_c t = \lambda/2$

2 $t = \lambda/(4n_c) = 550 \text{ nm}/(4 \times 1.38) = 99.6 \text{ nm}$

Giving only one reflection a phase shift leads to $2n_c t = \lambda$ and $t = 199 \text{ nm}$, the thickness that maximises reflection; the index sequence has to be checked at both surfaces.

(e) EXPLAIN · U1 [2]

When white light is reflected from the coated lens, the reflection looks purple. Explain why.

1 The path difference $2n_c t = 275 \text{ nm}$ is half of 550 nm, but only 0.39 of a wavelength at 700 nm and 0.69 of a wavelength at 400 nm, so the two reflected waves cancel best near the middle of the visible spectrum and only partly at its ends

2 The reflected light is therefore depleted in green and yellow and relatively strong at the red and violet ends, which together look purple

Attributing the colour to dispersion in the coating misses the mechanism: the coating is a quarter-wave layer for one wavelength only, and it is the phase condition that changes across the spectrum.

12 CORE COURSE

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[12]

The pendulum of a longcase clock is a light rigid rod carrying a solid brass sphere of mass 2.30 kg and radius $r = 0.0400 \text{ m}$, whose centre is $L = 0.990 \text{ m}$ below the pivot. The moment of inertia of a solid sphere about an axis through its centre is $(2/5)mr^2$. Take $g = 9.81 \text{ m/s}^2$. The clock's escapement gives the pendulum a small push once each swing to keep its amplitude at 4.00° .

(a) DERIVE · U2 [3]

Derive, from the rotational form of Newton's second law, an expression for the period of small oscillations of this pendulum in terms of L , r and g , stating the approximation you make.

1 Parallel-axis theorem: the moment of inertia about the pivot is $I = (2/5)mr^2 + mL^2$

2 Gravity exerts a torque $-mgL \sin \theta$ about the pivot, so $I d^2\theta/dt^2 = -mgL \sin \theta \approx -mgL\theta$, using $\sin \theta \approx \theta$ (θ in radians) for small swings

3 This is simple harmonic motion with $\omega^2 = mgL/I$, so $T_0 = 2\pi\sqrt{(I/(mgL))} = 2\pi\sqrt{((L^2 + 2r^2/5)/(gL))}$

Treating the bob as a point mass gives $I = mL^2$ and the simple-pendulum period; the sphere also turns about its own centre as it swings, and the parallel-axis term is the step that captures it.

(b) CALCULATE · U3 [2]

Calculate the period, and the fraction by which it exceeds the period of a simple pendulum of length 0.990 m.

1 $T_0 = 2\pi\sqrt{((0.990^2 + 0.4 \times 0.0400^2) \text{ m}^2)/(9.81 \text{ m/s}^2 \times 0.990 \text{ m})} = 1.9967 \text{ s}$, against $2\pi\sqrt{(0.990 \text{ m}/9.81 \text{ m/s}^2)} = 1.9960 \text{ s}$ for a simple pendulum

2 Fractional excess $= \sqrt{(1 + 2r^2/(5L^2))} - 1 \approx r^2/(5L^2) = 3.3 \times 10^{-4}$, about 28 s per day

Rounding both periods to 2.00 s hides the whole effect; the difference has to come from the ratio $r^2/(5L^2)$, not from subtracting rounded periods.

(c) DETERMINE · U4

[3]

With the escapement disconnected, the amplitude falls from 4.00° to 2.00° in 12.0 minutes. Determine the average power the escapement must supply to keep the amplitude at 4.00° , stating the assumption about the damping that your method requires.

- 1 For light damping proportional to angular velocity the amplitude decays exponentially, so the energy, proportional to the square of the amplitude, falls by a factor of 4 with time constant $\tau_E = 720 \text{ s}/\ln 4 = 519 \text{ s}$
- 2 Energy of the swing at 4.00° (0.0698 rad): $E = \frac{1}{2}I\omega_0^2\theta_0^2 = \frac{1}{2}mgL\theta_0^2 = \frac{1}{2} \times 2.30 \text{ kg} \times 9.81 \text{ m/s}^2 \times 0.990 \text{ m} \times (0.0698 \text{ rad})^2 = 5.44 \times 10^{-2} \text{ J}$
- 3 $P = E/\tau_E = 5.44 \times 10^{-2} \text{ J}/519 \text{ s} = 1.05 \times 10^{-4} \text{ W}$, replacing the 0.38% of the energy lost in each cycle

Using the amplitude ratio of 2 instead of the energy ratio of 4 doubles the time constant and halves the power; the energy goes as the square of the amplitude.

(d) CALCULATE · U3

[2]

For a finite amplitude θ_0 the period is $T \approx T_0(1 + \theta_0^2/16)$, with θ_0 in radians. The clock is adjusted to keep perfect time at an amplitude of 4.00° , but friction in its gear train lets the amplitude wander between 3.50° and 4.50° . Calculate the largest time the clock gains or loses in one day as a result.

- 1 Relative to 4.00° (0.06981 rad), the period at 4.50° (0.07854 rad) is longer by the fraction $(0.07854^2 - 0.06981^2)/16 = 8.09 \times 10^{-5}$, and at 3.50° (0.06109 rad) it is shorter by 7.14×10^{-5}
- 2 The largest error is $8.09 \times 10^{-5} \times 86\,400 \text{ s} = 7.0 \text{ s}$ lost per day at 4.50° (the clock gains 6.2 s per day at 3.50°)

Evaluating the whole correction at 4.50° , 33 s per day, ignores that the clock was regulated at 4.00° ; only the change in amplitude matters, and it enters as a difference of squares.

(e) EVALUATE · U1

[2]

The clock must keep time to within 1 s per day. Evaluate which idealisation of the simple-pendulum model, the point-mass bob or the small-angle limit, limits this clock's timekeeping.

- 1 The point-mass error, 3.3×10^{-4} or about 28 s per day, is constant, so it is removed once when the clock is regulated by raising or lowering the bob, and it does not limit the timekeeping
- 2 The amplitude-dependent error cannot be regulated out: it varies by up to 7 s per day as the amplitude wanders, seven times the tolerance, so the small-angle idealisation is the one that fails, and the remedy is a steadier or smaller amplitude rather than a smaller bob

Ranking the two corrections by size alone picks the bob; the discriminating point is that a constant error is calibrated away, while a varying one is not.

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CORE
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[10]

A pair of extraction fans of the same model stand 6.0 m apart on a flat roof. Each has 8 blades and radiates a steady tone at its blade-passing frequency, the number of blades multiplied by the fan's rotation rate in revolutions per second, and the controller of fan A shows a rotation rate of 24.50 revolutions per second. With both fans running, a technician's sound level meter, filtered to reject everything except a narrow band around this tone and quick enough to follow any change in the tone's amplitude, records a level that rises and falls smoothly and regularly between a maximum of 79.5 dB and a minimum of 71.9 dB, with successive maxima 5.00 s apart. The meter responds to the pressure at its microphone and reports the sound intensity level of a single plane wave with the same pressure amplitude. Treat the fans as small sources in open air and ignore reflections. Take the reference intensity as $1.00 \times 10^{-12} \text{ W/m}^2$, the density of air as 1.20 kg/m^3 , the speed of sound as 343 m/s and atmospheric pressure as $1.01 \times 10^5 \text{ Pa}$.

(a) CALCULATE · U3

[2]

Calculate the pressure amplitude of the sound at the meter when the level is at a maximum and when it is at a minimum.

- 1 At the maximum, $I = 1.00 \times 10^{-12} \text{ W/m}^2 \times 10^{(79.5/10)} = 8.91 \times 10^{-5} \text{ W/m}^2$, so $p_0 = \sqrt{2\rho v I} = \sqrt{(2 \times 1.20 \text{ kg/m}^3 \times 343 \text{ m/s} \times 8.91 \times 10^{-5} \text{ W/m}^2)} = 0.271 \text{ Pa}$
- 2 At the minimum, $I = 1.00 \times 10^{-12} \text{ W/m}^2 \times 10^{(71.9/10)} = 1.55 \times 10^{-5} \text{ W/m}^2$, so $p_0 = \sqrt{2\rho v I} = 0.113 \text{ Pa}$, a pressure-amplitude ratio of 2.40 between maximum and minimum

The 7.6 dB swing is an intensity ratio of 5.75 but a pressure-amplitude ratio of only $\sqrt{5.75} = 2.40$; candidates who scale pressure by $10^{(\beta/10)}$ instead of $10^{(\beta/20)}$ carry an amplitude ratio 2.4 times too large into (c).

(b) EXPLAIN · U1

[2]

The sound from the two fans reaches the meter travelling in different directions. Explain why the excess pressures of the two waves at the meter can be added as numbers, whereas the displacements of the air in the two waves cannot, and why the superposition principle holds for these waves.

- 1 Excess pressure is a scalar, so the two waves' pressures add algebraically at each instant; sound is longitudinal, so each wave displaces the air along its own direction of travel, and displacements arriving from different directions add as vectors, which is why a pressure-sensing meter, like the ear, responds to the summed pressure
- 2 Superposition needs a linear medium: even the largest pressure amplitude, 0.271 Pa, is only 2.7×10^{-6} of atmospheric pressure, so the air responds linearly and each wave passes through the other unchanged

Candidates who add displacement amplitudes as numbers have assumed, without noticing, that the waves travel along the same line; and asserting that sound waves superpose earns nothing without the comparison with atmospheric pressure that makes the medium linear.

(c) DETERMINE · U2

[4]

Determine the sound intensity level that the louder fan alone would produce at the meter, the level that the quieter fan alone would produce, and the difference between the two fans' rotation rates.

- 1 The two tones differ slightly in frequency, so their relative phase advances steadily: the pressures add in step at a maximum and in opposition at a minimum, $p_{\text{max}} = p_1 + p_2$ and $p_{\text{min}} = p_1 - p_2$
- 2 $p_1 = (0.271 \text{ Pa} + 0.113 \text{ Pa})/2 = 0.192 \text{ Pa}$ and $p_2 = (0.271 \text{ Pa} - 0.113 \text{ Pa})/2 = 0.0790 \text{ Pa}$, so the quieter fan's pressure amplitude is about 0.41 of the louder fan's
- 3 $I_1 = p_1^2/(2\rho v) = 4.47 \times 10^{-5} \text{ W/m}^2$, a level of 76.5 dB, and $I_2 = p_2^2/(2\rho v) = 7.58 \times 10^{-6} \text{ W/m}^2$, a level of 68.8 dB
- 4 The level peaks once for each 2π of relative phase, so the blade-passing frequencies differ by $1/(5.00 \text{ s}) = 0.200 \text{ Hz}$; dividing by the 8 blades, the rotation rates differ by 0.0250 revolutions per second (the beats alone cannot show which fan is faster)

The two amplitudes come from the half-sum and half-difference of the pressure amplitudes. Averaging the readings in decibels (75.7 dB), halving the sum and the difference of the intensities (77.2 dB and 75.7 dB), or reading the 71.9 dB minimum as the quieter fan alone gives neither fan's level: it is amplitudes, not intensities, that add, and at a minimum the quieter fan's sound is cancelling part of the louder fan's. Candidates who stop at 0.200 Hz have found the difference in blade-passing frequency, which is 8 times the difference in rotation rate.

The technician adjusts the controller of fan B until the level at the meter stops rising and falling. Describe how the steady level now depends on the fans and on where the meter is placed, giving the range of steady levels the meter could read at its present position, and the change in the difference between its distances from the two fans that takes the level from its highest to its lowest value, ignoring any change in each fan's own amplitude.

- 1 With equal frequencies the two tones keep a fixed phase difference, set by the relative angular positions of the two fans' blades and by the difference between the meter's distances from the fans, so the level is steady at some value between 71.9 dB (antiphase) and 79.5 dB (in phase), not necessarily at the 77.2 dB of the two intensities added
- 2 Changing the difference in distance by half a wavelength, $v/(2f) = 343 \text{ m/s}/(2 \times 196 \text{ Hz}) = 0.875 \text{ m}$, turns in-phase arrival into antiphase and takes the level from its highest to its lowest value, so the roof now carries a fixed pattern of loud and quiet places, and the 77.2 dB sum of intensities is only the average over them

Candidates who expect synchronised fans to be quieter everywhere, or the steady level to be the sum of the intensities, have missed that equal frequencies make the two sources coherent: the beat in time becomes an interference pattern in space.

In the laboratory frame S , three events are recorded on the x -axis. Event A: a detector at $x = 0$ fires at $t = 0$. Event B: a detector at $x = 900$ m fires at $t = 2.00 \mu\text{s}$. Event C: the same detector at $x = 900$ m fires again at $t = 4.00 \mu\text{s}$. Frame S' moves at $+0.800c$ along the x -axis relative to S , and its origin coincides with that of S at event A. Take $c = 3.00 \times 10^8$ m/s.

(a) CALCULATE · U3

[2]

Calculate the invariant interval $s^2 = (c\Delta t)^2 - (\Delta x)^2$ between events A and B, and classify the separation.

1 $c\Delta t = 3.00 \times 10^8 \text{ m/s} \times 2.00 \times 10^{-6} \text{ s} = 600 \text{ m}$, so $s^2 = (600 \text{ m})^2 - (900 \text{ m})^2 = -4.50 \times 10^5 \text{ m}^2$

2 $s^2 < 0$, so the separation is spacelike

Converting the time interval to a length, $c\Delta t$, before comparing it with Δx is the step that makes the classification visible; candidates who compare $2.00 \mu\text{s}$ with 900 m directly have nothing to compare.

(b) DETERMINE · U3

[3]

Determine the time interval $\Delta t'$ and the spatial separation $\Delta x'$ between A and B measured in S' , and state which event occurs first in S' .

1 $\gamma = 1/\sqrt{1 - 0.800^2} = 1.667$

2 $\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 1.667 \times (2.00 \mu\text{s} - 2.40 \mu\text{s}) = -0.667 \mu\text{s}$, so B occurs before A in S'

3 $\Delta x' = \gamma(\Delta x - v\Delta t) = 1.667 \times (900 \text{ m} - 480 \text{ m}) = 700 \text{ m}$, consistent with $(200 \text{ m})^2 - (700 \text{ m})^2 = -4.50 \times 10^5 \text{ m}^2$

Writing $\Delta t' = \gamma\Delta t$ is the error this part is set on: neither interval is a proper time, because the events occur at different places in both frames, so only the full Lorentz transformation will do.

(c) JUSTIFY · U1

[2]

Justify the claim that event A cannot have caused event B, giving two separate arguments.

1 A signal from A would have to cover 900 m in $2.00 \mu\text{s}$, a speed of $4.50 \times 10^8 \text{ m/s} = 1.5c$, faster than light; equivalently, the interval is spacelike

2 The time order of A and B depends on the frame (B precedes A in S'), whereas a cause must precede its effect in every inertial frame

Saying only that B comes first in S' does not yet show why that matters. The two arguments are one fact seen from the interval and from the transformation, and both must be stated.

(d) DETERMINE · U2

[2]

Determine the velocity, relative to S , of the inertial frame in which events A and B are simultaneous.

1 Setting $\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = 0$ gives $v = c^2\Delta t/\Delta x$

2 $v = c \times (600 \text{ m}/900 \text{ m}) = 0.667c = 2.00 \times 10^8 \text{ m/s}$, in the $+x$ direction

Only a spacelike pair has such a frame, since $c^2\Delta t/\Delta x$ is less than c only when $c\Delta t < \Delta x$; a candidate should notice that the result is automatically below c here.

(e) PREDICT · U1

[2]

Predict the order of events A and C in S' and in every other inertial frame, giving the reason.

1 For A and C, $c\Delta t = 1200 \text{ m}$ exceeds $\Delta x = 900 \text{ m}$, so $s^2 = +6.30 \times 10^5 \text{ m}^2$ and the separation is timelike

2 C follows A in every inertial frame: in S' , $\Delta t' = 1.667 \times (4.00 \mu\text{s} - 2.40 \mu\text{s}) = +2.67 \mu\text{s}$, and no $v < c$ can make $\Delta t - v\Delta x/c^2$ negative when $c\Delta t > \Delta x$

The contrast with (b) is the point. Candidates who conclude that relativity makes every time order frame-dependent have missed that the order of timelike pairs, and so causal order, is the same in all frames.

A radioisotope power source for a deep-space probe contains 3.50 kg of plutonium-238 (^{238}Pu , $Z = 94$), which decays by alpha emission to uranium-234 with a half-life of 87.7 years. Atomic masses: ^{238}Pu 238.049 560 u, ^{234}U 234.040 952 u, ^4He 4.002 603 u. Take $1 \text{ u} = 931.494 \text{ MeV}/c^2$, $1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}$, $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$, $1 \text{ year} = 3.156 \times 10^7 \text{ s}$, and the molar mass of ^{238}Pu as 238.05 g/mol.

(a) STATE · U1

[1]

State the decay equation, showing the nucleon number and proton number of every particle.

1 $^{238}_{94}\text{Pu} \rightarrow ^{234}_{92}\text{U} + ^4_2\text{He}$, with nucleon numbers $238 = 234 + 4$ and proton numbers $94 = 92 + 2$

Writing the daughter with proton number 94 or 93 shows that the candidate has balanced nucleons but not charge.

(b) EXPLAIN · U1

[2]

Explain why the Q value of this decay can be calculated directly from atomic masses, even though the decay is a nuclear process.

- 1 An atomic mass includes the atom's electrons: the ^{238}Pu atom carries 94 electron masses, and the products carry 92 in the uranium atom plus 2 in the helium atom, also 94
- 2 The electron masses therefore cancel in the mass difference, and the change in the atoms' total electron binding energy, at most a few tens of keV, is under 1% of a Q value of several MeV

This is a count, not a rule: in β^+ decay the same count leaves two electron masses uncanceled, and candidates who have memorised the alpha case as a rule are caught out there.

(c) CALCULATE · U3

[2]

Calculate the Q value of the decay in MeV.

1 $\Delta m = 238.049\,560 \text{ u} - 234.040\,952 \text{ u} - 4.002\,603 \text{ u} = 0.006\,005 \text{ u}$

2 $Q = 0.006\,005 \text{ u} \times 931.494 \text{ MeV/u} = 5.59 \text{ MeV}$

Rounding the masses before subtracting destroys the answer: the difference appears in the sixth significant figure, so every digit given must be carried.

(d) SHOW THAT · U2

[2]

Show that, for a parent nucleus at rest, the alpha particle carries about 98% of the Q value as kinetic energy.

- 1 Momentum conservation gives the alpha particle and the daughter equal and opposite momenta, so with $K = p^2/2m$ their kinetic energies are inversely proportional to their masses
- 2 $K_\alpha = Q \, m_{\text{U}}/(m_{\text{U}} + m_\alpha) = 5.59 \text{ MeV} \times 234.04/238.04 = 5.50 \text{ MeV}$, which is 98.3% of Q

Assuming the daughter takes no energy, or that Q is shared equally, are the errors; the non-relativistic treatment is justified because 5.5 MeV is tiny beside the alpha particle's rest energy of about 3.7 GeV.

(e) CALCULATE · U3

[3]

Calculate the initial thermal power of the source, assuming that all of the decay energy is absorbed within it.

1 $N = (3500 \text{ g}/238.05 \text{ g/mol}) \times 6.022 \times 10^{23} \text{ mol}^{-1} = 8.85 \times 10^{24} \text{ nuclei}$

2 $\lambda = \ln 2/(87.7 \times 3.156 \times 10^7 \text{ s}) = 2.50 \times 10^{-10} \text{ s}^{-1}$, so the activity is $A = \lambda N = 2.22 \times 10^{15} \text{ Bq}$

3 $P = AQ = 2.22 \times 10^{15} \text{ s}^{-1} \times 5.59 \text{ MeV} \times 1.602 \times 10^{-13} \text{ J/MeV} = 1.99 \times 10^3 \text{ W}$

Using K_α instead of Q undercounts by the recoil energy of the daughter, which is also absorbed in the source; the other common slip is leaving the half-life in years when the power is wanted in watts.

(f) DETERMINE · U3 [2]

Determine how long after the source is made its thermal power falls to 1500 W.

1 $P(t) = P_0 e^{(-\lambda t)}$, so $t = (T_{1/2}/\ln 2) \ln(P_0/P) = 87.7 \text{ years} \times \log_2(1987 \text{ W}/1500 \text{ W})$

2 $t = 87.7 \text{ years} \times 0.406 = 35.6 \text{ years}$

Assuming a linear decline at the initial rate gives 31 years; the power follows the exponential decay of the activity, and a fall to 75% of the initial value takes 0.406 half-lives.

16
CORE
COURSE

STRUCTURED · DEMANDING · UPIII-07.04 · UPIII-07.05 · UPIII-07.06 · UPIII-07.09

[13]

An engine takes 0.500 mol of an ideal diatomic gas ($C_V = 5R/2$, $\gamma = 1.40$) around a three-stage cycle. A $\rightarrow$ B: reversible isothermal compression at 300 K, in contact with a cold reservoir at 300 K, until the volume is one quarter of its value at A. B $\rightarrow$ C: heating at constant volume, by contact with a hot reservoir at 600 K, until the gas reaches temperature T_C , when contact is broken. C $\rightarrow$ A: reversible adiabatic expansion back to state A. Take $R = 8.314 \text{ J}/(\text{mol}\cdot\text{K})$.

(a) SHOW THAT · U2 [2]

Show that $T_C = 522 \text{ K}$.

1 Along the reversible adiabat C $\rightarrow$ A, $TV^{(\gamma-1)}$ is constant, and $V_C = V_B = V_A/4$

2 $T_C = T_A(V_A/V_C)^{(\gamma-1)} = 300 \text{ K} \times 4^{0.40} = 522 \text{ K}$

Using the exponent γ instead of $\gamma - 1$, as if temperature followed the pV^γ law, gives $300 \text{ K} \times 4^{1.40} = 2090 \text{ K}$; this part is set to catch that substitution.

(b) CALCULATE · U3 [4]

Calculate the heat Q absorbed by the gas and the work W done by the gas in each of the three stages, and the net work done by the gas per cycle.

1 A $\rightarrow$ B: $\Delta U = 0$, so $Q = W = nRT \ln(V_B/V_A) = 0.500 \text{ mol} \times 8.314 \text{ J}/(\text{mol}\cdot\text{K}) \times 300 \text{ K} \times \ln(1/4) = -1.73 \times 10^3 \text{ J}$

2 B $\rightarrow$ C: $W = 0$, so $Q = nC_V(T_C - T_B) = 0.500 \text{ mol} \times 20.79 \text{ J}/(\text{mol}\cdot\text{K}) \times 222 \text{ K} = +2.31 \times 10^3 \text{ J}$

3 C $\rightarrow$ A: $Q = 0$, so $W = -\Delta U = nC_V(T_C - T_A) = +2.31 \times 10^3 \text{ J}$

4 Net work = $-1729 \text{ J} + 0 + 2311 \text{ J} = +582 \text{ J}$ per cycle, equal to the net heat absorbed

Sign discipline is what is examined. A candidate who records the isothermal work as positive, or gives the adiabatic stage a heat term, cannot close the energy ledger, which requires the net work to equal $2311 \text{ J} - 1729 \text{ J}$.

(c) COMPARE · U3 [2]

Compare the thermal efficiency of this cycle with the Carnot efficiency of an engine operating between the same two reservoirs.

1 $\eta = W/Q_{\text{in}} = 582 \text{ J}/2311 \text{ J} = 0.252$ (25.2%), where only the B $\rightarrow$ C heat is heat input

2 $\eta_{\text{Carnot}} = 1 - 300 \text{ K}/600 \text{ K} = 0.500$, so the cycle achieves about half of the Carnot bound

Counting $|Q_{AB}|$ as input, or taking 522 K as the hot temperature (which gives 42.6%) when the heat actually comes from a 600 K reservoir, both misstate the bound the engine is measured against.

(d) DETERMINE · U3

[3]

Determine the total entropy change of the gas and the two reservoirs per cycle, and identify the stage in which entropy is generated.

- 1 Over a complete cycle $\Delta S_{\text{gas}} = 0$, because entropy is a state function
- 2 $\Delta S_{\text{hot}} = -2311 \text{ J}/600 \text{ K} = -3.85 \text{ J/K}$ and $\Delta S_{\text{cold}} = +1729 \text{ J}/300 \text{ K} = +5.76 \text{ J/K}$, so $\Delta S_{\text{total}} = +1.91 \text{ J/K}$
- 3 Entropy is generated in $B \rightarrow C$, where heat crosses a finite temperature difference from the 600 K reservoir into gas at 300 K to 522 K; $A \rightarrow B$ and $C \rightarrow A$ are reversible

Candidates often calculate the gas's entropy gain in $B \rightarrow C$ (+5.76 J/K) and call it the entropy generated. The generation is that gain minus the hot reservoir's loss, and it happens only where heat flows across a temperature gap.

(e) SHOW THAT · U2

[2]

Show that the work this cycle loses, compared with a Carnot engine that absorbs the same heat Q_{BC} from the 600 K reservoir, equals $T_{\text{cold}} \times S_{\text{gen}}$, where $T_{\text{cold}} = 300 \text{ K}$ and S_{gen} is the entropy generated per cycle.

- 1 Over a cycle $W = Q_{\text{BC}} - |Q_{\text{AB}}|$ and $S_{\text{gen}} = |Q_{\text{AB}}|/T_{\text{cold}} - Q_{\text{BC}}/T_{\text{hot}}$; eliminating $|Q_{\text{AB}}|$ gives $W = Q_{\text{BC}}(1 - T_{\text{cold}}/T_{\text{hot}}) - T_{\text{cold}} S_{\text{gen}}$
- 2 Numerically, $\eta_{\text{Carnot}} Q_{\text{BC}} - W = 1155 \text{ J} - 582 \text{ J} = 574 \text{ J}$, and $T_{\text{cold}} S_{\text{gen}} = 300 \text{ K} \times 1.91 \text{ J/K} = 574 \text{ J}$

This is the capstone of the entropy audit: each joule per kelvin of entropy generated costs T_{cold} joules of work. Showing only the numerical agreement, without the general algebra, earns the second mark but not the first.

In a low-energy electron diffraction experiment, electrons accelerated from rest through a potential difference V strike a clean copper crystal surface along its normal. The rows of surface atoms, a distance a apart, act as a reflection grating: first-order diffracted beams leave the surface at an angle ϕ to the normal, where $a \sin \phi = \lambda$ and λ is the de Broglie wavelength of the electrons. The angle ϕ is measured at several accelerating potential differences, and the energy scale of the electron gun is calibrated to $\pm 2\%$. Take $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$, $m_e = 9.109 \times 10^{-31} \text{ kg}$ and $e = 1.602 \times 10^{-19} \text{ C}$. In bulk copper the atoms of a plane of this orientation form a square array of side 0.2556 nm .

(a) SHOW THAT · U2

[2]

Show that the model predicts a straight line through the origin when $\sin \phi$ is plotted against $1/\sqrt{V}$, and give its gradient in terms of h , m_e , e and a .

- 1 The kinetic energy $eV = p^2/(2m_e)$ gives the de Broglie wavelength $\lambda = h/p = h/\sqrt{2m_e eV}$
- 2 Then $\sin \phi = \lambda/a = [h/(\sqrt{2m_e e})] \times (1/\sqrt{V})$, a straight line through the origin with gradient $G = h/(\sqrt{2m_e e})$

Using the photon relation $\lambda = hc/(eV)$ gives $\sin \phi$ proportional to $1/V$ and a curved plot; for a massive particle at these energies the wavelength goes as the inverse square root of the energy.

(b) DETERMINE · U4

[3]

Using all of the readings, determine the gradient of the graph of $\sin \phi$ against $1/\sqrt{V}$.

- 1 Values of $1/\sqrt{V}$ (in $\text{V}^{-1/2}$) and $\sin \phi$: 0.1414 and 0.681, 0.1155 and 0.550, 0.1000 and 0.483, 0.0894 and 0.426, 0.0816 and 0.394, 0.0756 and 0.360, 0.0707 and 0.340
- 2 A best-fit straight line through all seven points and the origin (or a least-squares fit), not a gradient taken from a single reading
- 3 Gradient $G = 4.80 \text{ V}^{-1/2}$ (least-squares line through the origin $4.798 \text{ V}^{-1/2}$; accept $4.75 \text{ V}^{-1/2}$ to $4.85 \text{ V}^{-1/2}$)

Plotting ϕ in degrees, or $\sin \phi$ against $1/V$, gives a curve whose apparent gradient depends on which readings are chosen; only $\sin \phi$ against $1/\sqrt{V}$ is linear in this model.

(c) CALCULATE · U4

[3]

Calculate the row spacing a , with its uncertainty arising from the calibration of the energy scale.

- 1 $a = h/(G\sqrt{2m_e e})$, with $\sqrt{2m_e e} = \sqrt{2 \times 9.109 \times 10^{-31} \text{ kg} \times 1.602 \times 10^{-19} \text{ C}} = 5.402 \times 10^{-25} \text{ kg}^{1/2} \text{ C}^{1/2}$
- 2 $a = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} / (4.80 \text{ V}^{-1/2} \times 5.402 \times 10^{-25} \text{ kg}^{1/2} \text{ C}^{1/2}) = 2.56 \times 10^{-10} \text{ m} = 0.256 \text{ nm}$ (accept 0.253 nm to 0.258 nm)
- 3 A $\pm 2\%$ error in V changes $1/\sqrt{V}$, and so G and a , by $\pm 1\%$, so $a = 0.256 \pm 0.003 \text{ nm}$

Carrying $\pm 2\%$ straight across to a misses the square root: the wavelength, and so the inferred spacing, depends on $V^{-1/2}$, which halves the relative uncertainty.

(d) EVALUATE · U4

[2]

A calculation predicts that the outermost layer of copper atoms relaxes towards the layer beneath it, reducing the spacing between those two layers by about 2% . Evaluate whether these data support a surface whose atoms keep their bulk arrangement within the layer, and whether this experiment could detect the predicted relaxation.

- 1 The measured $a = 0.256 \pm 0.003 \text{ nm}$ agrees with the bulk value of 0.2556 nm within its uncertainty, so the data support a surface layer whose atoms keep the bulk spacing within the plane
- 2 The predicted relaxation changes only the spacing perpendicular to the surface, which does not enter a $\sin \phi = \lambda$, so the beam angles cannot detect it at any precision; it would have to be sought in how the beam intensities vary with electron energy

The discriminating step is to ask what the measurement is sensitive to. An answer that compares the 2% relaxation with the $\pm 1\%$ uncertainty and calls it detectable has not used the model it was given.

In muonic hydrogen a negative muon, of mass 1.8835×10^{-28} kg, takes the place of the electron and is bound to a proton of mass 1.6726×10^{-27} kg; its energy levels are measured to find the size of the proton. The relative motion obeys the hydrogen-atom Schrödinger equation with the reduced mass $\mu = m_\mu m_p / (m_\mu + m_p)$ in place of the electron mass. For a state with no angular dependence ($l = 0$) the equation reads $-(\hbar^2/2\mu)(1/r^2) d/dr(r^2 d\psi/dr) - (e^2/(4\pi\epsilon_0 r))\psi = E\psi$. Take $\hbar = 1.0546 \times 10^{-34}$ J·s, $e = 1.602 \times 10^{-19}$ C, $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, the proton radius as 0.84 fm and the Bohr radius of ordinary hydrogen as 5.29×10^{-11} m. You may use $\int_0^\infty r^n e^{-\beta r} dr = n!/\beta^{n+1}$.

(a) DERIVE · U2 [5]

By substituting the trial solution $\psi(r) = Ce^{-r/a}$ into the Schrödinger equation, derive expressions for the length a and the energy E , and evaluate both for muonic hydrogen.

- 1 $d\psi/dr = -\psi/a$, so $r^2 d\psi/dr = -(r^2/a)\psi$
- 2 $(1/r^2) d/dr(r^2 d\psi/dr) = (1/a^2 - 2/(ar))\psi$
- 3 Substituting and dividing by ψ : $-(\hbar^2/2\mu)(1/a^2 - 2/(ar)) - e^2/(4\pi\epsilon_0 r) = E$, which must hold for every r , so the $1/r$ terms must cancel: $a = 4\pi\epsilon_0 \hbar^2 / (\mu e^2)$
- 4 The remaining constant terms give $E = -\hbar^2 / (2\mu a^2)$
- 5 $\mu = 1.693 \times 10^{-28}$ kg (185.8 electron masses), $a = 2.85 \times 10^{-13}$ m and $E = -4.05 \times 10^{-16}$ J = -2.53 keV

The step that separates candidates is recognising that an equation true for every r fixes two constants at once, one from the $1/r$ terms and one from the constant terms. Candidates who set $r = a$ and solve a single equation cannot find both.

(b) SHOW THAT · U2 [2]

Show that the normalised wavefunction has $C = 1/\sqrt{\pi a^3}$.

- 1 Normalisation over all space: $\int_0^\infty |\psi|^2 4\pi r^2 dr = 4\pi C^2 \int_0^\infty r^2 e^{-2r/a} dr = 1$
- 2 The integral is $2!/(2/a)^3 = a^3/4$, so $\pi a^3 C^2 = 1$ and $C = 1/\sqrt{\pi a^3}$

Omitting the volume element $4\pi r^2 dr$ and normalising $\int |\psi|^2 dr$ instead gives a constant with the wrong dimensions, which a units check exposes at once.

(c) DETERMINE · U3 [2]

Determine the most probable distance of the muon from the proton, in metres.

- 1 $dP/dr \propto (2r - 2r^2/a)e^{-2r/a} = 0$ gives $r = a$, the maximum ($r = 0$ is a minimum)
- 2 $r = a = 2.85 \times 10^{-13}$ m, about 186 times smaller than the corresponding distance in ordinary hydrogen

Setting $d\psi/dr = 0$ finds no interior maximum at all; the maximum belongs to $P(r)$, not to ψ .

(d) SKETCH · U1 [2]

Sketch the radial probability density $P(r) = 4\pi r^2 |\psi(r)|^2$ against r/a , marking the most probable radius found in (c) and the mean radius $\langle r \rangle = 1.5a$.

- 1 Axes labelled $P(r)$ and r/a ; the curve is zero at $r = 0$, rises initially as r^2 , and has a single maximum marked at $r/a = 1$
- 2 A long exponential tail towards large r , so the curve is asymmetric, with $\langle r \rangle = 1.5a$ marked to the right of the peak

Drawing $|\psi|^2$, which is largest at $r = 0$, is the commonest error: the probability per unit volume peaks at the nucleus, but the probability per unit radius vanishes there because the volume of a thin shell does.

(e) ESTIMATE · U4

[3]

Estimate the probability that the muon is found inside the proton, stating the assumption you make, and compare it with the corresponding probability for the electron in ordinary hydrogen.

- 1 Because the proton radius $R = 0.84$ fm is far smaller than a , take $|\psi|^2$ as constant at its value at $r = 0$ across the proton: $P \approx (4/3)\pi R^3 |\psi(0)|^2 = (4/3)(R/a)^3$
- 2 For muonic hydrogen $P \approx (4/3)(0.84 \times 10^{-15} \text{ m} / 2.85 \times 10^{-13} \text{ m})^3 = 3.4 \times 10^{-8}$
- 3 For ordinary hydrogen $P \approx (4/3)(0.84 \times 10^{-15} \text{ m} / 5.29 \times 10^{-11} \text{ m})^3 = 5.3 \times 10^{-15}$, so the muon is about 6×10^6 times more likely to be inside the proton, which is why muonic energy levels are far more sensitive to the proton's size

Integrating the exact probability costs time without changing the answer: with $R/a \approx 3 \times 10^{-3}$ the exponential factor is 1 to better than 1%. The skill examined is choosing that approximation and saying so.

19

CORE
COURSE

FREE RESPONSE · MODEL, SOLVE AND CHECK THE PRECISION · DISCRIMINATING · UPIII-08.09 · UPIII-11.02 · UPIII-11.04

[15]

The neutron's mass is determined most precisely from the gamma ray emitted when a slow neutron is captured by a proton at rest: $n + p \rightarrow d + \gamma$. A crystal-diffraction spectrometer measures the photon energy as $E_\gamma = 2.223\,25$ MeV, with an uncertainty of ± 0.4 eV. Take the rest energies $m_{pc^2} = 938.2721$ MeV and $m_{dc^2} = 1875.6129$ MeV, and treat the neutron's kinetic energy before capture as negligible. Write $M = m_n + m_p$ for the total rest mass before capture, and $B = (M - m_d)c^2$ for the binding energy of the deuteron.

(a) DERIVE · U2

[4]

Derive, from conservation of energy and momentum, an exact expression for E_γ in terms of M , m_d and c .

- 1 The initial system is at rest, so the deuteron recoils with momentum equal and opposite to the photon's, of magnitude E_γ/c
- 2 Energy conservation: $Mc^2 = E_\gamma + \sqrt{(m_dc^2)^2 + E_\gamma^2}$
- 3 Isolating the square root and squaring: $(Mc^2 - E_\gamma)^2 = (m_dc^2)^2 + E_\gamma^2$, so $M^2c^4 - 2Mc^2E_\gamma = m_d^2c^4$
- 4 $E_\gamma = (M^2 - m_d^2)c^2/(2M)$

Writing $E_\gamma = (M - m_d)c^2$ assumes the deuteron stays at rest, which momentum conservation forbids; the whole question turns on the keV that this assumption throws away.

(b) SHOW THAT · U2

[3]

Show that the result of (a) can be written exactly as $E_\gamma = B - B^2/(2Mc^2)$, and explain the physical meaning of the term $B^2/(2Mc^2)$.

- 1 $M^2 - m_d^2 = (M - m_d)(M + m_d)$, with $(M - m_d)c^2 = B$ and $M + m_d = 2M - B/c^2$
- 2 $E_\gamma = B(2Mc^2 - B)/(2Mc^2) = B - B^2/(2Mc^2)$, with no approximation
- 3 By energy conservation $Mc^2 = E_\gamma + m_dc^2 + K_d$, so $B^2/(2Mc^2) = B - E_\gamma$ is exactly the kinetic energy of the recoiling deuteron, about $E_\gamma^2/(2m_dc^2) = 1.32$ keV

Candidates who expand to first order and stop have shown an approximation where the question asks for an identity; the factorisation $M^2 - m_d^2 = (M - m_d)(M + m_d)$ makes the result exact in one line.

(c) DETERMINE · U3

[3]

Determine the rest energy of the neutron, m_{nc^2} , from the measured photon energy.

- 1 $B = E_\gamma + B^2/(2Mc^2)$; the correction is small, so evaluate it with $B \approx E_\gamma$ and $Mc^2 \approx 1877.8$ MeV: $(2.2233 \text{ MeV})^2/(2 \times 1877.8 \text{ MeV}) = 1.32$ keV
- 2 $B = 2.223\,25 \text{ MeV} + 0.001\,32 \text{ MeV} = 2.224\,57 \text{ MeV}$
- 3 $m_{nc^2} = m_{dc^2} + B - m_{pc^2} = 1875.6129 \text{ MeV} + 2.2246 \text{ MeV} - 938.2721 \text{ MeV} = 939.5654 \text{ MeV}$

One step of iteration suffices: the recoil correction changes by only about 1.6 eV between the first and second estimates, far below the 0.1 keV precision of the rest energies given.

(d) EVALUATE · U4

[2]

Evaluate whether the recoil term must be included when the neutron's rest energy is quoted from this measurement.

- 1 Neglecting recoil gives $m_n c^2 = 939.5641 \text{ MeV}$, lower by 1.32 keV, which is about 3×10^3 times the $\pm 0.4 \text{ eV}$ uncertainty in E_γ and larger than the 0.1 keV rounding of the rest energies given
- 2 So the recoil term is essential: without it the result is wrong by thousands of standard deviations of the photon measurement, and once it is included the precision of m_n is limited by the proton and deuteron rest energies rather than by the spectrometer

A correction of one part in 1700 of E_γ looks negligible until it is set against the measurement's own uncertainty; the judgement must be made against the error bar, not against the size of the quantity.

(e) COMPARE · U1

[3]

In the reverse process, photodisintegration $\gamma + d \rightarrow n + p$, a photon strikes a deuteron at rest. Compare the minimum photon energy for this process with the photon energy emitted in capture, and explain the difference.

- 1 At threshold the neutron and proton move off together with the photon's momentum, so the invariant mass must just reach M : $(E_{th} + m_d c^2)^2 - E_{th}^2 = M^2 c^4$, giving $E_{th} = (M^2 - m_d^2) c^2 / (2m_d) = B + B^2 / (2m_d c^2)$
- 2 $E_{th} = 2.224 57 \text{ MeV} + 0.001 32 \text{ MeV} = 2.225 89 \text{ MeV}$, which is 2.64 keV more than the 2.223 25 MeV emitted in capture
- 3 In capture the photon carries the binding energy less the deuteron's recoil energy; in photodisintegration the photon must supply the binding energy plus the kinetic energy that momentum conservation forces the n-p pair to carry, so the two energies differ by about two recoil energies, $B^2 / (m_d c^2)$

Candidates who expect the two energies to be equal, because one process is the reverse of the other, have treated the deuteron as infinitely heavy; the asymmetry is momentum conservation acting in opposite directions.

What the command words are asking for

Examiners mark to the command word as well as to the physics. An answer that explains where the question said state has spent time earning one mark; one that states a result where the question said derive has earned none, however correct the result.

State	Give the result, definition or law concisely. No justification is required.
Describe	Give the relevant features of a situation or process in words, precisely enough that someone else could reproduce it.
Explain	Give the physical reason a result or behaviour occurs, connecting it explicitly to the principle that governs it.
Calculate	Obtain a numerical value, showing the relation used, the substitution and the units of the result.
Determine	Obtain a result by any valid route the problem allows — analytic, graphical or numerical — with enough working to be followed.
Derive	Start from a named fundamental law or definition and reach the required result by explicit, justified steps.
Show that	Establish a result the question gives you. Every step must be shown, because the stated answer earns nothing on its own.
Estimate	Reach an order-of-magnitude value from stated, reasonable assumptions, and say what those assumptions are.
Sketch	Draw the shape of a graph or field with axes labelled and its key features marked — intercepts, asymptotes, turning points, limits. Exact scale is not required.
Justify	Support a claim or a choice with evidence, and with the reasoning that connects that evidence to the claim.
Evaluate	Weigh a model, argument or result against criteria, including its assumptions and domain of validity, and reach a stated judgement.
Compare	Identify the similarities and differences between two situations, models or results, relating each difference to its cause.
Predict	Say what will happen when a condition changes, reasoning from a principle rather than guessing, and give the reason.

WHAT IS BEING TESTED

University assessment objectives

Each course has the two papers a university course is examined by: a midterm on the first half of its units, and a cumulative final weighted towards the second half. Every paper combines short conceptual multiple-choice items with structured problems, a data or estimation question, and at least one extended derivation. Constants are given where a question needs them; a calculator and your own formula sheet are assumed.

U1 24 MARKS · 20%	Conceptual understanding Identify the governing principle and the conditions under which it holds, and reason qualitatively about what a result must look like before calculating it.
U2 35 MARKS · 30%	Mathematical modelling and derivation Translate a physical situation into mathematics: choose coordinates and approximations, set up the governing equations, solve them symbolically, and derive stated results from named laws.

U3

43 MARKS · 36%

Quantitative problem solving

Carry a multi-step calculation to a numerical result with correct units and sensible precision, and check it against dimensions, limiting cases and orders of magnitude.

U4

16 MARKS · 14%

Data, estimation and experimental reasoning

Analyse measurements and their uncertainties, linearise and fit data, estimate orders of magnitude from stated assumptions, and judge whether evidence supports a model.

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