

University Physics III · Midterm Examination

Oscillations, mechanical waves and wave phenomena, geometric and physical optics, and temperature and kinetic theory: derive the equation of motion, the total phase or the molecular flux before quoting a formula, then test each result against the approximation that made it valid.

SUGGESTED TIME 90 minutes	TOTAL MARKS 74	QUESTIONS 14	MULTIPLE CHOICE 8
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SECTION	UPIII · Midterm · University Physics III — Midterm examination
IN THE FORMAT OF	A university midterm: conceptual multiple choice, structured problems, a data-analysis question and an extended derivation
DEMAND	Routine 5 · Demanding 7 · Discriminating 2 <i>Pitched at what a strong course grade requires rather than at what a homework set rehearses. Routine substitutions carry few marks; most sit on problems that have to be set up before they can be solved, on derivations from a named law, and on checking a result against its limits. Every paper still opens with entry points, so you can tell which marks you should be securing first.</i>
COURSE UNITS	UPIII-01 · UPIII-02 · UPIII-03 · UPIII-04 · UPIII-05 · UPIII-06

INSTRUCTIONS

- Answer all questions in the spaces provided.
- Show your working. Method marks are awarded even where the final answer is wrong.
- Give your answers to a sensible number of significant figures, with a unit.
- The mark scheme for this paper follows the last question.

Written by GioPhysics from its own University Physics course maps. These are practice examinations in a university style: they are not the examinations of any university, contain no past-paper questions, and carry no academic credit. For a course you are taking, your instructor's syllabus and past papers remain the authority.

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University Physics III · Midterm Examination

Answer all questions. 74 marks in 90 minutes.

1 MULTIPLE CHOICE · ROUTINE · UPIII-01.03 · UPIII-01.04 [1]
CORE COURSE

A block on a light horizontal spring moves in simple harmonic motion on a frictionless surface. At one instant its speed is 60% of its maximum speed. What fraction of the oscillator's total mechanical energy is stored as elastic potential energy at that instant?

- A 0.36
- B 0.40
- C 0.80
- D 0.64

Answer

2 MULTIPLE CHOICE · DEMANDING · UPIII-01.09 [1]
CORE COURSE

A lightly damped oscillator of mass m and natural angular frequency ω_0 is driven by a force $F_0 \cos \omega t$ with $\omega = 3\omega_0$. Which statement correctly describes its steady-state displacement?

- A Its amplitude is about $F_0/(8m\omega_0^2)$, and it lags the driving force by nearly π .
- B Its amplitude is about $F_0/(8m\omega_0^2)$, and it lags the driving force by $\pi/2$.
- C Its amplitude is about $F_0/(9m\omega_0^2)$, and it lags the driving force by nearly π .
- D Its amplitude is about $F_0/(8m\omega_0^2)$, and it is nearly in phase with the driving force.

Answer

3 MULTIPLE CHOICE · DEMANDING · UPIII-02.05 · UPIII-02.08 [1]
CORE COURSE

A narrow tube is either open at both ends or closed at one end; you are not told which. Two of its successive resonant frequencies are 497 Hz and 639 Hz, with no resonance between them. Ignoring end corrections, what is the lowest resonant frequency of the air column?

- A 35.5 Hz
- B 71.0 Hz
- C 142 Hz
- D 355 Hz

Answer

4CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPIII-02.07 · UPIII-03.03

[1]

Two small loudspeakers are driven in phase by the same signal generator at a single frequency. A point P is the same distance from both, and reflections are negligible. With either loudspeaker operating alone, the sound-intensity level at P is 70.0 dB. What is the level at P when both operate together?

- A 70.0 dB
- B 73.0 dB
- C 76.0 dB
- D 140 dB

Answer

5CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-03.05

[1]

Two loudspeakers emit steady tones of equal amplitude at 396.0 Hz and 401.0 Hz, and a microphone records their superposition. Which statement describes the recorded signal?

- A A 398.5 Hz tone whose loudness peaks 2.5 times per second
- B A 797 Hz tone whose loudness peaks 5 times per second
- C A 398.5 Hz tone whose loudness peaks 5 times per second
- D A steady 5.0 Hz tone, below the range of hearing

Answer

6CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPIII-04.03 · UPIII-04.04

[1]

A step-index optical fibre has a core of refractive index 1.52 and a cladding of refractive index 1.46. Its flat entry face is perpendicular to its axis and is in air of refractive index 1.00. What is the largest angle to the fibre axis at which a ray in air can enter the core and still be guided by total internal reflection at the core-cladding boundary?

- A 16.2°
- B 41.1°
- C 73.8°
- D 25.0°

Answer

7CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-05.09

[1]

Unpolarized light of intensity I_0 passes in turn through three ideal linear polarizers whose transmission axes are at 30° , 90° and 135° to the vertical, all measured in the same rotational sense. What intensity emerges from the third polarizer?

- A $I_0/16$
- B $I_0/8$
- C $I_0/(4\sqrt{2})$
- D 0

Answer

8CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-06.02

[1]

An aluminium bar 1.50 m long fits snugly, and without stress, between two rigid supports at 12°C . It is heated uniformly to 47°C , and the supports prevent any change in its length. For aluminium, Young's modulus is $7.0 \times 10^{10} \text{ Pa}$ and the coefficient of linear expansion is $2.3 \times 10^{-5} \text{ K}^{-1}$. What is the compressive stress in the bar?

- A $3.8 \times 10^7 \text{ Pa}$
- B $5.6 \times 10^7 \text{ Pa}$
- C $8.5 \times 10^7 \text{ Pa}$
- D $5.2 \times 10^8 \text{ Pa}$

Answer

A vibrating-wire strain gauge bolted to a concrete beam contains a steel wire of length $L = 0.150$ m and linear mass density $\mu = 3.85 \times 10^{-4}$ kg/m, clamped at both ends. An electromagnet plucks the wire and a pick-up coil measures its fundamental frequency, which is 800 Hz when the gauge is installed.

(a) STATE

[1]

State the linear wave equation obeyed by small transverse displacements $y(x, t)$ of a wire under tension T with linear mass density μ , and the wave speed it implies.

.....

(b) CALCULATE

[2]

Calculate the speed of transverse waves on the wire and the tension in the wire when the gauge is installed.

.....

(c) PREDICT

[2]

Loading the beam raises the tension in the wire by 3.0%. Treating L and μ as unchanged, predict the new fundamental frequency.

.....

(d) CALCULATE

[3]

In a separate test, a long wire with the same μ and the same tension as in (b) carries a sinusoidal transverse travelling wave of frequency 800 Hz and amplitude 0.20 mm, and its far end is terminated so that no wave is reflected. Calculate the average power the wave transmits along the wire.

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A cylindrical instrument buoy floats upright in calm sea water. Its mass is $m = 850 \text{ kg}$, its horizontal cross-sectional area at the waterline is $A = 0.500 \text{ m}^2$, and its sides are vertical from 0.40 m below to 0.40 m above the calm waterline. Take the density of sea water as $\rho = 1025 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$. Model the water as exerting a hydrostatic buoyant force together with a linear damping force of magnitude $b|\dot{z}|$ that opposes the buoy's vertical velocity, ignore any water carried along with the buoy, and write $\gamma = b/m$ and $Q = \omega_0/\gamma$, where ω_0 is the undamped angular frequency.

(a) DERIVE

[3]

Ignoring damping, derive from Newton's second law and Archimedes' principle the equation of motion for a small vertical displacement z of the buoy from its floating equilibrium, and hence identify ω_0 .

(b) CALCULATE

[2]

Calculate the undamped period of the buoy's vertical oscillation.

(c) DETERMINE

[3]

The buoy is pushed down 0.120 m in calm water and released. After 5 complete oscillations its amplitude has fallen to 0.030 m . Determine γ and Q .

(d) PREDICT

[2]

A regular train of surface waves of amplitude $\eta_0 = 0.050 \text{ m}$, with period equal to your answer to (b), now passes the buoy. Taking the driving force as $\rho g A \eta$, where $\eta = \eta_0 \cos \omega t$ is the vertical displacement of the water surface, predict the amplitude of the buoy's steady-state oscillation.

(e) EVALUATE

[2]

Evaluate whether the model used in (d) can be trusted for these waves.

A discharge lamp contains a mixture of hydrogen and deuterium. Its blue-green Balmer emission consists of two lines, at $\lambda_1 = 486.13$ nm from hydrogen and $\lambda_2 = 486.00$ nm from deuterium. To show that both lines are present, a student may use either a transmission grating ruled with 1200 lines per millimetre over a width of 25.0 mm, used at normal incidence and able to be illuminated over any chosen width, or a Michelson interferometer whose movable mirror can be moved through up to 4.0 mm from the position at which the two arms are equal in length.

(a) **CALCULATE**

[2]

Calculate the chromatic resolving power needed to separate the two lines.

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(b) **DETERMINE**

[3]

Determine the highest diffraction order the grating produces at this wavelength, and hence the minimum width of grating that must be illuminated to resolve the lines.

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(c) **SHOW THAT**

[3]

As the mirror moves a distance x from the position of equal arm lengths, the detector records fringes whose visibility falls periodically to a minimum and recovers. Show that the mirror displacement between successive visibility minima is $\lambda_1\lambda_2/[2(\lambda_1 - \lambda_2)] \approx \lambda^2/(2\Delta\lambda)$.

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(d) **SKETCH**

[2]

Assuming the two lines are of equal intensity, sketch a graph of fringe visibility against mirror displacement from 0 to 4.0 mm, marking the displacements at which the visibility is a minimum.

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Reflective road markings contain transparent glass beads, each a sphere of radius $R = 0.300$ mm, half-embedded in white paint. Light from a car's headlamps enters the exposed front of a bead and is focused towards the back surface, where the paint scatters it; light scattered from a small bright patch there is sent back towards the car by the bead. Treat the incident light as a parallel beam travelling along an axis through the bead's centre, in air of refractive index 1.00. For refraction at a single spherical surface use $n_1/s + n_2/s' = (n_2 - n_1)/R$, where s is positive for a real object in front of the surface, s' is positive for a real image behind it, and R is positive when the centre of curvature lies behind the surface.

- (a) **SHOW THAT** [2]
 Show that, in the paraxial limit, refraction at the front surface alone would bring the beam to a focus a distance $nR/(n - 1)$ behind that surface, where n is the refractive index of the glass.
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- (b) **DETERMINE** [2]
 Determine the refractive index a bead must have for the paraxial focus to lie exactly on the paint at its back surface, and state where the paraxial focus lies for a bead of ordinary glass with $n = 1.50$.
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- (c) **SHOW THAT** [3]
 Take the centre of the bead as origin, with the incident light travelling in the $+x$ direction, so that the axis meets the back surface at $(R, 0)$. A ray entering parallel to the axis at height h above it has angle of incidence i , where $\sin i = h/R$, and angle of refraction r . Without making the paraxial approximation, show that this ray reaches the back surface at the point whose radius makes an angle $i - 2r$ with the $+x$ axis, on the opposite side of the axis from the point of entry.
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- (d) **CALCULATE** [2]
 For a bead with $n = 2.00$, calculate the perpendicular distance from the axis of the point at which the back surface is reached by the ray entering at $h = 0.50R$, and by the ray entering at $h = 0.90R$.
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- (e) **EVALUATE** [2]
 High-index road-marking beads are typically made from glass with n close to 1.9 rather than exactly 2.00. Evaluate this choice in the light of your results.
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To measure the speed of sound outdoors, a group mounts a buzzer of steady but unknown frequency f_0 on the roof of a car. On a calm day at 21 °C the car is driven at a series of steady speeds v_s along a straight runway directly towards a microphone at the same height, and a spectrum analyser records the frequency f received while the car is still far away. Each speed is logged by satellite navigation. In still air the moving-source Doppler relation is $f = f_0 v / (v - v_s)$, where v is the speed of sound.

Received frequency at each approach speed

Run	Approach speed v_s (m/s)	Received frequency f (Hz)	$1/f$ (10^{-4} s)
1	4.0	2175.6	4.5964
2	8.0	2201.0	4.5434
3	12.0	2227.8	4.4887
4	16.0	2254.6	4.4354
5	20.0	2282.9	4.3804
6	24.0	2311.2	4.3268

- (a) **SHOW THAT** [2]

Show that the relation predicts a straight line when $1/f$ is plotted against v_s , and identify its gradient and its intercept on the $1/f$ axis.

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- (b) **DETERMINE** [4]

Use the readings to determine f_0 and the speed of sound v .

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- (c) **JUSTIFY** [2]

Justify, using the readings, the use of the exact Doppler relation rather than the low-speed approximation $f \approx f_0(1 + v_s/v)$.

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- (d) **PREDICT** [2]

On a second day a steady wind of 5.0 m/s blows along the runway from the microphone towards the approaching car. Predict how the intercept and the gradient of the plot of $1/f$ against v_s would change, and the value of v that the analysis in (b) would then return.

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A sealed glass bulb of volume $V = 1.50 \times 10^{-3} \text{ m}^3$ contains argon at a temperature of 293 K and an initial pressure of 3.0 Pa. A circular hole 60 μm in diameter is then opened in a thin foil in the bulb's wall, leading directly to a high vacuum, while the bulb is held at 293 K. Model the escape as effusion, in which atoms leave through the hole independently of one another, and treat the argon as an ideal gas with a Maxwell-Boltzmann distribution of speeds, whose mean speed is $\langle v \rangle = \sqrt{(8kT/(\pi m))}$. Take the Boltzmann constant $k = 1.381 \times 10^{-23} \text{ J/K}$, the mass of an argon atom as 39.95 u with $1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$, and the effective diameter of an argon atom as about 0.36 nm.

(a) SHOW THAT

[4]

Consider a gas with n atoms per unit volume and an isotropic distribution of velocities. Show that the number of atoms striking unit area of the wall per unit time is $\Phi = \frac{1}{4}n\langle v \rangle$.

(b) DERIVE

[3]

Derive, from conservation of the number of atoms and the ideal-gas equation of state, an expression for the pressure in the bulb as a function of time t , identifying the time constant τ in terms of V , the area A of the hole and $\langle v \rangle$.

(c) CALCULATE

[3]

Calculate $\langle v \rangle$ for the argon, the time constant τ , and the time taken for the pressure to fall from 3.0 Pa to 0.30 Pa.

(d) ESTIMATE

[3]

Estimate the mean free path of the argon atoms at the initial pressure, stating the model you use, and hence decide whether treating the escape as effusion is justified.

(e) **EXPLAIN**

[2]

If the bulb were thermally insulated instead of held at 293 K, the argon remaining in it would cool as it leaked. Explain why, given that the gas does no work on any piston.

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University Physics III · Midterm Examination — mark scheme

One line per mark, as an examiner would award them. A note under a part is the error that part is set to catch.

1 CORE COURSE	MULTIPLE CHOICE · ROUTINE · UPIII-01.03 · UPIII-01.04	[1]
	A block on a light horizontal spring moves in simple harmonic motion on a frictionless surface. At one instant its speed is 60% of its maximum speed. What fraction of the oscillator's total mechanical energy is stored as elastic potential energy at that instant?	
	KEY D 0.64	
	<i>Kinetic energy goes as the square of the speed, so at 0.60 of the maximum speed the block holds $0.60^2 = 0.36$ of the total energy as kinetic energy, and the remaining 0.64 is elastic potential energy. A is that kinetic fraction, chosen by a candidate who answers for the wrong store. B shares the energy in proportion to speed, $1 - 0.60 = 0.40$, which would only be right if kinetic energy were linear in v. C is the displacement at that instant as a fraction of the amplitude, $x/A = \sqrt{1 - 0.36} = 0.80$: a correct number for a different quantity, left unconverted to energy, which goes as x^2.</i>	U1
2 CORE COURSE	MULTIPLE CHOICE · DEMANDING · UPIII-01.09	[1]
	A lightly damped oscillator of mass m and natural angular frequency ω_0 is driven by a force $F_0 \cos \omega t$ with $\omega = 3\omega_0$. Which statement correctly describes its steady-state displacement?	
	KEY A Its amplitude is about $F_0/(8m\omega_0^2)$, and it lags the driving force by nearly π.	
	<i>With light damping the steady-state amplitude is $F_0/(m\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}) \approx F_0/(m \omega_0^2 - 9\omega_0^2) = F_0/(8m\omega_0^2)$, and well above resonance the response is controlled by inertia, so the displacement is almost in antiphase with the force. B keeps the right amplitude but quotes the phase lag at resonance, where the damping alone limits the motion; above resonance the lag moves on towards π. C has the right phase but uses $F_0/(m\omega^2)$, the response of a free mass with no spring; it looks plausible because inertia dominates, but the spring term ω_0^2 still subtracts from ω^2 and turns 9 into 8. D describes driving well below resonance, where the spring controls the motion and the displacement follows the force; it attracts the candidate who reasons that a mass must move the way it is pushed.</i>	U1
3 CORE COURSE	MULTIPLE CHOICE · DEMANDING · UPIII-02.05 · UPIII-02.08	[1]
	A narrow tube is either open at both ends or closed at one end; you are not told which. Two of its successive resonant frequencies are 497 Hz and 639 Hz, with no resonance between them. Ignoring end corrections, what is the lowest resonant frequency of the air column?	
	KEY B 71.0 Hz	
	<i>If the tube were open at both ends, every integer harmonic would be present, successive resonances would differ by $f_1 = 639 - 497 = 142$ Hz, and 497 Hz would have to be a whole-number multiple of 142 Hz; it is not, since $497/142 = 3.5$. The tube must therefore be closed at one end, where only odd harmonics occur and successive resonances differ by $2f_1$, so $f_1 = 142/2 = 71.0$ Hz, and the given frequencies are the 7th and 9th harmonics ($497/71 = 7$, $639/71 = 9$). C is the spacing itself, the answer for the open tube that the whole-number test rules out. D is $497 - 142 = 355$ Hz, the 5th harmonic and the resonance just below 497 Hz, chosen by a candidate who steps down once and takes that as the lowest. A halves the spacing twice: once correctly, because successive odd harmonics are $2f_1$ apart, and again from recalling that closing one end halves an open tube's fundamental, a correction the first step already contains.</i>	U3

4

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPIII-02.07 · UPIII-03.03

[1]

Two small loudspeakers are driven in phase by the same signal generator at a single frequency. A point P is the same distance from both, and reflections are negligible. With either loudspeaker operating alone, the sound-intensity level at P is 70.0 dB. What is the level at P when both operate together?

KEY **C 76.0 dB**

The sources are coherent and the path difference to P is zero, so the pressure amplitudes add in phase: the amplitude doubles, the intensity quadruples, and the level rises by $10 \log_{10} 4 = 6.0$ dB to 76.0 dB. B is the correct answer for incoherent sources, such as two independent noise generators, whose intensities add because the interference term averages to zero; it is the most tempting option because intensity addition is the rule candidates meet first. D adds the two decibel levels, which are logarithms of intensity ratios and cannot be added. A treats the level as fixed by either source alone, as if the second contributed nothing at a point where it arrives exactly in step with the first.

U1

5

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-03.05

[1]

Two loudspeakers emit steady tones of equal amplitude at 396.0 Hz and 401.0 Hz, and a microphone records their superposition. Which statement describes the recorded signal?

KEY **C A 398.5 Hz tone whose loudness peaks 5 times per second**

Adding the two waves gives $2A \cos(2\pi \times 2.5t) \cos(2\pi \times 398.5t)$: an oscillation at the mean frequency whose amplitude is modulated by an envelope of frequency 2.5 Hz. Loudness depends on the magnitude of that envelope, which reaches a maximum twice in each of its cycles, so the loudness peaks $401.0 - 396.0 = 5.0$ times per second. A is the trap: it reads the envelope's positive and negative extremes as equally loud. B adds the two frequencies instead of averaging them. D mistakes the beat for a sound of its own: a linear superposition contains no component at the difference frequency, which appears only as a slow variation in loudness.

U1

6

CORE
COURSE

MULTIPLE CHOICE · DEMANDING · UPIII-04.03 · UPIII-04.04

[1]

A step-index optical fibre has a core of refractive index 1.52 and a cladding of refractive index 1.46. Its flat entry face is perpendicular to its axis and is in air of refractive index 1.00. What is the largest angle to the fibre axis at which a ray in air can enter the core and still be guided by total internal reflection at the core-cladding boundary?

KEY **D 25.0°**

The critical angle at the core-cladding boundary is $\theta_c = \sin^{-1}(1.46/1.52) = 73.8^\circ$, measured from the normal to that boundary, so a guided ray inside the core makes at most $90^\circ - \theta_c = 16.2^\circ$ with the axis. Snell's law at the entry face then gives $\sin \theta = 1.52 \sin(90^\circ - \theta_c) = \sqrt{1.52^2 - 1.46^2} = 0.423$, so $\theta = 25.0^\circ$. C is the critical angle itself, measured from the normal to the core-cladding boundary rather than from the fibre axis. A is the correct angle inside the core, chosen by a candidate who forgets to apply Snell's law at the entry face: the ray bends towards the normal as it passes from air into the glass, so its angle to the axis is larger in air than inside. B is the critical angle for a glass-air boundary, $\sin^{-1}(1/1.52)$, which would decide total internal reflection at the wall of an unclad fibre but plays no part at a core-cladding boundary.

U3

7CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-05.09

[1]

Unpolarized light of intensity I_0 passes in turn through three ideal linear polarizers whose transmission axes are at 30° , 90° and 135° to the vertical, all measured in the same rotational sense. What intensity emerges from the third polarizer?

KEY A $I_0/16$

The first polarizer transmits half of the unpolarized light, $I_0/2$, now polarized at 30° to the vertical. Malus's law depends only on the angle between the light's polarization and the next transmission axis: 60° at the second polarizer, giving a factor $\cos^2 60^\circ = 1/4$, and 45° at the third, giving $\cos^2 45^\circ = 1/2$, so the result is $(I_0/2)(1/4)(1/2) = I_0/16$. B applies Malus's law correctly twice but omits the factor $1/2$ at the first polarizer, treating unpolarized light as though it were already polarized along the first axis. C uses $\cos \theta$, the factor by which the field amplitude falls, in place of $\cos^2 \theta$, the factor for intensity: $(I_0/2)(1/2)(1/\sqrt{2}) = I_0/(4\sqrt{2})$. D measures each angle from the vertical rather than from the previous transmission axis, so the horizontal second polarizer appears crossed with the light and to transmit nothing; the light reaching it is polarized at 30° to the vertical, only 60° from that axis.

U3

8CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UPIII-06.02

[1]

An aluminium bar 1.50 m long fits snugly, and without stress, between two rigid supports at 12°C . It is heated uniformly to 47°C , and the supports prevent any change in its length. For aluminium, Young's modulus is $7.0 \times 10^{10} \text{ Pa}$ and the coefficient of linear expansion is $2.3 \times 10^{-5} \text{ K}^{-1}$. What is the compressive stress in the bar?

KEY B $5.6 \times 10^7 \text{ Pa}$

The supports prevent the free thermal strain $\alpha\Delta T$, so the stress is $\sigma = Y\alpha\Delta T = 7.0 \times 10^{10} \times 2.3 \times 10^{-5} \times 35 = 5.6 \times 10^7 \text{ Pa}$, whatever the bar's length or cross-section. C is Y multiplied by the free expansion $\Delta L = \alpha L\Delta T = 1.21 \times 10^{-3} \text{ m}$, the extension used as though it were the strain; A divides the strain by the length a second time. Both come from believing the length of the bar must matter, when strain is a fractional change and the length cancels. D uses the final absolute temperature, 320 K, in place of the 35 K rise: expansion responds to the change in temperature, and a change of 35°C is a change of 35 K.

U3

9

CORE
COURSE

STRUCTURED · ROUTINE · UPIII-02.02 · UPIII-02.03 · UPIII-02.05

[8]

A vibrating-wire strain gauge bolted to a concrete beam contains a steel wire of length $L = 0.150$ m and linear mass density $\mu = 3.85 \times 10^{-4}$ kg/m, clamped at both ends. An electromagnet plucks the wire and a pick-up coil measures its fundamental frequency, which is 800 Hz when the gauge is installed.

(a) STATE · U1

[1]

State the linear wave equation obeyed by small transverse displacements $y(x, t)$ of a wire under tension T with linear mass density μ , and the wave speed it implies.

1 $\partial^2 y / \partial t^2 = (T/\mu) \partial^2 y / \partial x^2$, so the wave speed is $v = \sqrt{T/\mu}$

(b) CALCULATE · U3

[2]

Calculate the speed of transverse waves on the wire and the tension in the wire when the gauge is installed.

1 both clamped ends are nodes, so the fundamental has $\lambda_1 = 2L = 0.300$ m and $v = f_1 \lambda_1 = 800 \times 0.300 = 240$ m/s

2 $T = \mu v^2 = 3.85 \times 10^{-4} \times 240^2 = 22.2$ N

(c) PREDICT · U3

[2]

Loading the beam raises the tension in the wire by 3.0%. Treating L and μ as unchanged, predict the new fundamental frequency.

1 with L and μ fixed, $f_1 = (1/2L)\sqrt{T/\mu}$, so f_1 is proportional to $\sqrt{T}$

2 $f_1 = 800 \times \sqrt{1.030} = 812$ Hz (811.9 Hz), a rise of about 1.5%

Candidates who scale the frequency in proportion to the tension predict 824 Hz; the square root is the whole of this part.

(d) CALCULATE · U3

[3]

In a separate test, a long wire with the same μ and the same tension as in (b) carries a sinusoidal transverse travelling wave of frequency 800 Hz and amplitude 0.20 mm, and its far end is terminated so that no wave is reflected. Calculate the average power the wave transmits along the wire.

1 $\omega = 2\pi \times 800 = 5.03 \times 10^3$ rad/s

2 uses the average transmitted power $P = \frac{1}{2}\mu\omega^2 A^2 v$

3 $P = \frac{1}{2} \times 3.85 \times 10^{-4} \times (5.03 \times 10^3)^2 \times (2.0 \times 10^{-4})^2 \times 240 = 4.7 \times 10^{-2}$ W (46.7 mW)

Omitting the $\frac{1}{2}$, which comes from averaging $\sin^2$ over a cycle, doubles the answer; using f in place of ω makes it about 39 times too small.

10

CORE
COURSE

STRUCTURED · DEMANDING · UPIII-01.01 · UPIII-01.02 · UPIII-01.08 · UPIII-01.09

[12]

A cylindrical instrument buoy floats upright in calm sea water. Its mass is $m = 850$ kg, its horizontal cross-sectional area at the waterline is $A = 0.500$ m², and its sides are vertical from 0.40 m below to 0.40 m above the calm waterline. Take the density of sea water as $\rho = 1025$ kg/m³ and $g = 9.81$ m/s². Model the water as exerting a hydrostatic buoyant force together with a linear damping force of magnitude $b|\dot{z}|$ that opposes the buoy's vertical velocity, ignore any water carried along with the buoy, and write $\gamma = b/m$ and $Q = \omega_0/\gamma$, where ω_0 is the undamped angular frequency.

- (a) **DERIVE** · U2 [3]
- Ignoring damping, derive from Newton's second law and Archimedes' principle the equation of motion for a small vertical displacement z of the buoy from its floating equilibrium, and hence identify ω_0 .
- 1 at equilibrium the buoyant force $\rho g V_0$ on the submerged volume V_0 balances the weight mg
 - 2 a further downward displacement z submerges an extra volume Az because the sides are vertical, so the net force is $-\rho g Az$: a restoring force proportional to the displacement
 - 3 Newton's second law gives $m d^2z/dt^2 = -\rho g Az$, the simple-harmonic equation $d^2z/dt^2 = -\omega_0^2 z$ with $\omega_0 = \sqrt{\rho g A/m}$
- Candidates who take $\rho g Az$ as the whole buoyant force, without first cancelling the weight against the equilibrium buoyancy, are left with a constant term in the equation of motion; the equilibrium step is what makes the restoring force linear.*
- (b) **CALCULATE** · U3 [2]
- Calculate the undamped period of the buoy's vertical oscillation.
- 1 $\rho g A = 1025 \times 9.81 \times 0.500 = 5.03 \times 10^3 \text{ N/m}$, so $\omega_0 = \sqrt{(5.03 \times 10^3/850)} = 2.43 \text{ rad/s}$
 - 2 $T = 2\pi/\omega_0 = 2.58 \text{ s}$
- Only the waterline area enters. Candidates who bring in the buoy's height or total volume have lost track of which quantity sets how the buoyant force changes with z .*
- (c) **DETERMINE** · U4 [3]
- The buoy is pushed down 0.120 m in calm water and released. After 5 complete oscillations its amplitude has fallen to 0.030 m. Determine γ and Q .
- 1 for light damping the amplitude decays as $z_0 e^{-(\gamma t/2)}$, and 5 oscillations take $5T = 12.9 \text{ s}$ (the damped period differs from T by about 0.1%)
 - 2 $\gamma = 2 \ln(0.120/0.030)/(5T) = 2 \times 1.386/12.92 = 0.215 \text{ s}^{-1}$
 - 3 $Q = \omega_0/\gamma = 2.43/0.215 = 11.3$
- The factor of 2 is what this part is set to catch: the amplitude decays at $\gamma/2$ and the energy at γ . Candidates who write the amplitude as $z_0 e^{-(\gamma t)}$ obtain $Q \approx 22.7$.*
- (d) **PREDICT** · U3 [2]
- A regular train of surface waves of amplitude $\eta_0 = 0.050 \text{ m}$, with period equal to your answer to (b), now passes the buoy. Taking the driving force as $\rho g A \eta$, where $\eta = \eta_0 \cos \omega t$ is the vertical displacement of the water surface, predict the amplitude of the buoy's steady-state oscillation.
- 1 at resonance the steady-state amplitude is $F_0/(b\omega_0) = F_0/(m\gamma\omega_0)$, with $F_0 = \rho g A \eta_0 = 5.03 \times 10^3 \times 0.050 = 251 \text{ N}$
 - 2 since $\rho g A = m\omega_0^2$, this is $Q\eta_0 = 11.3 \times 0.050 = 0.57 \text{ m}$
- Candidates who use the undamped response $F_0/(m|\omega_0^2 - \omega^2|)$ find an infinite amplitude and stop; at resonance only the damping limits the motion.*
- (e) **EVALUATE** · U1 [2]
- Evaluate whether the model used in (d) can be trusted for these waves.
- 1 the predicted 0.57 m amplitude exceeds the 0.40 m over which the sides are vertical (relative to the moving surface the excursion is also 0.57 m, since z lags η by $\pi/2$ at resonance and $\sqrt{(0.57^2 + 0.050^2)} = 0.57 \text{ m}$), so the waterline area, and with it the restoring force per unit displacement, would change during each cycle: the linear restoring force the model rests on breaks down
 - 2 judgement: the 0.57 m prediction is not reliable; at resonance the linear model holds only for wave amplitudes below about $0.40 \text{ m}/Q \approx 0.035 \text{ m}$
- The discriminating step is to test the answer against the range of validity given in the stem rather than accept it: a Q of 11 turns a modest wave into a displacement the model was never built for.*

A discharge lamp contains a mixture of hydrogen and deuterium. Its blue-green Balmer emission consists of two lines, at $\lambda_1 = 486.13$ nm from hydrogen and $\lambda_2 = 486.00$ nm from deuterium. To show that both lines are present, a student may use either a transmission grating ruled with 1200 lines per millimetre over a width of 25.0 mm, used at normal incidence and able to be illuminated over any chosen width, or a Michelson interferometer whose movable mirror can be moved through up to 4.0 mm from the position at which the two arms are equal in length.

(a) CALCULATE · U3

[2]

Calculate the chromatic resolving power needed to separate the two lines.

1 $R = \lambda/\Delta\lambda$, with $\lambda \approx 486.07$ nm and $\Delta\lambda = 486.13 - 486.00 = 0.13$ nm

2 $R = 486.07/0.13 = 3.7 \times 10^3$ (3739)

Candidates who invert the definition and quote $\Delta\lambda/\lambda \approx 2.7 \times 10^{-4}$ have written the fractional separation of the lines, not the resolving power, which must be a large number.

(b) DETERMINE · U3

[3]

Determine the highest diffraction order the grating produces at this wavelength, and hence the minimum width of grating that must be illuminated to resolve the lines.

1 $d = 1/(1200 \text{ mm}^{-1}) = 833$ nm, and at normal incidence $\sin \theta = m\lambda/d \leq 1$ requires $m \leq d/\lambda = 1.71$, so only the first order exists

2 $R = mN$ with $m = 1$ requires at least $N = 3739$ illuminated lines

3 minimum width = $Nd = 3739 \times 833$ nm = 3.12 mm, well within the 25.0 mm available

The trap is to halve the width by working in second order. At 1200 lines/mm that order would need $\sin \theta = 1.17$, so it does not exist at this wavelength.

(c) SHOW THAT · U2

[3]

As the mirror moves a distance x from the position of equal arm lengths, the detector records fringes whose visibility falls periodically to a minimum and recovers. Show that the mirror displacement between successive visibility minima is $\lambda_1\lambda_2/[2(\lambda_1 - \lambda_2)] \approx \lambda^2/(2\Delta\lambda)$.

1 moving the mirror by x changes the path difference by $2x$, so the interference orders of the two lines at the detector are $2x/\lambda_1$ and $2x/\lambda_2$

2 the two fringe systems are in step (maximum visibility) when the orders differ by a whole number and exactly out of step (minimum visibility) when they differ by a half-integer; the difference is $2x/\lambda_2 - 2x/\lambda_1 = 2x(\lambda_1 - \lambda_2)/(\lambda_1\lambda_2)$

3 successive minima are reached each time this difference grows by 1, so their spacing is $\Delta x = \lambda_1\lambda_2/[2(\lambda_1 - \lambda_2)] \approx \lambda^2/(2\Delta\lambda)$

Candidates who forget that the light crosses the extra arm length twice take the path difference as x and obtain $\lambda^2/\Delta\lambda$, twice the correct spacing.

(d) SKETCH · U1

[2]

Assuming the two lines are of equal intensity, sketch a graph of fringe visibility against mirror displacement from 0 to 4.0 mm, marking the displacements at which the visibility is a minimum.

1 visibility is 1 at zero displacement and follows $|\cos(2\pi x\Delta\lambda/\lambda^2)|$: rounded maxima, falling to a sharp zero at 0.45 mm and recovering to 1 at 0.91 mm

2 further zeros marked at 1.36, 2.27 and 3.18 mm (spacing 0.91 mm), with maxima of 1 at 1.82, 2.73 and 3.63 mm: four minima within the 4.0 mm travel

The first minimum lies at half the spacing, not at the spacing: at equal arm lengths the two lines are in step, so a sketch with its first zero at 0.91 mm is half a period out everywhere.

Reflective road markings contain transparent glass beads, each a sphere of radius $R = 0.300$ mm, half-embedded in white paint. Light from a car's headlamps enters the exposed front of a bead and is focused towards the back surface, where the paint scatters it; light scattered from a small bright patch there is sent back towards the car by the bead. Treat the incident light as a parallel beam travelling along an axis through the bead's centre, in air of refractive index 1.00. For refraction at a single spherical surface use $n_1/s + n_2/s' = (n_2 - n_1)/R$, where s is positive for a real object in front of the surface, s' is positive for a real image behind it, and R is positive when the centre of curvature lies behind the surface.

(a) SHOW THAT · U2

[2]

Show that, in the paraxial limit, refraction at the front surface alone would bring the beam to a focus a distance $nR/(n - 1)$ behind that surface, where n is the refractive index of the glass.

- 1 for a parallel beam $s \rightarrow \infty$, so $n_1/s = 0$, with $n_1 = 1.00$, $n_2 = n$, and R positive because the centre of curvature of the front surface lies inside the bead, behind that surface
- 2 $n/s' = (n - 1)/R$, so $s' = nR/(n - 1)$

The sign of R is what this part checks. A candidate who takes R as negative for a surface bulging towards the light finds a virtual focus in front of the bead.

(b) DETERMINE · U3

[2]

Determine the refractive index a bead must have for the paraxial focus to lie exactly on the paint at its back surface, and state where the paraxial focus lies for a bead of ordinary glass with $n = 1.50$.

- 1 the back surface is a full diameter, $2R$, behind the front surface, so $n/(n - 1) = 2$, giving $n = 2.00$
- 2 for $n = 1.50$, $s' = 3R = 0.900$ mm, which is 0.300 mm beyond the back surface, inside the paint, so the illuminated patch on the paint is broad

Candidates who place the focus at the centre of the bead, a distance R behind the front surface, are led to $n/(n - 1) = 1$, which no glass satisfies; that impossibility should send them back to the geometry.

(c) SHOW THAT · U2

[3]

Take the centre of the bead as origin, with the incident light travelling in the $+x$ direction, so that the axis meets the back surface at $(R, 0)$. A ray entering parallel to the axis at height h above it has angle of incidence i , where $\sin i = h/R$, and angle of refraction r . Without making the paraxial approximation, show that this ray reaches the back surface at the point whose radius makes an angle $i - 2r$ with the $+x$ axis, on the opposite side of the axis from the point of entry.

- 1 the normal at the entry point lies along the radius, so that radius makes an angle i with the $-x$ axis, i.e. an angle $180^\circ - i$ measured from the $+x$ axis on the side of the entry point
- 2 the path inside the bead is a chord, which forms an isosceles triangle with the radii to its two ends; both base angles equal r , so the angle at the centre between the entry and exit radii is $180^\circ - 2r$
- 3 the ray is bent towards the axis, so the exit radius lies $180^\circ - 2r$ further round, at $(180^\circ - i) - (180^\circ - 2r) = -(i - 2r)$ from the $+x$ axis: an angle $i - 2r$ on the opposite side of the axis from the entry point

With no figure, the geometry must be built from the words. The isosceles triangle formed by the chord and two radii is the step almost every candidate who fails this part has missed.

(d) CALCULATE · U3

[2]

For a bead with $n = 2.00$, calculate the perpendicular distance from the axis of the point at which the back surface is reached by the ray entering at $h = 0.50R$, and by the ray entering at $h = 0.90R$.

- 1 $h = 0.50R$: $i = 30.0^\circ$, $r = \sin^{-1}(0.25) = 14.48^\circ$, $i - 2r = 1.04^\circ$, so the distance is $R \sin(i - 2r) = 0.300 \times 0.0182 = 5.5 \times 10^{-3}$ mm ($5.5 \mu\text{m}$) beyond the axis
- 2 $h = 0.90R$: $i = 64.16^\circ$, $r = \sin^{-1}(0.45) = 26.74^\circ$, $i - 2r = 10.67^\circ$, so the distance is $0.300 \times \sin 10.67^\circ = 5.6 \times 10^{-2}$ mm ($56 \mu\text{m}$) beyond the axis

The paraxial model says both rays land on the axis. The exact trace puts the outer ray ten times further off than the inner one: spherical aberration made concrete.

(e) EVALUATE · U1

[2]

High-index road-marking beads are typically made from glass with n close to 1.9 rather than exactly 2.00. Evaluate this choice in the light of your results.

- 1 at $n = 2.00$ only near-axis rays reach the axis point; rays entering further out cross the axis before the back surface and land up to $56 \mu\text{m}$ from it at $h = 0.90R$, so satisfying the paraxial condition exactly does not give the smallest patch on the paint
- 2 lowering n to 1.90 moves the paraxial focus just beyond the back surface ($s' = 2.11R$), so near-axis rays land slightly on the entry side of the axis while outer rays land less far on the opposite side: at $n = 1.90$ the ray entering at $h = 0.90R$ has $r = 28.27^\circ$ and $i - 2r = 7.61^\circ$, landing $0.300 \times \sin 7.61^\circ = 40 \mu\text{m}$ beyond the axis against $56 \mu\text{m}$ at $n = 2.00$, so the patch is narrower (about $44 \mu\text{m}$ across for $h \leq 0.90R$), returning a tighter, brighter cone to the driver, and the choice is sound

The judgement rewarded is that a paraxial design condition is a starting point, not an optimum. Calling $n = 1.9$ a manufacturing compromise, without reference to the aberration just calculated, earns nothing.

13
CORE
COURSE

DATA ANALYSIS · DEMANDING · UPIII-03.06 · UPIII-03.07 · UPIII-03.09

[10]

To measure the speed of sound outdoors, a group mounts a buzzer of steady but unknown frequency f_0 on the roof of a car. On a calm day at 21°C the car is driven at a series of steady speeds v_s along a straight runway directly towards a microphone at the same height, and a spectrum analyser records the frequency f received while the car is still far away. Each speed is logged by satellite navigation. In still air the moving-source Doppler relation is $f = f_0 v / (v - v_s)$, where v is the speed of sound.

(a) SHOW THAT · U2

[2]

Show that the relation predicts a straight line when $1/f$ is plotted against v_s , and identify its gradient and its intercept on the $1/f$ axis.

- 1 inverting, $1/f = (v - v_s)/(f_0 v) = 1/f_0 - v_s/(f_0 v)$, which is linear in v_s
- 2 gradient = $-1/(f_0 v)$; intercept = $1/f_0$

Candidates who plot f against v_s see a line that is almost straight and stop there; its slight curvature is exactly what the reciprocal plot removes.

(b) DETERMINE · U4

[4]

Use the readings to determine f_0 and the speed of sound v .

- 1 a best-fit straight line of $1/f$ against v_s , with its gradient read from widely separated points on the line, gives gradient = $-1.35 \times 10^{-6} \text{ s}^2 \text{ m}^{-1}$ (accept -1.34×10^{-6} to $-1.36 \times 10^{-6} \text{ s}^2 \text{ m}^{-1}$)
- 2 the intercept at $v_s = 0$ is $1/f_0 = 4.651 \times 10^{-4} \text{ s}$, so $f_0 = 2150 \text{ Hz}$ (accept 2148 Hz to 2152 Hz)
- 3 $v = -\text{intercept}/\text{gradient} = 1/(f_0 \times |\text{gradient}|)$
- 4 $v = 4.651 \times 10^{-4} / 1.35 \times 10^{-6} = 344 \text{ m/s}$ (accept 341 m/s to 348 m/s), consistent with the 344 m/s expected in air at 21°C

The intercept lies outside the range of the readings and must be found by extrapolation. Candidates who take the first reading, 2175.6 Hz , as f_0 carry a 1% error into v and find about 340 m/s .

(c) JUSTIFY · U4

[2]

Justify, using the readings, the use of the exact Doppler relation rather than the low-speed approximation $f \approx f_0(1 + v_s/v)$.

- 1 the approximation makes f linear in v_s , rising by the same $f_0 \times 4.0/v \approx 25.0$ Hz for each 4.0 m/s step and reaching 2300.0 Hz at 24.0 m/s; the readings rise by 25.4, 26.8, 26.8, 28.3 and 28.3 Hz and reach 2311.2 Hz
- 2 the steadily growing steps and the 11 Hz excess at 24.0 m/s are many times the scatter of the readings about the reciprocal straight line (about 0.3 Hz), so the data reject the approximation, which drops terms in $(v_s/v)^2$ and higher, and the exact relation is needed

The marks are for setting the size of a discrepancy against the size of the noise. Saying the exact relation is more accurate earns nothing without both numbers.

(d) PREDICT · U1

[2]

On a second day a steady wind of 5.0 m/s blows along the runway from the microphone towards the approaching car. Predict how the intercept and the gradient of the plot of $1/f$ against v_s would change, and the value of v that the analysis in (b) would then return.

- 1 in the frame of the air the microphone moves away from the car at 5.0 m/s and the car approaches at $v_s + 5.0$ m/s, so $f = f_0(v - 5.0)/(v - 5.0 - v_s)$; at $v_s = 0$ the buzzer and microphone share the same motion through the air, $f = f_0$, and the intercept is unchanged
- 2 $1/f = 1/f_0 - v_s/[f_0(v - 5.0)]$, so the gradient is steeper and the analysis returns $v - 5.0 = 339$ m/s: the speed of sound relative to the ground along the path, not the speed of sound in the air

The Doppler relation holds in the frame of the medium. Candidates who let the wind change only the source speed predict a shifted intercept, which contradicts the fact that a buzzer and a microphone at rest relative to each other must agree on the frequency.

14
CORE
COURSE

FREE RESPONSE · MODEL, SOLVE AND CHECK THE LIMITS · DISCRIMINATING · UPIII-06.03 · UPIII-06.04 · UPIII-06.05 · UPIII-06.06 · UPIII-06.07

[15]

A sealed glass bulb of volume $V = 1.50 \times 10^{-3} \text{ m}^3$ contains argon at a temperature of 293 K and an initial pressure of 3.0 Pa. A circular hole 60 μm in diameter is then opened in a thin foil in the bulb's wall, leading directly to a high vacuum, while the bulb is held at 293 K. Model the escape as effusion, in which atoms leave through the hole independently of one another, and treat the argon as an ideal gas with a Maxwell-Boltzmann distribution of speeds, whose mean speed is $\langle v \rangle = \sqrt{8kT/(\pi m)}$. Take the Boltzmann constant $k = 1.381 \times 10^{-23} \text{ J/K}$, the mass of an argon atom as 39.95 u with $1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$, and the effective diameter of an argon atom as about 0.36 nm.

(a) SHOW THAT · U2

[4]

Consider a gas with n atoms per unit volume and an isotropic distribution of velocities. Show that the number of atoms striking unit area of the wall per unit time is $\Phi = \frac{1}{4}n\langle v \rangle$.

- 1 isotropy: the fraction of atoms moving at between θ and $\theta + d\theta$ to the inward normal of the wall is $2\pi \sin \theta d\theta/4\pi = \frac{1}{2} \sin \theta d\theta$
- 2 atoms with speeds between v and $v + dv$ moving in that direction reach an area A of wall within a time dt only if they lie in a slanted cylinder of volume $Av \cos \theta dt$, so they make $n g(v)dv \times \frac{1}{2} \sin \theta d\theta \times Av \cos \theta dt$ impacts, where $g(v)$ is the normalised speed distribution
- 3 integrating over the directions heading towards the wall, $\int_0^{\pi/2} \frac{1}{2} \sin \theta \cos \theta d\theta = \frac{1}{4}$
- 4 integrating over speeds, $\int_0^\infty v g(v) dv = \langle v \rangle$, so $\Phi = \frac{1}{4}n\langle v \rangle$

The $\cos \theta$ is where the derivation is usually lost: atoms arriving obliquely come from a thinner slab. Omitting it gives $\frac{1}{2}n\langle v \rangle$, and sending one sixth of the atoms along each axis direction gives $n\langle v \rangle/6$.

(b) DERIVE · U2 [3]

Derive, from conservation of the number of atoms and the ideal-gas equation of state, an expression for the pressure in the bulb as a function of time t , identifying the time constant τ in terms of V , the area A of the hole and $\langle v \rangle$.

- 1 atoms leave through the hole at the rate ΦA and none return from the vacuum, so $dN/dt = -\frac{1}{4}(N/V)\langle v \rangle A$
- 2 at constant T , $\langle v \rangle$ is constant, so integrating gives $N = N_0 e^{-(t/\tau)}$ with $\tau = 4V/(A\langle v \rangle)$
- 3 $pV = NkT$ at constant V and T makes p proportional to N , so $p = p_0 e^{-(t/\tau)}$

Candidates who hold the rate of escape constant predict a linear fall in pressure. The rate is proportional to the number density left in the bulb, which is what makes the decay exponential and τ independent of the starting pressure.

(c) CALCULATE · U3 [3]

Calculate $\langle v \rangle$ for the argon, the time constant τ , and the time taken for the pressure to fall from 3.0 Pa to 0.30 Pa.

- 1 $m = 39.95 \times 1.661 \times 10^{-27} = 6.636 \times 10^{-26}$ kg, so $\langle v \rangle = \sqrt{(8 \times 1.381 \times 10^{-23} \times 293)/(\pi \times 6.636 \times 10^{-26})} = 394$ m/s
- 2 $A = \pi(30 \times 10^{-6})^2 = 2.83 \times 10^{-9}$ m², so $\tau = 4 \times 1.50 \times 10^{-3}/(2.83 \times 10^{-9} \times 394) = 5.39 \times 10^3$ s (about 1.5 h)
- 3 $t = \tau \ln(3.0/0.30) = 5.39 \times 10^3 \times 2.303 = 1.24 \times 10^4$ s (about 3.4 h)

Candidates who use the rms speed, 428 m/s, obtain a time constant about 8% too short. The flux is set by the mean speed, which is why the derivation in (a) produced $\langle v \rangle$ and not a mean of squares.

(d) ESTIMATE · U4 [3]

Estimate the mean free path of the argon atoms at the initial pressure, stating the model you use, and hence decide whether treating the escape as effusion is justified.

- 1 model: hard spheres of diameter $d \approx 0.36$ nm with Maxwell-Boltzmann relative speeds, so $\lambda = kT/(\sqrt{2}\pi d^2 p)$, with number density $p/kT = 7.4 \times 10^{20}$ m⁻³
- 2 $\lambda = 1.381 \times 10^{-23} \times 293/(\sqrt{2} \times \pi \times (0.36 \times 10^{-9})^2 \times 3.0) \approx 2.3 \times 10^{-3}$ m (about 2 mm; accept about 3 mm if the $\sqrt{2}$ is omitted)
- 3 λ is about 40 times the 60 μ m diameter of the hole, so atoms pass through it without colliding with one another; the effusion model is justified, and becomes more so as the pressure falls

An estimate earns its marks only with its model stated. The decisive comparison is the mean free path against the hole, not against the bulb: that is what separates effusion from bulk flow through the opening.

(e) EXPLAIN · U1 [2]

If the bulb were thermally insulated instead of held at 293 K, the argon remaining in it would cool as it leaked. Explain why, given that the gas does no work on any piston.

- 1 the flux weights atoms by their speed, since the rate at which atoms of speed v arrive at the hole is proportional to $v g(v)$; faster atoms escape preferentially, and each escaping atom carries a mean kinetic energy of $2kT$, more than the average of $3kT/2$ per atom in the bulb
- 2 the gas left behind therefore loses more than the average energy per atom removed, so its mean translational kinetic energy per atom, $(3/2)kT$, falls and its temperature falls with it

Candidates who say the gas cools because it expands have borrowed the adiabatic-expansion argument. There is no piston; the cooling is a velocity-selection effect of the flux alone.

What the command words are asking for

Examiners mark to the command word as well as to the physics. An answer that explains where the question said state has spent time earning one mark; one that states a result where the question said derive has earned none, however correct the result.

State	Give the result, definition or law concisely. No justification is required.
Describe	Give the relevant features of a situation or process in words, precisely enough that someone else could reproduce it.
Explain	Give the physical reason a result or behaviour occurs, connecting it explicitly to the principle that governs it.
Calculate	Obtain a numerical value, showing the relation used, the substitution and the units of the result.
Determine	Obtain a result by any valid route the problem allows — analytic, graphical or numerical — with enough working to be followed.
Derive	Start from a named fundamental law or definition and reach the required result by explicit, justified steps.
Show that	Establish a result the question gives you. Every step must be shown, because the stated answer earns nothing on its own.
Estimate	Reach an order-of-magnitude value from stated, reasonable assumptions, and say what those assumptions are.
Sketch	Draw the shape of a graph or field with axes labelled and its key features marked — intercepts, asymptotes, turning points, limits. Exact scale is not required.
Justify	Support a claim or a choice with evidence, and with the reasoning that connects that evidence to the claim.
Evaluate	Weigh a model, argument or result against criteria, including its assumptions and domain of validity, and reach a stated judgement.
Compare	Identify the similarities and differences between two situations, models or results, relating each difference to its cause.
Predict	Say what will happen when a condition changes, reasoning from a principle rather than guessing, and give the reason.

WHAT IS BEING TESTED

University assessment objectives

Each course has the two papers a university course is examined by: a midterm on the first half of its units, and a cumulative final weighted towards the second half. Every paper combines short conceptual multiple-choice items with structured problems, a data or estimation question, and at least one extended derivation. Constants are given where a question needs them; a calculator and your own formula sheet are assumed.

U1 15 MARKS · 20%	Conceptual understanding Identify the governing principle and the conditions under which it holds, and reason qualitatively about what a result must look like before calculating it.
U2 20 MARKS · 27%	Mathematical modelling and derivation Translate a physical situation into mathematics: choose coordinates and approximations, set up the governing equations, solve them symbolically, and derive stated results from named laws.

U3

27 MARKS · 36%

Quantitative problem solving

Carry a multi-step calculation to a numerical result with correct units and sensible precision, and check it against dimensions, limiting cases and orders of magnitude.

U4

12 MARKS · 16%

Data, estimation and experimental reasoning

Analyse measurements and their uncertainties, linearise and fit data, estimate orders of magnitude from stated assumptions, and judge whether evidence supports a model.

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