

University Quantum Field Theory I · Midterm Examination

The first eight units of Quantum Field Theory I on one paper: the Feynman propagator derived by contour, the Gell-Mann–Low formula and Wick's theorem, symmetry factors and Feynman rules read off a Lagrangian, Källén–Lehmann and LSZ, decay rates and cross-sections carried to seconds and barns, one-loop ϕ^4 in dimensional regularisation with its beta function and Landau pole, and the signs and traces of Yukawa theory with Dirac fermions. The data question fits a resonance against the s-wave unitarity limit, and the extended derivation quantises the photon covariantly, derives its propagator in every covariant gauge and checks the Ward identity on Compton scattering by a charged scalar, down to Thomson's cross-section.

SUGGESTED TIME	TOTAL MARKS	QUESTIONS	MULTIPLE CHOICE
90 minutes	73	14	8

SECTION **UQFTI · Midterm · University Quantum Field Theory I — Midterm examination**

IN THE FORMAT OF A university midterm: conceptual multiple choice, structured problems, a data-analysis question and an extended derivation

DEMAND **Recall 2 · Routine 7 · Demanding 3 · Discriminating 2**

Pitched at what a strong course grade requires rather than at what a homework set rehearses. Routine substitutions carry few marks; most sit on problems that have to be set up before they can be solved, on derivations from a named law, and on checking a result against its limits. Every paper still opens with entry points, so you can tell which marks you should be securing first.

COURSE UNITS UQFTI-01 · UQFTI-02 · UQFTI-03 · UQFTI-04 · UQFTI-05 · UQFTI-06 · UQFTI-07 · UQFTI-08

INSTRUCTIONS

- Answer all questions in the spaces provided.
- Show your working. Method marks are awarded even where the final answer is wrong.
- Give your answers to a sensible number of significant figures, with a unit.
- The mark scheme for this paper follows the last question.

Written by GioPhysics from its own University Physics course maps. These are practice examinations in a university style: they are not the examinations of any university, contain no past-paper questions, and carry no academic credit. For a course you are taking, your instructor's syllabus and past papers remain the authority.

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University Quantum Field Theory I · Midterm Examination

Answer all questions. 73 marks in 90 minutes.

1

CORE COURSE

MULTIPLE CHOICE · RECALL · UQFTI-02.01 · UQFTI-06.01

[1]

With $\hbar = c = 1$ the action is dimensionless, so in d spacetime dimensions the Lagrangian density has mass dimension d . The kinetic terms then give a real scalar field ϕ the mass dimension $(d - 2)/2$ and a Dirac field ψ the mass dimension $(d - 1)/2$. For which interaction is the coupling constant dimensionless, so that the theory is renormalisable by power counting, in $d = 6$?

- A $(\lambda/4!)\phi^4$
- B $(g/6!)\phi^6$
- C $(g/3!)\phi^3$
- D $g\phi\bar{\psi}\psi$

Answer

2

CORE COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-02.05 · UQFTI-02.06

[1]

The Gell-Mann-Low formula expresses a correlation function in the interacting vacuum $|\Omega\rangle$ through free fields: $\langle\Omega|T\phi_H(x)\phi_H(y)|\Omega\rangle = \langle 0|T[\phi_I(x)\phi_I(y) \exp(-i\int H_I dt)]|0\rangle / \langle 0|T[\exp(-i\int H_I dt)]|0\rangle$, where $|0\rangle$ is the free vacuum, ϕ_I and H_I are in the interaction picture, the time integrals run from $-T$ to T , and the limit is taken as $T \rightarrow \infty(1 - i\epsilon)$. What does taking the limit along the slightly tilted direction $T(1 - i\epsilon)$ accomplish?

- A It damps every eigenstate of the full Hamiltonian above the ground state by $e^{-(E_n - E_0)\epsilon T}$ relative to $|\Omega\rangle$, so the long evolution projects $|0\rangle$ onto $|\Omega\rangle$, provided $\langle\Omega|0\rangle \neq 0$ and there is a gap above E_0 .
- B It switches the interaction on and off slowly, so that $|0\rangle$ evolves into $|\Omega\rangle$ with $\langle\Omega|0\rangle = 1$, and the denominator is then needed only to cancel the ultraviolet divergences of the numerator.
- C It makes the Dyson series converge by damping the high-frequency modes of H_I , while the projection of $|0\rangle$ onto $|\Omega\rangle$ is supplied separately by normal ordering H_I .
- D It makes the contractions retarded rather than Feynman propagators, so that the formula respects causality, while the time ordering itself projects $|0\rangle$ onto $|\Omega\rangle$.

Answer

3CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-03.02 · UQFTI-03.03

[1]

In ϕ^4 theory with $\mathcal{L}_{\text{int}} = -(\lambda/4!)\phi^4$, two connected vacuum diagrams appear at order λ^2 . In diagram (i) two vertices are joined by four propagators. In diagram (ii) two vertices are joined by two propagators, and each vertex also carries one propagator that begins and ends on it. Each diagram contributes its spacetime integral times $(-i\lambda)^2/S$. What are the symmetry factors S of (i) and (ii)?

- A 24 and 8
- B 24 and 16
- C 48 and 8
- D 48 and 16

Answer

4CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-04.02 · UQFTI-04.03

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. A real scalar field ϕ , canonically normalised so that $[\phi(x, t), \pi(y, t)] = i\delta^3(x - y)$, is interacting and invariant under $\phi \rightarrow -\phi$. Its lightest particle has physical mass m , and the field can create states of any odd number of these particles. Its exact two-point function has the Källén-Lehmann form $\int_0^\infty (d\mu^2/2\pi) \rho(\mu^2) i/(p^2 - \mu^2 + i\epsilon)$, with $\rho(\mu^2) = 2\pi Z\delta(\mu^2 - m^2)$ plus a continuum. Which statement is correct?

- A The continuum begins at $\mu^2 = 4m^2$, and $Z < 1$ because ϕ also creates two-particle states.
- B The continuum begins at $\mu^2 = 9m^2$, $\rho(\mu^2) \geq 0$, and the canonical commutator fixes the total weight $\int \rho d\mu^2/2\pi = 1$, so $0 \leq Z \leq 1$.
- C The continuum begins at $\mu^2 = 9m^2$, and Z may exceed 1 if the interaction is attractive, because ρ is then negative above threshold.
- D The continuum begins at $\mu^2 = 3m^2$, and $Z = 1$ exactly, because the canonical commutator fixes the residue of the pole.

Answer

5CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-04.06 · UQFTI-04.07

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. In the scalar Yukawa model, $\mathcal{L}_{\text{int}} = -g\psi^\dagger\psi\phi$, with ψ a complex scalar of mass m_ψ and ϕ a real scalar of mass $m_\phi < 2m_\psi$. To order g^2 the amplitude for $\psi(p_1)\bar{\psi}(p_2) \rightarrow \psi(p_3)\bar{\psi}(p_4)$ is $M = -g^2[1/(s - m_\phi^2) + 1/(t - m_\phi^2)]$, with $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$, the first term from annihilation through ϕ and the second from ϕ exchange. Defining s , t and u in the same way for $\psi(p_1)\psi(p_2) \rightarrow \psi(p_3)\psi(p_4)$, what is the order- g^2 amplitude for that process?

- A $-g^2[1/(s - m_\phi^2) + 1/(t - m_\phi^2)]$
- B $-g^2[1/(t - m_\phi^2) - 1/(u - m_\phi^2)]$
- C $-g^2[1/(s - m_\phi^2) + 1/(t - m_\phi^2) + 1/(u - m_\phi^2)]$
- D $-g^2[1/(t - m_\phi^2) + 1/(u - m_\phi^2)]$

Answer

6CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-01.01 · UQFTI-05.04

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. In ϕ^4 theory with $\mathcal{L}_{\text{int}} = -(\lambda/4!)\phi^4$ and $\lambda = 0.60$, two particles collide at a centre-of-mass energy $\sqrt{s} = 4.0$ GeV, far enough above threshold that their mass can be neglected. At tree level the amplitude for $\phi\phi \rightarrow \phi\phi$ is $M = -\lambda$, and for a $2 \rightarrow 2$ process with equal masses in and out, $d\sigma/d\Omega = |M|^2/(64\pi^2 s)$ in the centre-of-mass frame. Take $(\hbar c)^2 = 0.3894$ GeV² mb. What is the total cross-section for $\phi\phi \rightarrow \phi\phi$?

- A** 87 nb
- B** 174 nb
- C** 14 nb
- D** 0.15 nb

Answer

7CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-07.02 · UQFTI-07.05

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. Two identical Dirac fermions of mass m scatter, $\psi(p_1)\psi(p_2) \rightarrow \psi(p_3)\psi(p_4)$, through the Yukawa interaction $\mathcal{L}_{\text{int}} = -g\phi\bar{\psi}\psi$, where ϕ is a real scalar of mass m_ϕ . At tree level two diagrams contribute, one with momentum transfer squared $t = (p_1 - p_3)^2$ and one with $u = (p_1 - p_4)^2$. In the non-relativistic limit $\bar{u}(p_3)u(p_1) \rightarrow 2m$ times a Kronecker delta between the spin labels. Which statement is correct?

- A** The two diagrams add with the same sign, as they would for bosons, because the particle exchanged between the fermion lines is the boson ϕ .
- B** The two diagrams have a relative minus sign, which makes scalar exchange repulsive between two fermions although it attracts a fermion and an antifermion.
- C** The two diagrams have a relative minus sign, because their final states differ by an exchange of anticommuting operators; the t -channel term alone is the Born amplitude of an attractive Yukawa potential.
- D** Only the t -channel diagram contributes, because the u -channel diagram counts the same final state again and is removed by the $1/2$ for identical particles.

Answer

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. In Feynman gauge the four photon operators of one momentum k , taken along z , obey $[a^\lambda(k), a^{\lambda'\dagger}(k')] = -g^{\lambda\lambda'}(2\pi)^3 2|k| \delta^3(k - k')$ for $\lambda, \lambda' = 0, 1, 2, 3$, where $\lambda = 0$ labels the timelike polarisation $\epsilon^0 = (1, 0, 0, 0)$, $\lambda = 1, 2$ the transverse ones and $\lambda = 3$ the longitudinal one, $\epsilon^3 = (0, 0, 0, 1)$. The Gupta-Bleuler condition requires the positive-frequency part of $\partial_\mu A^\mu$ to annihilate every physical state, which for this momentum reads $(a^0 - a^3)|\psi\rangle = 0$. Which statement is correct?

- A** The condition removes timelike and longitudinal photons from every physical state, which may contain only photons created by $a^{1\dagger}$ and $a^{2\dagger}$.
- B** Physical states may contain $(a^{0\dagger} - a^{3\dagger})$ acting on a transverse state; such a state has zero norm and adds nothing to the energy or to any gauge-invariant expectation value.
- C** A longitudinal photon $a^{3\dagger}|0\rangle$ has negative norm like the timelike one, and the condition admits only states in which the two negative norms cancel.
- D** The full condition $\partial_\mu A^\mu|\psi\rangle = 0$, creation part included, would serve equally well; only its positive-frequency part is kept, to simplify the algebra.

Answer

A free real scalar field of mass m is written, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$, as $\phi(x) = \int d^3p / ((2\pi)^3 2E_p) [a(p)e^{-ip \cdot x} + a^\dagger(p)e^{ip \cdot x}]$, where $E_p = \sqrt{|\mathbf{p}|^2 + m^2}$, $p^0 = E_p$ in the exponents and $[a(p), a^\dagger(q)] = (2\pi)^3 2E_p \delta^3(\mathbf{p} - \mathbf{q})$. Its vacuum two-point function is $D(x - y) = \langle 0 | \phi(x) \phi(y) | 0 \rangle = \int d^3p / ((2\pi)^3 2E_p) e^{-ip \cdot (x - y)}$. Take $\hbar c = 197.327 \text{ MeV fm}$.

(a) DERIVE

[3]

Starting from the definition $D_F(x - y) = \langle 0 | T \phi(x) \phi(y) | 0 \rangle$, derive the representation $D_F(x - y) = \int d^4p / (2\pi)^4 i e^{-ip \cdot (x - y)} / (p^2 - m^2 + i\epsilon)$, evaluating the p^0 integral with the contour closed in whichever half-plane each time ordering requires.

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(b) SKETCH

[2]

The retarded propagator is $D_R(x - y) = \theta(x^0 - y^0) \langle 0 | [\phi(x), \phi(y)] | 0 \rangle$, whose momentum-space form is $i / ((p^0 + i\epsilon)^2 - |\mathbf{p}|^2 - m^2)$. Sketch the complex p^0 -plane at fixed $\mathbf{p}$, marking the poles of D_F and of D_R and the contour used when $x^0 < y^0$, and state where in spacetime each propagator is non-zero.

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(c) CALCULATE

[2]

Two static point sources, $j(x) = g[\delta^3(x - x_1) + \delta^3(x - x_2)]$, coupled through $\mathcal{L}_{\text{int}} = j\phi$ and held a distance $r = |x_1 - x_2|$ apart for a long time T , give a vacuum persistence amplitude $\langle 0 | 0 \rangle \propto e^{-iE(r)T}$, with interaction energy $E(r) = -(g^2 / (4\pi r)) e^{-mr}$. For a scalar of mass $m = 550 \text{ MeV}$, $g^2 / (4\pi) = 0.80$ and $r = 0.70 \text{ fm}$, calculate $E(r)$ in MeV and state the range of the force in femtometres.

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(d) EXPLAIN

[1]

$D_F(x - y)$ is non-zero when $x - y$ is spacelike. Explain why this does not allow a signal to travel faster than light.

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Two real scalar fields, ϕ of mass m_ϕ and χ of mass m_χ , live in four spacetime dimensions and interact through $\mathcal{L}_{\text{int}} = -(g/2)\phi\chi^2 - (\kappa/2)\phi\partial_\mu\chi\partial^\mu\chi$, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. Treat both couplings to the lowest order at which each process occurs. For the two-body decay of a particle of mass m_ϕ into particles of momentum p in its rest frame, Lorentz-invariant phase space gives $\int d\Pi_2 = p/(4\pi m_\phi)$ when the two final particles are distinguishable, and $\Gamma = (1/(2m_\phi))\int |M|^2 d\Pi_2$. Take $\hbar = 6.582 \times 10^{-25}$ GeV s.

(a) DETERMINE

[2]

Determine the Feynman rule for the $\phi\chi\chi$ vertex, taking every momentum as incoming and p_2 and p_3 as the momenta of the two χ lines, and give the mass dimensions of g and κ .

(b) SHOW THAT

[2]

Set $\kappa = 0$. By counting the Wick contractions in the order- g^2 term of the Gell-Mann-Low numerator, show that the one-loop ϕ self-energy, a loop of two χ propagators, carries a symmetry factor of 2, and that the one-loop χ self-energy, a loop of one ϕ and one χ propagator, carries a symmetry factor of 1. Check each by counting the diagram's automorphisms.

(c) DETERMINE

[2]

Still with $\kappa = 0$, determine the tree-level amplitude M for $\chi(p_1)\chi(p_2) \rightarrow \chi(p_3)\chi(p_4)$ in terms of the Mandelstam variables s , t and u , check its symmetry under $t \leftrightarrow u$ and the value of $s + t + u$, and say why $\sqrt{Z} = 1$ may be used on each external leg at this order.

(d) PREDICT

[2]

The masses are now chosen so that $m_\phi > 2m_\chi$. Predict what becomes of the s -channel term of the amplitude in (c) as s approaches m_ϕ^2 , and of the ϕ particle's place in the LSZ formula, giving the reason, and say whether the t - and u -channel poles are affected.

(e) CALCULATE

[2]

Restore κ . With $m_\phi = 0.90$ GeV, $m_\chi = 0.14$ GeV, $g = 0.50$ GeV and $\kappa = 0.80$ GeV $^{-1}$, calculate the decay rate $\Gamma(\phi \rightarrow \chi\chi)$ in MeV and the lifetime of ϕ in seconds.

A real spin-0 boson ϕ of mass m_ϕ couples to a Dirac fermion ψ of mass m either as a scalar, $\mathcal{L}_S = -g\phi\bar{\psi}\psi$, or as a pseudoscalar, $\mathcal{L}_P = -ig\phi\bar{\psi}\gamma^5\psi$, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. Use the spin sums $\sum u(k)\bar{u}(k) = \not{K} + m$ and $\sum v(k)\bar{v}(k) = \not{K} - m$; the identities $\text{tr } 1 = 4$ and $\text{tr}(ab) = 4a \cdot b$; the vanishing of the trace of an odd number of γ matrices; $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$, $(\gamma^5)^2 = 1$ and $\gamma^0(\gamma^5)^\dagger\gamma^0 = -\gamma^5$. For the decay $\phi \rightarrow \psi(k_1)\bar{\psi}(k_2)$ write $\beta = \sqrt{1 - 4m^2/m_\phi^2}$, the speed of either fermion in the ϕ rest frame; then $\int d\Pi_2 = \beta/(8\pi)$, and $\Gamma = (1/(2m_\phi)) \times (\text{spin-summed } |M|^2) \times \int d\Pi_2$. Take $\hbar = 6.582 \times 10^{-25} \text{ GeV s}$.

(a) DERIVE

[2]

For the scalar coupling the amplitude is $M_S = -g \bar{u}(k_1)v(k_2)$. Using Casimir's trick with the spin sums, derive the spin-summed $|M_S|^2$ in terms of g , m_ϕ and β .

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(b) SHOW THAT

[2]

For the pseudoscalar coupling the amplitude is $M_P = -g \bar{u}(k_1)i\gamma^5v(k_2)$. Show that the spin-summed $|M_P|^2$ is $2g^2m_\phi^2$, with no factor of β^2 .

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(c) CALCULATE

[2]

For $m_\phi = 1.50 \text{ GeV}$, $m = 0.60 \text{ GeV}$ and $g = 0.30$, calculate $\Gamma(\phi \rightarrow \psi\bar{\psi})$ and the ϕ lifetime in seconds for each coupling.

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(d) EXPLAIN

[2]

Explain the powers β^3 and β in the two rates by the parity of ϕ and the orbital angular momentum of the fermion pair near threshold.

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(e) JUSTIFY

[2]

In the one-loop ϕ self-energy built from a ψ loop, Wick's theorem supplies a factor -1 . Justify this factor from the anticommutation of the fields, and confirm it from the optical theorem, which requires $\text{Im } M(\phi \rightarrow \phi) = m_\phi\Gamma(\phi \rightarrow \psi\bar{\psi})$ once $m_\phi > 2m$.

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Real scalar ϕ^4 theory, $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - (\lambda/4!)\phi^4$, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$, is regularised dimensionally in $d = 4 - 2\epsilon$ dimensions, with λ replaced by $\lambda\mu^{2\epsilon}$ so that λ stays dimensionless, μ being an arbitrary scale. Use the Feynman-parameter identity $1/(AB) = \int_0^1 dx [xA + (1-x)B]^{-2}$, the Euclidean master integral $\int d^d\ell_E/(2\pi)^d (\ell_E^2 + \Delta)^{-2} = \Gamma(2 - d/2)(4\pi)^{-d/2}\Delta^{d/2-2}$, and $\Gamma(\epsilon) = 1/\epsilon - \gamma_E + O(\epsilon)$. At one loop the field-strength counterterm vanishes, so the bare coupling is $\lambda_0 = \mu^{2\epsilon}(\lambda + \delta_\lambda)$, where the counterterm vertex is $-i\delta_\lambda$.

(a) SHOW THAT

[3]

The s-channel one-loop diagram for $\phi\phi \rightarrow \phi\phi$, with incoming momenta p_1 and p_2 , $p = p_1 + p_2$ and $s = p^2$, carries a symmetry factor of 2. Show that, below the threshold $s = 4m^2$, its contribution to iM , with an overall factor $\mu^{2\epsilon}$ removed, is $(i\lambda^2/(32\pi^2))\int_0^1 dx [1/\epsilon - \gamma_E + \ln 4\pi - \ln(\Delta_s/\mu^2)] + O(\epsilon)$, where $\Delta_s = m^2 - x(1-x)s$.

(b) SHOW THAT

[2]

Including the t- and u-channel diagrams and the counterterm, the renormalised coupling λ is defined by the condition $M = -\lambda$ at $s = 4m^2$, $t = u = 0$. Show that $M(s, t, u) = -\lambda - (\lambda^2/(32\pi^2))\sum \int_0^1 dx \ln[\Delta(q^2)/\Delta(q_0^2)]$, the sum running over the three channels $q^2 = s, t, u$ with reference values $q_0^2 = 4m^2, 0, 0$ and $\Delta(q^2) = m^2 - x(1-x)q^2$, and that neither ϵ nor μ survives in it.

(c) DERIVE

[2]

In minimal subtraction δ_λ contains only the poles of the three channels. Derive the one-loop beta function $\beta(\lambda) = \mu d\lambda/d\mu$ from the μ -independence of the bare coupling λ_0 .

(d) CALCULATE

[2]

In a model whose coupling is $\lambda(\mu_0) = 1.5$ at $\mu_0 = 1.0$ TeV, calculate the scale at which the one-loop running coupling has doubled and the scale of its Landau pole, both in GeV.

(e) ESTIMATE

[2]

Estimate the highest scale at which this one-loop running can be trusted, stating the criterion you use, and say what the Landau pole of (d) then tells you about the theory.

Two distinct spinless particles, a of mass 0.400 GeV and b of mass 0.600 GeV , scatter elastically, $a + b \rightarrow a + b$, and near a centre-of-mass energy of 1.85 GeV the cross-section is dominated by an s -channel resonance X of mass m_X and total width Γ . Parts (a) to (d) treat X as a spin-0 particle coupled to ab through $\mathcal{L}_{\text{int}} = -gXab$, which gives $\Gamma(X \rightarrow ab) = g^2 p / (8\pi m_X^2)$, where p is the momentum of a or b in the centre-of-mass frame at $\sqrt{s} = m_X$; near the peak the amplitude is $M = -g^2 / (s - m_X^2 + im_X\Gamma)$. For two particles in and two out with these masses, $\sigma = |M|^2 / (16\pi s)$. To the accuracy of these data the line shape is $\sigma(\sqrt{s}) = \sigma_{\text{peak}}(\Gamma^2/4) / [(\sqrt{s} - m_X)^2 + \Gamma^2/4]$. The optical theorem reads $\text{Im } M(\text{forward}) = 2\sqrt{s} p \sigma_{\text{tot}}$. Work in natural units with the metric $(+, -, -, -)$, and take $(\hbar c)^2 = 0.3894\text{ GeV}^2\text{ mb}$ and $\hbar = 6.582 \times 10^{-25}\text{ GeV s}$. The table gives the measured elastic cross-section at seven energies.

Elastic cross-section for $a + b \rightarrow a + b$ near the resonance

$\sqrt{s}\text{ (GeV)}$	$\sigma\text{ (mb)}$	Uncertainty in $\sigma\text{ (mb)}$
1.826	0.198	0.005
1.834	0.363	0.009
1.842	0.805	0.020
1.850	1.300	0.033
1.858	0.809	0.020
1.866	0.364	0.009
1.874	0.190	0.005

(a) **SHOW THAT** [2]

Show that $\sigma_{\text{peak}} = (4\pi/p^2)B^2$, where $B = \Gamma(X \rightarrow ab)/\Gamma$ is the branching fraction to ab , so that $4\pi/p^2$ is the largest cross-section an s -wave resonance can give in this channel.

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(b) **DETERMINE** [3]

By plotting a suitable linear function of the data, determine m_X , Γ and σ_{peak} .

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(c) **CALCULATE** [2]

Calculate p at $\sqrt{s} = m_X$ and the s -wave unitarity limit $4\pi/p^2$ in millibarns, and hence B and the lifetime of X in seconds.

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(d) DETERMINE [2]

Use the optical theorem to determine the total and the inelastic cross-sections at the peak, and determine $\Gamma(X \rightarrow ab)$ and g .

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(e) EVALUATE [2]

A separate experiment counts the decays of X and finds $B = 0.43 \pm 0.05$. For a resonance of spin J the peak elastic cross-section is $(2J + 1)(4\pi/p^2)B^2$. Evaluate whether the data support $J = 0$ against $J = 1$, and say what further measurement on the same events would settle the spin directly.

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14
CORE
COURSE

FREE RESPONSE · EXTENDED DERIVATION · DISCRIMINATING · UQFTI-08.01 · UQFTI-08.02 · UQFTI-08.04 · UQFTI-08.05 · UQFTI-08.06 · UQFTI-08.08 · UQFTI-08.09

[15]

The free electromagnetic field, in natural units $\hbar = c = 1$ and Heaviside-Lorentz units with $e^2 = 4\pi\alpha$ and the metric $(+, -, -, -)$, has the Maxwell Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. A complex scalar ϕ of mass m and charge e couples to it through $\mathcal{L}_\phi = (D_\mu\phi)^*(D^\mu\phi) - m^2\phi^*\phi$ with $D_\mu = \partial_\mu + ieA_\mu$, whose Feynman rules are a vertex $-ie(p + p')^\mu$, with p and p' the incoming and outgoing scalar momenta, and a two-photon seagull vertex $2ie^2g^{\mu\nu}$. Write Π^μ for the momentum conjugate to A_μ . Take $\alpha = 1/137.036$, $(\hbar c)^2 = 0.3894 \text{ GeV}^2 \text{ mb}$, and $m = 139.570 \text{ MeV}$, the mass of the charged pion, in (f).

(a) SHOW THAT [2]

Show that the Maxwell Lagrangian gives A_0 no conjugate momentum, so that canonical quantisation of all four components fails, and that adding the gauge-fixing term $-(1/(2\xi))(\partial_\mu A^\mu)^2$ supplies one.

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(b) DERIVE [3]

From the Euler-Lagrange equations, derive the field equation of the gauge-fixed Lagrangian and, from its quadratic action in momentum space, the photon propagator in a general covariant gauge.

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- (c) **SHOW THAT** [3]
- Two conserved currents, $k_\mu j_1^\mu(k) = k_\mu j_2^\mu(k) = 0$, exchange a photon with amplitude proportional to $j_1^\mu D_{\mu\nu}(k) j_2^\nu$. Show that ξ drops out and, taking k along z with $|k| = \kappa$, that the exchange splits into an instantaneous Coulomb term $i j_1^0 j_2^0 / \kappa^2$ and a transverse term $i j_1^T \cdot j_2^T / (\kappa^2 + i\epsilon)$, where j^T is the part of the three-vector current perpendicular to k .
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- (d) **COMPARE** [2]
- A massive vector field of mass M_V , the Proca field, has the propagator $-i[g_{\mu\nu} - k_\mu k_\nu / M_V^2] / (k^2 - M_V^2 + i\epsilon)$. Compare its polarisations and propagator with the photon's, and explain from the little group of a massless particle why the photon must couple to a conserved current while the Proca field need not.
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- (e) **SHOW THAT** [3]
- For Compton scattering on the charged scalar, $\gamma(k_1) + \phi(p) \rightarrow \gamma(k_2) + \phi(p')$, write $iM^\mu(\mu\nu)$, with μ belonging to the incoming photon and ν to the outgoing one, as the sum of the s -channel, u -channel and seagull diagrams. Show that $k_{1\mu} M^\mu(\mu\nu) = 0$ for the sum, but not for any one diagram or for the two exchange diagrams alone.
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-
-
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- (f) **CALCULATE** [2]
- In the rest frame of the initial scalar choose both photon polarisations with $\epsilon^0 = 0$, so that $p \cdot \epsilon = k \cdot \epsilon = 0$ for each photon. For photon energies $\omega \ll m$, where $d\sigma/d\Omega = |M|^2 / (64\pi^2 m^2)$, calculate the total cross-section for a point-like scalar with the charged pion's mass, in microbarns, averaging over initial and summing over final polarisations.
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-
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University Quantum Field Theory I · Midterm Examination — mark scheme

One line per mark, as an examiner would award them. A note under a part is the error that part is set to catch.

1 MULTIPLE CHOICE · RECALL · UQFTI-02.01 · UQFTI-06.01 **[1]**

CORE
COURSE

With $\hbar = c = 1$ the action is dimensionless, so in d spacetime dimensions the Lagrangian density has mass dimension d . The kinetic terms then give a real scalar field ϕ the mass dimension $(d - 2)/2$ and a Dirac field ψ the mass dimension $(d - 1)/2$. For which interaction is the coupling constant dimensionless, so that the theory is renormalisable by power counting, in $d = 6$?

KEY **C** $(g/3!)\phi^3$

A is the renormalisable self-coupling of four dimensions: $[\lambda] = d - 4(d - 2)/2 = 4 - d$ vanishes there, but in six dimensions λ has mass dimension -2 , the superficial degree of divergence grows with every vertex added, and ϕ^4 is non-renormalisable. B has $[g] = d - 6(d - 2)/2 = 6 - 2d$, which vanishes in three dimensions, where ϕ^6 is the renormalisable self-coupling; in six it is -6 . D is the Yukawa coupling, with $[g] = d - (d - 2)/2 - (d - 1) = (4 - d)/2$: dimensionless in four dimensions, which is why Yukawa theory is renormalisable there, but of dimension -1 in six. C is correct: $[g] = d - 3(d - 2)/2 = (6 - d)/2$, which is $+1$ in four dimensions, where ϕ^3 is super-renormalisable, and 0 in six, where it is the renormalisable cubic theory.

U1

2 MULTIPLE CHOICE · ROUTINE · UQFTI-02.05 · UQFTI-02.06 **[1]**

CORE
COURSE

The Gell-Mann-Low formula expresses a correlation function in the interacting vacuum $|\Omega\rangle$ through free fields: $\langle\Omega|T\phi_H(x)\phi_H(y)|\Omega\rangle = \langle 0|T[\phi_I(x)\phi_I(y) \exp(-i\int H_I dt)]|0\rangle / \langle 0|T[\exp(-i\int H_I dt)]|0\rangle$, where $|0\rangle$ is the free vacuum, ϕ_I and H_I are in the interaction picture, the time integrals run from $-T$ to T , and the limit is taken as $T \rightarrow \infty(1 - i\epsilon)$. What does taking the limit along the slightly tilted direction $T(1 - i\epsilon)$ accomplish?

KEY **A** **It damps every eigenstate of the full Hamiltonian above the ground state by $e^{-(E_n - E_0)\epsilon T}$ relative to $|\Omega\rangle$, so the long evolution projects $|0\rangle$ onto $|\Omega\rangle$, provided $\langle\Omega|0\rangle \neq 0$ and there is a gap above E_0 .**

B confuses the tilt with adiabatic switching, and even adiabatic switching does not give unit overlap: the evolved state is $|\Omega\rangle$ multiplied by a phase and by $\langle\Omega|0\rangle$, and in infinite volume that overlap goes to zero, which is what Haag's theorem says about this derivation. The denominator, the sum of vacuum bubbles, removes exactly the phase and the overlap; it is present in a finite theory too and has nothing to do with ultraviolet divergences. C attaches the damping to the wrong operator: the factor $e^{-(iHT(1 - i\epsilon))}$ acts on the energy eigenstates of the full Hamiltonian, not on modes of H_I , and normal ordering H_I removes contractions within one vertex but cannot turn the free vacuum into the interacting one. D reverses the job of the $i\epsilon$: time-ordered contractions of interaction-picture fields are Feynman propagators, whose $i\epsilon$ is inherited from this same tilt, and the retarded propagator belongs to the classical response to a source. A is correct: writing $|0\rangle = \sum_n |n\rangle\langle n|0\rangle$, the evolution weights each term by $e^{-(iE_n T(1 - i\epsilon))}$, so every state above E_0 dies away relative to $|\Omega\rangle$; the non-zero overlap and the gap are the two assumptions, and the phase and overlap left over cancel between numerator and denominator.

U1

3

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-03.02 · UQFTI-03.03

[1]

In ϕ^4 theory with $\mathcal{L}_{\text{int}} = -(\lambda/4!)\phi^4$, two connected vacuum diagrams appear at order λ^2 . In diagram (i) two vertices are joined by four propagators. In diagram (ii) two vertices are joined by two propagators, and each vertex also carries one propagator that begins and ends on it. Each diagram contributes its spacetime integral times $(-i\lambda)^2/S$. What are the symmetry factors S of (i) and (ii)?

KEY **D 48 and 16**

A counts, for (i), only the permutations of the four lines, $4! = 24$, and for (ii) only the swap of the two connecting lines and the reversal of each self-loop, $2 \times 2 \times 2 = 8$; both forget that the two vertices can also be exchanged: when a diagram is unchanged by that swap, the $1/2!$ of the second-order Dyson term is not cancelled by relabelling the vertices. B and C each make that omission in one diagram but not the other. D is correct. By contractions, the second-order term carries $1/(2! \times 4! \times 4!) = 1/1152$; diagram (i) arises in $4! = 24$ ways, giving $24/1152 = 1/48$, and diagram (ii) in $6 \times 6 \times 2 = 72$ ways, choosing two of the four fields at each vertex for its self-loop and then pairing the remaining two with two, giving $72/1152 = 1/16$. Counting automorphisms agrees: $4! \times 2 = 48$ for (i) and $2 \times 2 \times 2 \times 2 = 16$ for (ii).

U2

4

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-04.02 · UQFTI-04.03

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. A real scalar field ϕ , canonically normalised so that $[\phi(x, t), \pi(y, t)] = i\delta^3(x - y)$, is interacting and invariant under $\phi \rightarrow -\phi$. Its lightest particle has physical mass m , and the field can create states of any odd number of these particles. Its exact two-point function has the Källén-Lehmann form $\int_0^\infty (d\mu^2/2\pi) \rho(\mu^2) i/(p^2 - \mu^2 + i\epsilon)$, with $\rho(\mu^2) = 2\pi Z\delta(\mu^2 - m^2)$ plus a continuum. Which statement is correct?

KEY **B The continuum begins at $\mu^2 = 9m^2$, $\rho(\mu^2) \geq 0$, and the canonical commutator fixes the total weight $\int \rho d\mu^2/2\pi = 1$, so $0 \leq Z \leq 1$.**

A ignores the symmetry: under $\phi \rightarrow -\phi$ a two-particle state is even and ϕ is odd, so $\langle \Omega | \phi(0) | \text{two particles} \rangle = 0$, and the lightest multiparticle state ϕ can create has three particles. C contradicts the derivation itself: $\rho(\mu^2) = \sum_\lambda 2\pi\delta(\mu^2 - m_\lambda^2) |\langle \Omega | \phi(0) | \lambda_0 \rangle|^2$ is a sum of squared moduli, non-negative whatever the sign of the interaction, so Z cannot exceed 1. D makes two errors. The threshold is the square of the summed masses, $(3m)^2 = 9m^2$, not the sum of the squares; and the canonical commutator fixes the sum $Z + \int \rho_{\text{cont}} d\mu^2/2\pi = 1$, not Z alone, so $Z = 1$ only when the continuum is empty, for a free field. B is correct: the threshold is $(3m)^2$, positivity holds term by term, and the sum rule with a non-negative continuum gives $0 \leq Z \leq 1$.

U1

5

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-04.06 · UQFTI-04.07

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. In the scalar Yukawa model, $\mathcal{L}_{\text{int}} = -g\psi^\dagger\psi\phi$, with ψ a complex scalar of mass m_ψ and ϕ a real scalar of mass $m_\phi < 2m_\psi$. To order g^2 the amplitude for $\psi(p_1)\bar{\psi}(p_2) \rightarrow \psi(p_3)\bar{\psi}(p_4)$ is $M = -g^2[1/(s - m_\phi^2) + 1/(t - m_\phi^2)]$, with $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$, the first term from annihilation through ϕ and the second from ϕ exchange. Defining s , t and u in the same way for $\psi(p_1)\psi(p_2) \rightarrow \psi(p_3)\psi(p_4)$, what is the order- g^2 amplitude for that process?

KEY **D $-g^2[1/(t - m_\phi^2) + 1/(u - m_\phi^2)]$**

A copies the $\psi\bar{\psi}$ amplitude without crossing it. Crossing the incoming antiparticle into an outgoing particle and the outgoing antiparticle into an incoming one exchanges the roles of s and u , so the annihilation term becomes a u -channel exchange. C keeps an s -channel pole as well, but two ψ particles carry ψ number 2, which the vertex $\psi^\dagger\psi\phi$ conserves, so they cannot annihilate into a single ϕ . B treats the identical particles as fermions; ψ is a scalar, so the diagrams with the final particles exchanged add, and the amplitude must be symmetric, not antisymmetric, under $t \leftrightarrow u$. D is correct: ϕ is exchanged in the t and u channels with the same sign, the amplitude is symmetric under $t \leftrightarrow u$ as Bose symmetry requires, and $s + t + u = 4m_\psi^2$ in both processes.

U2

6

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-01.01 · UQFTI-05.04

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. In ϕ^4 theory with $\mathcal{L}_{\text{int}} = -(\lambda/4!)\phi^4$ and $\lambda = 0.60$, two particles collide at a centre-of-mass energy $\sqrt{s} = 4.0$ GeV, far enough above threshold that their mass can be neglected. At tree level the amplitude for $\phi\phi \rightarrow \phi\phi$ is $M = -\lambda$, and for a $2 \rightarrow 2$ process with equal masses in and out, $d\sigma/d\Omega = |M|^2/(64\pi^2 s)$ in the centre-of-mass frame. Take $(\hbar c)^2 = 0.3894$ GeV² mb. What is the total cross-section for $\phi\phi \rightarrow \phi\phi$?

KEY **A 87 nb**

B integrates $d\sigma/d\Omega$ over the full sphere without the factor $1/2$ for two identical particles in the final state, so every final configuration is counted twice, once at θ and once at $\pi - \theta$. C is $d\sigma/d\Omega$ itself, 13.9 nb per steradian, never integrated over angles. D takes the vertex to be $-i\lambda/4!$, reading it straight off the Lagrangian; the $4!$ ways of attaching four external lines to the four identical fields at the vertex cancel the $1/4!$, which is why the coupling is written with it, and leaving it in divides σ by 576. A is correct: $\sigma = \frac{1}{2} \times 4\pi \times \lambda^2/(64\pi^2 s) = \lambda^2/(32\pi s) = 0.36/(32\pi \times 16 \text{ GeV}^2) = 2.24 \times 10^{-4} \text{ GeV}^{-2}$, and multiplying by $(\hbar c)^2 = 0.3894 \text{ GeV}^2 \text{ mb}$ gives $8.72 \times 10^{-5} \text{ mb} = 87 \text{ nb}$.

U3

7

CORE
COURSE

MULTIPLE CHOICE · ROUTINE · UQFTI-07.02 · UQFTI-07.05

[1]

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. Two identical Dirac fermions of mass m scatter, $\psi(p_1)\psi(p_2) \rightarrow \psi(p_3)\psi(p_4)$, through the Yukawa interaction $\mathcal{L}_{\text{int}} = -g\phi\bar{\psi}\psi$, where ϕ is a real scalar of mass m_ϕ . At tree level two diagrams contribute, one with momentum transfer squared $t = (p_1 - p_3)^2$ and one with $u = (p_1 - p_4)^2$. In the non-relativistic limit $\bar{u}(p_3)u(p_1) \rightarrow 2m$ times a Kronecker delta between the spin labels. Which statement is correct?

KEY **C The two diagrams have a relative minus sign, because their final states differ by an exchange of anticommuting operators; the t-channel term alone is the Born amplitude of an attractive Yukawa potential.**

A takes the sign from the wrong line: the relative sign is fixed by the external fermions, not by the exchanged boson, since the two Wick contractions pair the final-state operators with the vertex fields in orders that differ by one transposition of anticommuting operators. B mistakes an exchange sign for the sign of a force. The force is read from the t-channel term alone, which in the non-relativistic limit is the Born amplitude of $V(r) = -(g^2/4\pi r)e^{-(m_\phi r)}$; the u-channel term is the exchange term that antisymmetrises the two-fermion state, and scalar exchange attracts every pair, fermion-fermion and fermion-antifermion alike. D confuses the amplitude with the phase-space integral: the u-channel contraction is a distinct term of the amplitude and must be included, while the $1/2$ for identical particles belongs to the integral over final states in the total cross-section. C is correct: M is antisymmetric under exchange of the two final fermions, and each term on its own describes an attractive Yukawa force.

U1

Work in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. In Feynman gauge the four photon operators of one momentum k , taken along z , obey $[a^\lambda(k), a^{\lambda'\dagger}(k')] = -g^{\lambda\lambda'}(2\pi)^3 2|k| \delta^3(k - k')$ for $\lambda, \lambda' = 0, 1, 2, 3$, where $\lambda = 0$ labels the timelike polarisation $\epsilon^0 = (1, 0, 0, 0)$, $\lambda = 1, 2$ the transverse ones and $\lambda = 3$ the longitudinal one, $\epsilon^3 = (0, 0, 0, 1)$. The Gupta-Bleuler condition requires the positive-frequency part of $\partial_\mu A^\mu$ to annihilate every physical state, which for this momentum reads $(a^0 - a^3)|\psi\rangle = 0$. Which statement is correct?

KEY B Physical states may contain $(a^{0\dagger} - a^{3\dagger})$ acting on a transverse state; such a state has zero norm and adds nothing to the energy or to any gauge-invariant expectation value.

A overstates the condition: it does not remove timelike and longitudinal photons but admits them in the combination created by $a^{0\dagger} - a^{3\dagger}$, which commutes with $a^0 - a^3$ because $[a^0, a^{0\dagger}] = -1$ and $[a^3, a^{3\dagger}] = +1$ cancel. C has the sign of the longitudinal norm wrong: $-g^{33} = +1$, so $a^{3\dagger}|0\rangle$ has positive norm, and the zero norm of the admitted combination comes from a positive norm cancelling the timelike photon's negative one. D asks the creation part, proportional to $a^{0\dagger} - a^{3\dagger}$, to annihilate the state as well; no non-zero state of the Fock space, the vacuum included, satisfies that, which is why Gupta and Bleuler kept only the positive-frequency half, enough to make $\langle\psi|\partial_\mu A^\mu|\psi\rangle = 0$. B is correct: $(a^{0\dagger} - a^{3\dagger})|T\rangle$ with $|T\rangle$ transverse satisfies the condition, has norm $\langle T|(a^0 - a^3)(a^{0\dagger} - a^{3\dagger})|T\rangle = 0$, is orthogonal to every physical state, and drops out of $\langle H\rangle$ because $a^{3\dagger}a^3 - a^{0\dagger}a^0$ has zero expectation value whenever $(a^0 - a^3)|\psi\rangle = 0$.

U1

A free real scalar field of mass m is written, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$, as $\phi(x) = \int d^3p / ((2\pi)^3 2E_p) [a(p)e^{-ip\cdot x} + a^\dagger(p)e^{ip\cdot x}]$, where $E_p = \sqrt{p^2 + m^2}$, $p^0 = E_p$ in the exponents and $[a(p), a^\dagger(q)] = (2\pi)^3 2E_p \delta^3(p - q)$. Its vacuum two-point function is $D(x - y) = \langle 0|\phi(x)\phi(y)|0\rangle = \int d^3p / ((2\pi)^3 2E_p) e^{-ip\cdot(x - y)}$. Take $\hbar c = 197.327$ MeV fm.

(a) DERIVE · U2

Starting from the definition $D_F(x - y) = \langle 0|T\phi(x)\phi(y)|0\rangle$, derive the representation $D_F(x - y) = \int d^4p / (2\pi)^4 i e^{-ip\cdot(x - y)} / (p^2 - m^2 + i\epsilon)$, evaluating the p^0 integral with the contour closed in whichever half-plane each time ordering requires.

- 1 $p^2 - m^2 + i\epsilon = (p^0)^2 - E_p^2 + i\epsilon$ vanishes at $p^0 = \pm(E_p - i\epsilon')$, with $\epsilon' = \epsilon/(2E_p)$: the positive-energy pole lies just below the real axis and the negative-energy pole just above it
- 2 for $x^0 > y^0$ the factor $e^{-ip^0(x^0 - y^0)}$ decays in the lower half-plane, so the contour is closed below, clockwise, enclosing $p^0 = E_p$: $\int dp^0 / (2\pi) i e^{-ip^0(x^0 - y^0)} / [(p^0 - E_p)(p^0 + E_p)] = (-2\pi i / 2\pi) \times i e^{-iE_p(x^0 - y^0)} / (2E_p) = e^{-iE_p(x^0 - y^0)} / (2E_p)$, and the remaining d^3p integral is $D(x - y)$
- 3 for $x^0 < y^0$ the contour is closed above, enclosing $p^0 = -E_p$, which gives $e^{iE_p(x^0 - y^0)} / (2E_p)$; relabelling $p \rightarrow -p$ turns the d^3p integral into $D(y - x)$, so the integral equals $\theta(x^0 - y^0)D(x - y) + \theta(y^0 - x^0)D(y - x) = \langle 0|T\phi(x)\phi(y)|0\rangle$

The usual slip is to close the contour in the wrong half-plane for a given time order, which yields $D(y - x)$ for $x^0 > y^0$, an anti-time-ordered product. The sign of $x^0 - y^0$ decides where the exponential decays, and nothing else decides the half-plane.

(b) SKETCH · U1

[2]

The retarded propagator is $D_R(x - y) = \theta(x^0 - y^0) \langle 0 | [\phi(x), \phi(y)] | 0 \rangle$, whose momentum-space form is $i/((p^0 + i\epsilon)^2 - |\mathbf{p}|^2 - m^2)$. Sketch the complex p^0 -plane at fixed $\mathbf{p}$, marking the poles of D_F and of D_R and the contour used when $x^0 < y^0$, and state where in spacetime each propagator is non-zero.

- 1 the sketch shows the real p^0 axis with the poles of D_F at $+E_p - i\epsilon'$, below, and $-E_p + i\epsilon'$, above; the poles of D_R both below the axis, at $\pm E_p - i\epsilon'$; and the contour for $x^0 < y^0$ closed by a large semicircle in the upper half-plane, enclosing the negative-energy pole of D_F and neither pole of D_R
- 2 D_R is therefore zero for $x^0 < y^0$ and, being a commutator, also zero at spacelike separation, so it is supported on and inside the forward light cone; D_F is non-zero for both time orders and at spacelike separation, where it falls off as $e^{(-mr)}$, r being the invariant spacelike distance

(c) CALCULATE · U3

[2]

Two static point sources, $j(x) = g[\delta^3(x - x_1) + \delta^3(x - x_2)]$, coupled through $\mathcal{L}_{\text{int}} = j\phi$ and held a distance $r = |x_1 - x_2|$ apart for a long time T , give a vacuum persistence amplitude $\langle 0|0 \rangle \propto e^{(-iE(r)T)}$, with interaction energy $E(r) = -(g^2/(4\pi r))e^{(-mr)}$. For a scalar of mass $m = 550$ MeV, $g^2/(4\pi) = 0.80$ and $r = 0.70$ fm, calculate $E(r)$ in MeV and state the range of the force in femtometres.

- 1 in natural units $r = 0.70 \text{ fm}/(197.327 \text{ MeV fm}) = 3.547 \times 10^{-3} \text{ MeV}^{-1}$, so $1/r = 281.9 \text{ MeV}$, and $mr = 550 \times 0.70/197.327 = 1.951$, with $e^{(-1.951)} = 0.1421$
- 2 $E = -0.80 \times 281.9 \text{ MeV} \times 0.1421 = -32.1 \text{ MeV}$, negative because scalar exchange between like sources attracts; the range is $1/m = 197.327 \text{ MeV fm}/550 \text{ MeV} = 0.359 \text{ fm}$

(d) EXPLAIN · U1

[1]

$D_F(x - y)$ is non-zero when $x - y$ is spacelike. Explain why this does not allow a signal to travel faster than light.

- 1 whether a measurement at y can affect one at x is decided by the commutator $[\phi(x), \phi(y)] = D(x - y) - D(y - x)$, not by an amplitude; for spacelike $x - y$ a proper Lorentz transformation carries $x - y$ into $-(x - y)$ and leaves D invariant, so the commutator vanishes, the amplitude for propagation from y to x cancelling that from x to y , while D_F , an amplitude, need not vanish

Answers that call D_F 'exponentially small' outside the light cone miss the point: a small non-zero commutator would still be a violation. Causality lives in the commutator, and the retarded propagator built from it is the one that describes the response to a source.

10
CORE
COURSE

STRUCTURED · DEMANDING · UQFTI-02.01 · UQFTI-02.09 · UQFTI-03.02 · UQFTI-03.07 · UQFTI-04.05 · UQFTI-04.07 · UQFTI-04.09 · UQFTI-05.03

[10]

Two real scalar fields, ϕ of mass m_ϕ and χ of mass m_χ , live in four spacetime dimensions and interact through $\mathcal{L}_{\text{int}} = -(g/2)\phi\chi^2 - (\kappa/2)\phi\partial_\mu\chi\partial^\mu\chi$, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. Treat both couplings to the lowest order at which each process occurs. For the two-body decay of a particle of mass m_ϕ into particles of momentum $\mathbf{p}$ in its rest frame, Lorentz-invariant phase space gives $\int d\Pi_2 = p/(4\pi m_\phi)$ when the two final particles are distinguishable, and $\Gamma = (1/(2m_\phi)) \int |\mathcal{M}|^2 d\Pi_2$. Take $\hbar = 6.582 \times 10^{-25} \text{ GeV s}$.

(a) DETERMINE · U2

[2]

Determine the Feynman rule for the $\phi\chi\chi$ vertex, taking every momentum as incoming and p_2 and p_3 as the momenta of the two χ lines, and give the mass dimensions of g and κ .

- 1 the term $-(g/2)\phi\chi^2$ gives $i \times (-g/2) \times 2 = -ig$, the 2 counting the ways the two χ fields meet the two χ lines; $[g] = 4 - 3 = 1$, a super-renormalisable coupling
- 2 each ∂_μ acting on an incoming χ line of momentum p gives $-ip_\mu$, so $-(\kappa/2)\phi\partial_\mu\chi\partial^\mu\chi$ gives $i \times (-\kappa/2) \times 2 \times (-ip_2) \cdot (-ip_3) = i\kappa p_2 \cdot p_3$; the full vertex is $-ig + i\kappa p_2 \cdot p_3$, with $[\kappa] = 4 - 1 - 2 - 2 = -1$, a non-renormalisable coupling

Reading the vertex straight off the coefficient gives $-ig/2$ and loses the factor 2 from the identical χ fields, in the derivative term as well. Candidates who attach a momentum to only one χ line miss that the derivative vertex depends on both, and on the product of the two.

(b) SHOW THAT · U2

[2]

Set $\kappa = 0$. By counting the Wick contractions in the order- g^2 term of the Gell-Mann-Low numerator, show that the one-loop ϕ self-energy, a loop of two χ propagators, carries a symmetry factor of 2, and that the one-loop χ self-energy, a loop of one ϕ and one χ propagator, carries a symmetry factor of 1. Check each by counting the diagram's automorphisms.

- 1 the order- g^2 term is $(1/2!)(-ig/2)^2 \int \phi(z)\chi(z)^2\phi(w)\chi(w)^2$, a prefactor $(-ig)^2/8$; in $\langle 0 | T\phi(x)\phi(y) \dots | 0 \rangle$, $\phi(x)$ joins $\phi(z)$ or $\phi(w)$ (2 ways), $\phi(y)$ the other, and the two $\chi(z)$ fields pair with the two $\chi(w)$ fields in 2 ways, so 4 contractions give $(-ig)^2 \times 4/8 = (-ig)^2/2$: $S = 2$, matching the one automorphism that swaps the two χ lines
- 2 in $\langle 0 | T\chi(x)\chi(y) \dots | 0 \rangle$, $\chi(x)$ joins any of the four χ fields (4 ways), $\chi(y)$ either χ field at the other vertex (2 ways), and the remaining χ and ϕ fields pair uniquely, so 8 contractions give $(-ig)^2 \times 8/8$: $S = 1$, as the loop's two lines, one ϕ and one χ , cannot be exchanged

Candidates who carry over from ϕ^4 the habit of a $1/2$ for every bubble put it on the χ self-energy too. A loop of two lines of different fields has no symmetry, and the direct count shows the $1/2!$ and both $1/2$ s cancelled completely.

(c) DETERMINE · U2

[2]

Still with $\kappa = 0$, determine the tree-level amplitude M for $\chi(p_1)\chi(p_2) \rightarrow \chi(p_3)\chi(p_4)$ in terms of the Mandelstam variables s , t and u , check its symmetry under $t \leftrightarrow u$ and the value of $s + t + u$, and say why $\sqrt{Z} = 1$ may be used on each external leg at this order.

- 1 ϕ is exchanged in all three channels, each giving $(-ig)^2 \times i/(q^2 - m_\phi^2)$, so $M = -g^2[1/(s - m_\phi^2) + 1/(t - m_\phi^2) + 1/(u - m_\phi^2)]$, dimensionless as a $2 \rightarrow 2$ amplitude in four dimensions must be, since $[g^2] = 2$
- 2 M is symmetric under $t \leftrightarrow u$, as identical bosons in the final state require, and $s + t + u = 4m_\chi^2$; because $Z = 1 + O(g^2)$ and the mass shift is $O(g^2)$, $\sqrt{Z}$ on each leg and the physical mass change M only at order g^4

The commonest error is to drop the s -channel by analogy with $\psi\psi \rightarrow \psi\psi$ in the scalar Yukawa model, where ψ number forbids it. Here χ carries only the charge of the symmetry $\chi \rightarrow -\chi$, which two χ particles do not carry, so they can annihilate into one ϕ .

(d) PREDICT · U1

[2]

The masses are now chosen so that $m_\phi > 2m_\chi$. Predict what becomes of the s-channel term of the amplitude in (c) as s approaches m_ϕ^2 , and of the ϕ particle's place in the LSZ formula, giving the reason, and say whether the t- and u-channel poles are affected.

- 1 the pole at $s = m_\phi^2$ now lies inside the physical region $s \geq 4m_\chi^2$, so the tree amplitude diverges there; the ϕ self-energy acquires an imaginary part from the open $\chi\chi$ channel, and resumming it turns $1/(s - m_\phi^2)$ into $1/(s - m_\phi^2 + im_\phi\Gamma)$, a Breit-Wigner peak of finite height
- 2 ϕ is no longer stable, so it is not an asymptotic state: its pole leaves the real axis, LSZ's isolated one-particle pole is lost and ϕ appears only as a resonance in $\chi\chi$ scattering; the t- and u-channel poles at $t = m_\phi^2$ and $u = m_\phi^2$ stay outside the physical region, where $t \leq 0$ and $u \leq 0$, and still describe a force of range $1/m_\phi$

Answers that keep ϕ as an external state of LSZ with a real mass have missed the assumption the reduction formula states: stable particles. A pole inside the physical region is the signature of a particle that can be produced on shell, and therefore decays.

(e) CALCULATE · U3

[2]

Restore κ . With $m_\phi = 0.90$ GeV, $m_\chi = 0.14$ GeV, $g = 0.50$ GeV and $\kappa = 0.80$ GeV⁻¹, calculate the decay rate $\Gamma(\phi \rightarrow \chi\chi)$ in MeV and the lifetime of ϕ in seconds.

- 1 on shell $2p_2 \cdot p_3 = m_\phi^2 - 2m_\chi^2$, so $M = -g + \kappa(m_\phi^2 - 2m_\chi^2)/2 = -0.500$ GeV + 0.308 GeV = -0.192 GeV, and the χ momentum is $p = \sqrt{(m_\phi^2/4 - m_\chi^2)} = 0.428$ GeV
- 2 with 1/2 for two identical χ , $\Gamma = (1/(2m_\phi)) \times |M|^2 \times \frac{1}{2} \times p/(4\pi m_\phi) = |M|^2 p/(16\pi m_\phi^2) = 3.86 \times 10^{-4}$ GeV = 0.386 MeV, and $\tau = \hbar/\Gamma = 1.71 \times 10^{-21}$ s

Two slips are set for here. Omitting the 1/2 for identical final particles doubles Γ to 0.772 MeV. Giving the derivative vertex the wrong sign makes the two couplings reinforce, $\Gamma = 6.86$ MeV, eighteen times too large: the sign fixed in (a) decides whether they cancel or add.

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[10]

A real spin-0 boson ϕ of mass m_ϕ couples to a Dirac fermion ψ of mass m either as a scalar, $\mathcal{L}_S = -g\phi\bar{\psi}\psi$, or as a pseudoscalar, $\mathcal{L}_P = -ig\phi\bar{\psi}\gamma^5\psi$, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$. Use the spin sums $\sum u(k)\bar{u}(k) = \not{K} + m$ and $\sum v(k)\bar{v}(k) = \not{K} - m$; the identities $\text{tr } 1 = 4$ and $\text{tr}(\not{a}\not{b}) = 4a \cdot b$; the vanishing of the trace of an odd number of γ matrices; $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$, $(\gamma^5)^2 = 1$ and $\gamma^0(\gamma^5)^\dagger\gamma^0 = -\gamma^5$. For the decay $\phi \rightarrow \psi(k_1)\bar{\psi}(k_2)$ write $\beta = \sqrt{1 - 4m^2/m_\phi^2}$, the speed of either fermion in the ϕ rest frame; then $\int d\Pi_2 = \beta/(8\pi)$, and $\Gamma = (1/(2m_\phi)) \times (\text{spin-summed } |M|^2) \times \int d\Pi_2$. Take $\hbar = 6.582 \times 10^{-25}$ GeV s.

(a) DERIVE · U2

[2]

For the scalar coupling the amplitude is $M_S = -g \bar{u}(k_1)v(k_2)$. Using Casimir's trick with the spin sums, derive the spin-summed $|M_S|^2$ in terms of g , m_ϕ and β .

- 1 $(\bar{u} v)^* = \bar{v} u$, so $\sum |M_S|^2 = g^2 \sum \bar{u}(k_1)v(k_2)\bar{v}(k_2)u(k_1) = g^2 \text{tr}[(\not{K}_1 + m)(\not{K}_2 - m)] = g^2(4k_1 \cdot k_2 - 4m^2)$, the terms linear in m containing an odd number of γ matrices
- 2 on shell $2k_1 \cdot k_2 = m_\phi^2 - 2m^2$, so $\sum |M_S|^2 = 2g^2(m_\phi^2 - 4m^2) = 2g^2m_\phi^2\beta^2$

Candidates who use $\sum v\bar{v} = \not{K} + m$ lose the sign of the mass term and obtain $2g^2m_\phi^2$, which is the pseudoscalar result, for the scalar; the β^2 is the whole physics of the part.

(b) SHOW THAT · U2

[2]

For the pseudoscalar coupling the amplitude is $M_P = -g \bar{u}(k_1)i\gamma^5v(k_2)$. Show that the spin-summed $|M_P|^2$ is $2g^2m_\phi^2$, with no factor of β^2 .

- 1 $(\bar{u} i\gamma^5 v)^* = \bar{v} \gamma^0(i\gamma^5)^\dagger\gamma^0 u = \bar{v}(-i)(-\gamma^5)u = \bar{v} i\gamma^5 u$, so $\sum |M_P|^2 = g^2 \text{tr}[(\not{K}_1 + m)i\gamma^5(\not{K}_2 - m)i\gamma^5] = -g^2 \text{tr}[(\not{K}_1 + m)\gamma^5(\not{K}_2 - m)\gamma^5]$
- 2 anticommuting γ^5 through, $\gamma^5(\not{K}_2 - m)\gamma^5 = -\not{K}_2 - m$, so $\sum |M_P|^2 = g^2 \text{tr}[(\not{K}_1 + m)(\not{K}_2 + m)] = g^2(4k_1 \cdot k_2 + 4m^2) = 2g^2m_\phi^2$: the γ^5 reverses the sign of the mass term

The error to catch is forgetting that $\gamma^0(\gamma^5)^\dagger\gamma^0 = -\gamma^5$ when forming the conjugate amplitude. It flips the overall sign and gives $-2g^2m_\phi^2$, which a candidate should recognise at once as impossible for a sum of squared moduli.

(c) CALCULATE · U3

[2]

For $m_\phi = 1.50$ GeV, $m = 0.60$ GeV and $g = 0.30$, calculate $\Gamma(\phi \rightarrow \psi\bar{\psi})$ and the ϕ lifetime in seconds for each coupling.

- 1 $\beta = \sqrt{1 - 4 \times 0.36/2.25} = 0.600$, and $\Gamma_S = (1/(2m_\phi)) \times 2g^2m_\phi^2\beta^2 \times \beta/(8\pi) = g^2m_\phi\beta^3/(8\pi) = 1.16 \times 10^{-3}$ GeV = 1.16 MeV, so $\tau_S = \hbar/\Gamma_S = 5.67 \times 10^{-22}$ s
- 2 $\Gamma_P = g^2m_\phi\beta/(8\pi) = 3.22$ MeV and $\tau_P = 2.04 \times 10^{-22}$ s; the ratio $\Gamma_P/\Gamma_S = 1/\beta^2 = 2.78$

Candidates who include a factor 1/2 for identical particles halve both rates; ψ and $\bar{\psi}$ are distinct particles, so no such factor applies.

(d) EXPLAIN · U1

[2]

Explain the powers β^3 and β in the two rates by the parity of ϕ and the orbital angular momentum of the fermion pair near threshold.

- 1 a fermion and its antifermion have opposite intrinsic parities, so a pair with orbital angular momentum L has parity $(-1)^{L+1}$; total $J = 0$ requires the pair's spin S to equal L , so S and L are both 0 or both 1
- 2 the scalar coupling makes ϕ a 0^+ state, which needs L odd, the $^3P_0$ pair with $L = S = 1$, and a P-wave rate carries $p^{2L+1} \propto \beta^3$; the pseudoscalar is 0^- , which allows the $^1S_0$ pair with $L = 0$ and a rate $\propto \beta$, so near threshold the pseudoscalar decays faster by $1/\beta^2$

A frequent answer attributes β^3 to 'suppression by the fermion mass' without saying why only the scalar is suppressed. The difference is angular momentum: parity forbids the scalar from producing the pair in an S-wave.

(e) JUSTIFY · U1

[2]

In the one-loop ϕ self-energy built from a ψ loop, Wick's theorem supplies a factor -1 . Justify this factor from the anticommutation of the fields, and confirm it from the optical theorem, which requires $\text{Im } M(\phi \rightarrow \phi) = m_\phi \Gamma(\phi \rightarrow \psi\bar{\psi})$ once $m_\phi > 2m$.

- 1 the two vertices supply $\bar{\psi}(x)\psi(x)\bar{\psi}(y)\psi(y)$; writing the contractions in the order of a trace, $\psi(x)\bar{\psi}(y)\psi(y)\bar{\psi}(x) \rightarrow \text{tr}[S_F(x-y)S_F(y-x)]$, means moving $\bar{\psi}(x)$ from the front to the back past three anticommuting fields, which gives $(-1)^3 = -1$
- 2 on the cut the fermion carries k_1 and the antifermion line carries $-k_2$, so the numerator trace is $\text{tr}[(\not{k}_1 + m)(-\not{k}_2 + m)] = -\text{tr}[(\not{k}_1 + m)(\not{k}_2 - m)] = -\Sigma|M_S|^2/g^2$; only the loop's -1 makes $2 \text{Im } M(\phi \rightarrow \phi) = \int d\Pi_2 \Sigma|M_S|^2 = 2m_\phi \Gamma$ positive, as unitarity demands

Candidates often treat the -1 as a convention that could be absorbed into g^2 . It cannot: g^2 is positive in both places, and unitarity fixes the sign of the imaginary part. Commuting fields would give a negative width, a propagator growing in time: the perturbative face of the spin-statistics theorem.

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[11]

Real scalar ϕ^4 theory, $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - (\lambda/4!)\phi^4$, in natural units $\hbar = c = 1$ with the metric $(+, -, -, -)$, is regularised dimensionally in $d = 4 - 2\epsilon$ dimensions, with λ replaced by $\lambda\mu^{2\epsilon}$ so that λ stays dimensionless, μ being an arbitrary scale. Use the Feynman-parameter identity $1/(AB) = \int_0^1 dx [xA + (1-x)B]^{-2}$, the Euclidean master integral $\int d^d\ell E/(2\pi)^d (\ell^2 + \Delta)^{-2} = \Gamma(2 - d/2) (4\pi)^{-(d/2)} \Delta^{-(d/2 - 2)}$, and $\Gamma(\epsilon) = 1/\epsilon - \gamma_E + O(\epsilon)$. At one loop the field-strength counterterm vanishes, so the bare coupling is $\lambda_0 = \mu^{2\epsilon}(\lambda + \delta_\lambda)$, where the counterterm vertex is $-i\delta_\lambda$.

(a) SHOW THAT · U2

[3]

The s-channel one-loop diagram for $\phi\phi \rightarrow \phi\phi$, with incoming momenta p_1 and p_2 , $p = p_1 + p_2$ and $s = p^2$, carries a symmetry factor of 2. Show that, below the threshold $s = 4m^2$, its contribution to iM , with an overall factor $\mu^\epsilon(2\epsilon)$ removed, is $(i\lambda^2/(32\pi^2)) \int_0^1 dx [1/\epsilon - \gamma_E + \ln 4\pi - \ln(\Delta_s/\mu^2)] + O(\epsilon)$, where $\Delta_s = m^2 - x(1-x)s$.

- 1 two vertices, each $-i\lambda\mu^\epsilon(2\epsilon)$, two propagators and the factor $1/2$ give $iM_s = \frac{1}{2}(-i\lambda\mu^\epsilon(2\epsilon))^2 \int d^d\ell/(2\pi)^d [i/(\ell^2 - m^2)][i/((\ell + p)^2 - m^2)] = \frac{1}{2}\lambda^2\mu^\epsilon(4\epsilon) \int d^d\ell/(2\pi)^d 1/[(\ell^2 - m^2)((\ell + p)^2 - m^2)]$
- 2 Feynman parameters turn the denominator into $[(\ell + xp)^2 - \Delta_s]^2$ with $\Delta_s = m^2 - x(1-x)s$; shifting $\ell \rightarrow \ell - xp$ and rotating $\ell^0 = i\ell_E^0$, so that $\ell^2 = -\ell_E^2$ and $d^d\ell = i d^d\ell_E$, gives the loop integral as $i \int_0^1 dx \Gamma(\epsilon)(4\pi)^{-(2+\epsilon)} \Delta_s^{-\epsilon}(-\epsilon)$
- 3 expanding, $(4\pi)^{-(2+\epsilon)} = (1 + \epsilon \ln 4\pi)/(16\pi^2)$ and $\mu^\epsilon(2\epsilon)\Delta_s^{-\epsilon}(-\epsilon) = 1 - \epsilon \ln(\Delta_s/\mu^2)$, so $iM_s = \mu^\epsilon(2\epsilon)(i\lambda^2/(32\pi^2)) \int_0^1 dx [1/\epsilon - \gamma_E + \ln 4\pi - \ln(\Delta_s/\mu^2)] + O(\epsilon)$: one $\mu^\epsilon(2\epsilon)$ belongs with the tree-level coupling and the other makes the logarithm dimensionless

The factor i from the Wick rotation is the one most often lost, and with it the relative phase between the loop and the tree amplitude $-i\lambda$, so the result can no longer be combined with the counterterm. Candidates who expand $\Gamma(\epsilon)$ but not $\Delta^{-\epsilon}(-\epsilon)$ are left with the logarithm of a dimensionful quantity, which the $\mu^\epsilon(2\epsilon)$ is there to prevent.

(b) SHOW THAT · U2

[2]

Including the t- and u-channel diagrams and the counterterm, the renormalised coupling λ is defined by the condition $M = -\lambda$ at $s = 4m^2$, $t = u = 0$. Show that $M(s, t, u) = -\lambda - (\lambda^2/(32\pi^2)) \sum \int_0^1 dx \ln[\Delta(q^2)/\Delta(q_0^2)]$, the sum running over the three channels $q^2 = s, t, u$ with reference values $q_0^2 = 4m^2, 0, 0$ and $\Delta(q^2) = m^2 - x(1-x)q^2$, and that neither ϵ nor μ survives in it.

- 1 at one loop $M = -\lambda + (\lambda^2/(32\pi^2)) \sum \int dx [1/\epsilon - \gamma_E + \ln 4\pi - \ln(\Delta(q^2)/\mu^2)] - \delta_\lambda$, and the condition at the reference point fixes $\delta_\lambda = (\lambda^2/(32\pi^2)) \sum \int dx [1/\epsilon - \gamma_E + \ln 4\pi - \ln(\Delta(q_0^2)/\mu^2)]$, which carries the whole pole
- 2 subtracting, $1/\epsilon, \gamma_E, \ln 4\pi$ and μ^2 cancel channel by channel, leaving $M = -\lambda - (\lambda^2/(32\pi^2)) \sum \int dx \ln[\Delta(q^2)/\Delta(q_0^2)]$: finite, independent of the regulator and of μ , and a relation between the amplitude at (s, t, u) and the coupling measured at the reference point

Candidates who fix δ_λ by minimal subtraction and then present the amplitude as a prediction have left $\ln \mu^2$ in it. It becomes a prediction only when λ is tied to a measurement at a stated point, or when the μ -dependence of λ is itself accounted for, which is what (c) does.

(c) DERIVE · U2

[2]

In minimal subtraction δ_λ contains only the poles of the three channels. Derive the one-loop beta function $\beta(\lambda) = \mu d\lambda/d\mu$ from the μ -independence of the bare coupling λ_0 .

- 1 each channel contributes a pole $\lambda^2/(32\pi^2\epsilon)$, so $\delta_\lambda = 3\lambda^2/(32\pi^2\epsilon)$, and $\mu d\lambda_0/d\mu = 0$ gives $0 = 2\epsilon(\lambda + 3\lambda^2/(32\pi^2\epsilon)) + \beta(1 + 6\lambda/(32\pi^2\epsilon))$
- 2 solving to order λ^2 , $\beta = -2\epsilon\lambda - 6\lambda^2/(32\pi^2) + 12\lambda^2/(32\pi^2) = -2\epsilon\lambda + 3\lambda^2/(16\pi^2)$, so as $\epsilon \rightarrow 0$, $\beta(\lambda) = 3\lambda^2/(16\pi^2)$, positive: the coupling grows with the scale

The $-2\epsilon\lambda$ term is not optional. Candidates who drop it before solving lose the cross term that turns $-6\lambda^2/(32\pi^2)$ into $+6\lambda^2/(32\pi^2)$, and obtain a beta function of the wrong sign: an asymptotically free ϕ^4 theory.

(d) CALCULATE · U3

[2]

In a model whose coupling is $\lambda(\mu_0) = 1.5$ at $\mu_0 = 1.0$ TeV, calculate the scale at which the one-loop running coupling has doubled and the scale of its Landau pole, both in GeV.

1 integrating $d\lambda/\lambda^2 = (3/(16\pi^2))d \ln \mu$ gives $1/\lambda(\mu) = 1/\lambda(\mu_0) - (3/(16\pi^2))\ln(\mu/\mu_0)$; $\lambda = 3.0$ when $\ln(\mu/\mu_0) = (16\pi^2/3)(1/1.5 - 1/3.0) = 17.55$, that is at $\mu = 1000 \text{ GeV} \times e^{17.55} = 4.2 \times 10^{10} \text{ GeV}$

2 $1/\lambda$ vanishes when $\ln(\mu_L/\mu_0) = 16\pi^2/(3 \times 1.5) = 35.09$, so the Landau pole is at $\mu_L = 1000 \text{ GeV} \times e^{35.09} = 1.7 \times 10^{18} \text{ GeV}$

Candidates who integrate $d\lambda/d \ln \mu = 3\lambda^2/(16\pi^2)$ as if one of the two factors of λ on the right were fixed at λ_0 obtain a power law, $\lambda \propto \mu^{(3\lambda_0/(16\pi^2))}$, which grows for ever and misses the pole. Using $\log_{10}$ for the logarithm moves both scales by many orders of magnitude.

(e) ESTIMATE · U4

[2]

Estimate the highest scale at which this one-loop running can be trusted, stating the criterion you use, and say what the Landau pole of (d) then tells you about the theory.

1 a stated criterion, for example $\lambda(\mu) \leq 4\pi$, where the one-loop term $3\lambda/(16\pi^2)$ has grown to about 0.24: $\ln(\mu/\mu_0) = (16\pi^2/3)(1/1.5 - 1/(4\pi)) = 30.9$, so $\mu \approx 3 \times 10^{16} \text{ GeV}$; whatever the criterion $\lambda \leq \lambda_{\text{max}}$, the trusted range stops short of the pole by $\ln(\mu_L/\mu) = 16\pi^2/(3\lambda_{\text{max}})$, which is $4\pi/3 = 4.19$ for $\lambda_{\text{max}} = 4\pi$, a factor $e^{(4.19)} \approx 66$ in scale and small beside the 15 orders of magnitude between μ_0 and μ_L

2 the pole lies beyond the scale at which the formula that predicts it holds, so one-loop running does not establish a singularity; it signals strong coupling, and with the non-perturbative evidence for triviality it means ϕ^4 makes sense only as an effective theory with a cutoff below the pole

The judgement tested is that the Landau pole marks where the approximation fails rather than a computed property of the theory. A candidate who quotes μ_L as a physical scale, or trusts the one-loop formula up to $\lambda \rightarrow \infty$, has not asked where the expansion parameter stops being small.

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[11]

Two distinct spinless particles, a of mass 0.400 GeV and b of mass 0.600 GeV, scatter elastically, $a + b \rightarrow a + b$, and near a centre-of-mass energy of 1.85 GeV the cross-section is dominated by an s-channel resonance X of mass m_X and total width Γ . Parts (a) to (d) treat X as a spin-0 particle coupled to ab through $\mathcal{L}_{\text{int}} = -gXab$, which gives $\Gamma(X \rightarrow ab) = g^2 p / (8\pi m_X^2)$, where p is the momentum of a or b in the centre-of-mass frame at $\sqrt{s} = m_X$; near the peak the amplitude is $M = -g^2 / (s - m_X^2 + im_X\Gamma)$. For two particles in and two out with these masses, $\sigma = |M|^2 / (16\pi s)$. To the accuracy of these data the line shape is $\sigma(\sqrt{s}) = \sigma_{\text{peak}}(\Gamma^2/4) / [(s - m_X^2)^2 + \Gamma^2/4]$. The optical theorem reads $\text{Im } M(\text{forward}) = 2\sqrt{s} p \sigma_{\text{tot}}$. Work in natural units with the metric (+, −, −, −), and take $(\hbar c)^2 = 0.3894 \text{ GeV}^2 \text{ mb}$ and $\hbar = 6.582 \times 10^{-25} \text{ GeV s}$. The table gives the measured elastic cross-section at seven energies.

(a) SHOW THAT · U2

[2]

Show that $\sigma_{\text{peak}} = (4\pi/p^2)B^2$, where $B = \Gamma(X \rightarrow ab)/\Gamma$ is the branching fraction to ab, so that $4\pi/p^2$ is the largest cross-section an s-wave resonance can give in this channel.

1 at $s = m_X^2$, $|M|^2 = g^4 / (m_X^2 \Gamma^2)$, so $\sigma_{\text{peak}} = g^4 / (16\pi m_X^4 \Gamma^2)$

2 with $g^2 = 8\pi m_X^2 \Gamma(X \rightarrow ab)/p$, $g^4 = 64\pi^2 m_X^4 \Gamma(X \rightarrow ab)^2 / p^2$, so $\sigma_{\text{peak}} = (4\pi/p^2)[\Gamma(X \rightarrow ab)/\Gamma]^2 = (4\pi/p^2)B^2$, which reaches $4\pi/p^2$ only for a purely elastic resonance, $B = 1$

Candidates who do not set $s = m_X^2$ at the peak keep a stray m_X^2/s , harmless here but a sign that the $1/s$ of the flux has not been followed. The point to see is that the bound arises because the same coupling sets both the width and the amplitude.

(b) DETERMINE · U4 [3]

By plotting a suitable linear function of the data, determine m_X , Γ and σ_{peak} .

- 1 the readings are symmetric about 1.850 GeV within their uncertainties, so $m_X = 1.850$ GeV, and the line shape gives $1/\sigma = (1/\sigma_{\text{peak}})[1 + 4(\sqrt{s} - m_X)^2/\Gamma^2]$, linear in $x = (\sqrt{s} - m_X)^2$
- 2 $1/\sigma$ at $x = 0, 64, 256$ and 576 MeV², averaging each symmetric pair, is 0.769, 1.239, 2.751 and 5.157 mb⁻¹; a straight-line fit gives an intercept of 0.768 mb⁻¹ and a slope of 7.64×10^{-3} mb⁻¹ MeV⁻²
- 3 $\sigma_{\text{peak}} = 1/(0.768 \text{ mb}^{-1}) = 1.30$ mb, and $\Gamma = 2\sqrt{(\text{intercept/slope})} = 2\sqrt{(0.768/0.00764)} \text{ MeV} = 20.1$ MeV, accepting 19.5 to 20.5 MeV

Reading the half-maximum points by straight-line interpolation between readings 8 MeV apart biases Γ upwards, to about 21.7 MeV here, because the line shape is convex there, beyond its inflection points at $|\sqrt{s} - m_X| = \Gamma/(2\sqrt{3}) \approx 6$ MeV, so each chord lies above the curve; the linearisation uses every reading. Candidates who plot $1/\sigma$ against $\sqrt{s} - m_X$ instead of its square obtain two branches that no single line fits.

(c) CALCULATE · U3 [2]

Calculate p at $\sqrt{s} = m_X$ and the s-wave unitarity limit $4\pi/p^2$ in millibarns, and hence B and the lifetime of X in seconds.

- 1 $p = \sqrt{[(s - (m_a + m_b)^2)(s - (m_a - m_b)^2)]/(2\sqrt{s})} = \sqrt{[(3.4225 - 1.0000)(3.4225 - 0.0400)]/3.700} \text{ GeV} = 0.774 \text{ GeV}$, so $4\pi/p^2 = 21.0 \text{ GeV}^{-2} = 21.0 \times 0.3894 \text{ mb} = 8.18 \text{ mb}$
- 2 $B = \sqrt{(\sigma_{\text{peak}}/(4\pi/p^2))} = \sqrt{(1.30/8.18)} = 0.399$, and $\tau = \hbar/\Gamma = 6.582 \times 10^{-25} \text{ GeV s}/0.0201 \text{ GeV} = 3.3 \times 10^{-23} \text{ s}$

Converting with $\hbar c$ instead of $(\hbar c)^2$, or comparing the peak in millibarns with $4\pi/p^2$ left in GeV^{-2} , gives $B = 0.56$ or $B = 0.25$; the second is close enough to the spin-1 value in (e) to mislead the whole analysis.

(d) DETERMINE · U2 [2]

Use the optical theorem to determine the total and the inelastic cross-sections at the peak, and determine $\Gamma(X \rightarrow ab)$ and g .

- 1 at the peak $M = ig^2/(m_X \Gamma)$, so $\text{Im } M = g^2/(m_X \Gamma) = 2m_X p \sigma_{\text{tot}}$ gives $\sigma_{\text{tot}} = g^2/(2m_X^2 p \Gamma) = (4\pi/p^2)B = 8.18 \text{ mb} \times 0.399 = 3.26 \text{ mb}$, and $\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{peak}} = 1.96 \text{ mb}$
- 2 $\Gamma(X \rightarrow ab) = B\Gamma = 0.399 \times 20.1 \text{ MeV} = 8.0 \text{ MeV}$, and $g^2 = 8\pi m_X^2 \Gamma(X \rightarrow ab)/p = 8\pi \times 3.4225 \text{ GeV}^2 \times 0.0080 \text{ GeV}/0.774 \text{ GeV} = 0.89 \text{ GeV}^2$, so $g = 0.94 \text{ GeV}$

The total cross-section at the peak is linear in B and the elastic one quadratic, so the inelastic share is $(4\pi/p^2)B(1 - B)$. Candidates who take σ_{tot} to be the measured elastic peak have assumed $B = 1$, which (c) has just excluded.

(e) EVALUATE · U4 [2]

A separate experiment counts the decays of X and finds $B = 0.43 \pm 0.05$. For a resonance of spin J the peak elastic cross-section is $(2J + 1)(4\pi/p^2)B^2$. Evaluate whether the data support $J = 0$ against $J = 1$, and say what further measurement on the same events would settle the spin directly.

- 1 the measured peak requires $B = \sqrt{(1.30/8.18)} = 0.40$ for $J = 0$ and $B = \sqrt{(1.30/(3 \times 8.18))} = 0.23$ for $J = 1$; against 0.43 ± 0.05 these lie 0.6σ and 4.0σ away, so $J = 0$ is supported and $J = 1$ excluded at about 4σ
- 2 the angular distribution in the centre-of-mass frame settles it directly: a spin-0 resonance decays isotropically, so $d\sigma/d\Omega$ is flat at the peak, while spin 1 gives $d\sigma/d\Omega \propto [P_1(\cos \theta)]^2 = \cos^2 \theta$

Candidates who reject $J = 1$ because its peak 'would exceed the unitarity limit' have the inequality backwards: a higher spin raises the limit, so the same peak needs a smaller B . The test needs the independent branching fraction, because the peak alone fixes only $(2J + 1)B^2$.

The free electromagnetic field, in natural units $\hbar = c = 1$ and Heaviside-Lorentz units with $e^2 = 4\pi\alpha$ and the metric $(+, -, -, -)$, has the Maxwell Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. A complex scalar ϕ of mass m and charge e couples to it through $\mathcal{L}_\phi = (D_\mu\phi)^*(D^\mu\phi) - m^2\phi^*\phi$ with $D_\mu = \partial_\mu + ieA_\mu$, whose Feynman rules are a vertex $-ie(p + p')^\mu$, with p and p' the incoming and outgoing scalar momenta, and a two-photon seagull vertex $2ie^2g^{\mu\nu}$. Write Π^μ for the momentum conjugate to A_μ . Take $\alpha = 1/137.036$, $(\hbar c)^2 = 0.3894 \text{ GeV}^2 \text{ mb}$, and $m = 139.570 \text{ MeV}$, the mass of the charged pion, in (f).

(a) SHOW THAT · U2

[2]

Show that the Maxwell Lagrangian gives A_0 no conjugate momentum, so that canonical quantisation of all four components fails, and that adding the gauge-fixing term $-(1/(2\xi))(\partial_\mu A^\mu)^2$ supplies one.

- 1 $\Pi^\mu = \partial\mathcal{L}/\partial(\partial_0 A_\mu) = -F^{\mu 0}$, and $F^{\mu 0} = 0$ by antisymmetry, so $\Pi^0 = 0$ identically and $[A_0(x), \Pi^0(y)] = i\delta^3(x - y)$ cannot be imposed: A_0 enters as a Lagrange multiplier enforcing Gauss's law, not as a dynamical field
- 2 with the gauge-fixing term $\Pi^\mu = -F^{\mu 0} - (1/\xi)g^{\mu 0}\partial_\nu A^\nu$, so $\Pi^0 = -(1/\xi)\partial_\nu A^\nu$, which is non-zero and makes A_0 dynamical

Candidates who argue only that a gauge field has redundant components have not located the failure. It is the vanishing of one canonical momentum, which no choice of commutator can repair without changing the Lagrangian.

(b) DERIVE · U2

[3]

From the Euler-Lagrange equations, derive the field equation of the gauge-fixed Lagrangian and, from its quadratic action in momentum space, the photon propagator in a general covariant gauge.

- 1 $\partial\mathcal{L}/\partial(\partial_\mu A_\nu) = -F^{\mu\nu} - (1/\xi)g^{\mu\nu}\partial_\rho A^\rho$, so $\partial_\mu[\partial\mathcal{L}/\partial(\partial_\mu A_\nu)] = 0$ gives $\partial_\mu F^{\mu\nu} + (1/\xi)\partial^\nu(\partial_\rho A^\rho) = 0$, that is $\partial^2 A^\nu - (1 - 1/\xi)\partial^\nu(\partial_\rho A^\rho) = 0$, so that at $\xi = 1$ each component obeys $\partial^2 A^\nu = 0$
- 2 integrating by parts, the action is $\frac{1}{2}\int A_\mu[g^{\mu\nu}\partial^2 - (1 - 1/\xi)\partial^\mu\partial^\nu]A_\nu d^4x$, whose momentum-space kernel is $-k^2g^{\mu\nu} + (1 - 1/\xi)k^\mu k^\nu$, and the propagator must satisfy $[-k^2g^{\mu\nu} + (1 - 1/\xi)k^\mu k^\nu]D_{\nu\rho}(k) = i\delta^\mu_\rho$
- 3 the ansatz $D_{\nu\rho} = (-i/k^2)[g_{\nu\rho} + c k_\nu k_\rho/k^2]$ leaves a $k^\mu k_\rho$ term that cancels only if $(1 - 1/\xi)(1 + c) = c$, that is $c = \xi - 1$, so $D_{\mu\nu}(k) = -i[g_{\mu\nu} - (1 - \xi)k_\mu k_\nu/k^2]/(k^2 + i\epsilon)$, reducing to $-ig_{\mu\nu}/(k^2 + i\epsilon)$ in Feynman gauge

Without the gauge-fixing term the kernel $-k^2g^{\mu\nu} + k^\mu k^\nu$ annihilates k_ν and has no inverse. Candidates who try to invert it, or quote the Feynman-gauge propagator without deriving the general- ξ form, miss the step on which (c) depends.

(c) SHOW THAT · U2

[3]

Two conserved currents, $k_\mu j_1^\mu(k) = k_\mu j_2^\mu(k) = 0$, exchange a photon with amplitude proportional to $j_1^\mu D_{\mu\nu}(k) j_2^\nu$. Show that ξ drops out and, taking k along z with $|k| = \kappa$, that the exchange splits into an instantaneous Coulomb term $i j_1^0 j_2^0/\kappa^2$ and a transverse term $i j_1^T \cdot j_2^T/(k^2 + i\epsilon)$, where j^T is the part of the three-vector current perpendicular to k .

- 1 the ξ -dependent term carries $k_\mu j_1^\mu$ and $k_\nu j_2^\nu$, both zero, so $j_1^\mu D_{\mu\nu} j_2^\nu = -i j_1 \cdot j_2/(k^2 + i\epsilon)$ in every covariant gauge
- 2 with $k^\mu = (k^0, 0, 0, \kappa)$, conservation reads $k^0 j^0 = \kappa j^3$, so $j^3 = (k^0/\kappa)j^0$ and $j_1 \cdot j_2 = j_1^0 j_2^0[1 - (k^0)^2/\kappa^2] - j_1^T \cdot j_2^T = -(k^2/\kappa^2)j_1^0 j_2^0 - j_1^T \cdot j_2^T$
- 3 hence $-i j_1 \cdot j_2/k^2 = i j_1^0 j_2^0/\kappa^2 + i j_1^T \cdot j_2^T/(k^2 + i\epsilon)$: the first term does not depend on k^0 , so it acts instantaneously and, as $1/\kappa^2$, Fourier-transforms to the Coulomb interaction $1/(4\pi r)$; only the second has the photon pole, carried by the two transverse polarisations, which is Coulomb gauge recovered

Candidates often show the ξ -independence and stop, or eliminate j^0 rather than j^3 and obtain a term that is not instantaneous. The split works because current conservation trades the timelike and longitudinal photons, which have no pole of their own once combined, for an interaction local in time.

(d) COMPARE · U1

[2]

A massive vector field of mass M_V , the Proca field, has the propagator $-i[g_{\mu\nu} - k_\mu k_\nu / M_V^2] / (k^2 - M_V^2 + i\epsilon)$. Compare its polarisations and propagator with the photon's, and explain from the little group of a massless particle why the photon must couple to a conserved current while the Proca field need not.

- 1 a massive vector carries the three polarisations of spin 1 under its little group $SO(3)$, with $\Sigma \epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + k_\mu k_\nu / M_V^2$, and its propagator has no gauge parameter; its $k_\mu k_\nu / M_V^2$ term grows with energy unless it meets conserved currents, when it drops out as the photon's ξ term does
- 2 a massless particle's little group is $ISO(2)$, whose translations leave the helicity ± 1 unchanged but shift $\epsilon^\mu \rightarrow \epsilon^\mu + c k^\mu$, so ϵ^μ is a four-vector only up to a multiple of k^μ and amplitudes are Lorentz invariant only if $k_\mu M^\mu = 0$ for every external photon, which requires coupling to a conserved current; no such shift exists when $M_V \neq 0$

Answers that say the photon has two polarisations because it is massless have restated the fact. The little group explains it, and shows that $k_\mu M^\mu = 0$ is a requirement on every amplitude, not a choice of gauge.

(e) SHOW THAT · U2

[3]

For Compton scattering on the charged scalar, $\gamma(k_1) + \phi(p) \rightarrow \gamma(k_2) + \phi(p')$, write $iM^{\mu\nu}$, with μ belonging to the incoming photon and ν to the outgoing one, as the sum of the s-channel, u-channel and seagull diagrams. Show that $k_{1\mu} M^{\mu\nu} = 0$ for the sum, but not for any one diagram or for the two exchange diagrams alone.

- 1 $iM^{\mu\nu} = (-ie)^2[(2p + k_1)^\mu (2p' + k_2)^\nu i/(s - m^2) + (2p' - k_1)^\mu (2p - k_2)^\nu i/(u - m^2)] + 2ie^2 g^{\mu\nu} = -ie^2[(2p + k_1)^\mu (2p' + k_2)^\nu / (s - m^2) + (2p' - k_1)^\mu (2p - k_2)^\nu / (u - m^2) - 2g^{\mu\nu}]$, with $s = (p + k_1)^2$ and $u = (p - k_2)^2$
- 2 with $k_1^2 = 0$, $k_1 \cdot (2p + k_1) = 2p \cdot k_1 = s - m^2$ and $k_1 \cdot (2p' - k_1) = 2p' \cdot k_1 = m^2 - u$, so contracting with $k_{1\mu}$ cancels each propagator: the s-channel gives $-ie^2(2p' + k_2)^\nu$, the u-channel $+ie^2(2p - k_2)^\nu$ and the seagull $+2ie^2 k_1^\nu$, each non-zero
- 3 the two exchange diagrams together give $-2ie^2(p' - p + k_2)^\nu = -2ie^2 k_1^\nu$, using $p + k_1 = p' + k_2$, which is not zero; the seagull's $+2ie^2 k_1^\nu$ cancels it exactly, so $k_{1\mu} M^{\mu\nu} = 0$ holds only for the sum of all three

The seagull is the term candidates omit, having learned Compton scattering on a Dirac electron, where there is none; the Ward identity then fails by exactly $2ie^2 k_1^\nu$. The identity also needs the external lines on shell, $k_1^2 = 0$ and $p^2 = p'^2 = m^2$, which every step above uses.

(f) CALCULATE · U3

[2]

In the rest frame of the initial scalar choose both photon polarisations with $\epsilon^0 = 0$, so that $p \cdot \epsilon = k \cdot \epsilon = 0$ for each photon. For photon energies $\omega \ll m$, where $d\sigma/d\Omega = |M|^2 / (64\pi^2 m^2)$, calculate the total cross-section for a point-like scalar with the charged pion's mass, in microbarns, averaging over initial and summing over final polarisations.

- 1 with $p = (m, 0)$ and these polarisations, $(2p + k_1) \cdot \epsilon_1 = 0$ and $(2p - k_2) \cdot \epsilon_2^* = 0$, so only the seagull survives: $|M|^2 = 4e^4 |\epsilon_1 \cdot \epsilon_2^*|^2$, $d\sigma/d\Omega = (\alpha/m)^2 |\epsilon_1 \cdot \epsilon_2^*|^2$, and the polarisation average gives $(\alpha/m)^2 (1 + \cos^2\theta)/2$
- 2 integrating over angles, $\sigma = (8\pi/3)(\alpha/m)^2 = (8\pi/3)[1/(137.036 \times 0.13957 \text{ GeV})]^2 = 0.0229 \text{ GeV}^{-2}$, and $0.0229 \times 0.3894 \text{ mb} = 8.92 \text{ } \mu\text{b}$: Thomson's cross-section with the pion's mass in place of the electron's

In this frame and gauge both exchange diagrams vanish identically, so the classical Thomson result is carried entirely by the seagull. Summing over the initial polarisations instead of averaging gives twice the answer, $17.8 \text{ } \mu\text{b}$.

What the command words are asking for

Examiners mark to the command word as well as to the physics. An answer that explains where the question said state has spent time earning one mark; one that states a result where the question said derive has earned none, however correct the result.

State	Give the result, definition or law concisely. No justification is required.
Describe	Give the relevant features of a situation or process in words, precisely enough that someone else could reproduce it.
Explain	Give the physical reason a result or behaviour occurs, connecting it explicitly to the principle that governs it.
Calculate	Obtain a numerical value, showing the relation used, the substitution and the units of the result.
Determine	Obtain a result by any valid route the problem allows — analytic, graphical or numerical — with enough working to be followed.
Derive	Start from a named fundamental law or definition and reach the required result by explicit, justified steps.
Show that	Establish a result the question gives you. Every step must be shown, because the stated answer earns nothing on its own.
Estimate	Reach an order-of-magnitude value from stated, reasonable assumptions, and say what those assumptions are.
Sketch	Draw the shape of a graph or field with axes labelled and its key features marked — intercepts, asymptotes, turning points, limits. Exact scale is not required.
Justify	Support a claim or a choice with evidence, and with the reasoning that connects that evidence to the claim.
Evaluate	Weigh a model, argument or result against criteria, including its assumptions and domain of validity, and reach a stated judgement.
Compare	Identify the similarities and differences between two situations, models or results, relating each difference to its cause.
Predict	Say what will happen when a condition changes, reasoning from a principle rather than guessing, and give the reason.

WHAT IS BEING TESTED

University assessment objectives

Each course has the two papers a university course is examined by: a midterm on the first half of its units, and a cumulative final weighted towards the second half. Every paper combines short conceptual multiple-choice items with structured problems, a data or estimation question, and at least one extended derivation. Constants are given where a question needs them; a calculator and your own formula sheet are assumed.

U1 16 MARKS • 22%	Conceptual understanding Identify the governing principle and the conditions under which it holds, and reason qualitatively about what a result must look like before calculating it.
U2 37 MARKS • 51%	Mathematical modelling and derivation Translate a physical situation into mathematics: choose coordinates and approximations, set up the governing equations, solve them symbolically, and derive stated results from named laws.

U3

13 MARKS · 18%

Quantitative problem solving

Carry a multi-step calculation to a numerical result with correct units and sensible precision, and check it against dimensions, limiting cases and orders of magnitude.

U4

7 MARKS · 10%

Data, estimation and experimental reasoning

Analyse measurements and their uncertainties, linearise and fit data, estimate orders of magnitude from stated assumptions, and judge whether evidence supports a model.

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