AP · COLLEGE BOARD AP PHYSICS · FRAMEWORKS FIRST EXAMINED IN 2025

Every formula

AP Physics

> *Every AP physics exam supplies a table of equations, so the useful question is not what exists but what the table leaves you to remember. Anything not marked GIVEN is one to carry in your head. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not a College Board document; the official syllabus is the authority.*

AP 1 AP Physics 1: Algebra-Based

1 Kinematics

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Average velocity

$$v_{avg} = \Delta x / \Delta t$$

- v_{avg} — average velocity (m/s)
- Δx — displacement, not distance travelled (m)
- Δt — time interval (s)

A definition rather than a result. Displacement on top is the whole difference between velocity and speed.

ALGEBRA

Average acceleration

$$a_{avg} = \Delta v / \Delta t$$

- a_{avg} — average acceleration (m/s²)
- Δv — change in velocity (m/s)
- Δt — time interval (s)

Also a definition. Acceleration is a change in velocity, so a turn at constant speed is still an acceleration.

ALGEBRA

Velocity under constant acceleration

$$v = v_0 + at$$

- v — velocity at time t (m/s)
- v_0 — velocity at $t = 0$ (m/s)
- a — constant acceleration (m/s²)
- t — time (s)

ALGEBRA

Position under constant acceleration

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

- x — position at time t (m)
- x_0 — position at $t = 0$ (m)
- v_0 — velocity at $t = 0$ (m/s)
- a — constant acceleration (m/s²)
- t — time (s)

ALGEBRA

Velocity without time

$$v^2 = v_0^2 + 2a(x - x_0)$$

- v, v_0 — final and initial velocity (m/s)
- a — constant acceleration (m/s²)
- $x - x_0$ — displacement (m)

The one to reach for when the question never mentions time.

ALGEBRA

Projectile motion by components

$$a_x = 0 \cdot a_y = -g$$

- a_x — horizontal acceleration, with drag neglected (m/s²)
- a_y — vertical acceleration, downwards (m/s²)
- t — the same time for both components (s)

Not an equation the table needs to print: it is the method. Horizontal and vertical motion are solved separately and share only the time.

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Reading the motion graphs

$$\text{velocity} = \text{slope of } x\text{–}t \cdot \text{acceleration} = \text{slope of } v\text{–}t \cdot \Delta x = \text{area under } v\text{–}t$$

- slope — of a tangent where the graph is curved
- area — counted negative below the time axis

AP asks for these as often as it asks for the equations, and a graph question rarely gives you numbers to substitute.

2 Force and Translational Dynamics

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Newton's second law

$$a = F_{net} / m$$

$F_{net} = ma$

- F_{net} — vector sum of the forces on the object (N)
- m — mass (kg)
- a — acceleration, along F_{net} (m/s²)

Applied one axis at a time, off a free-body diagram.

ALGEBRA

Weight

$$F_g = mg$$

- F_g — gravitational force on the object (N)
- m — mass (kg)
- g — gravitational field strength (N/kg or m/s²)

ALGEBRA

Friction

$$F_f \le \mu_s F_N \ \ (\text{static}) \cdot F_f = \mu_k F_N \ \ (\text{kinetic})$$

- F_f — friction force, opposing relative sliding (N)
- μ_s, μ_k — coefficients of static and kinetic friction
- F_N — normal force (N)

Static friction is an inequality: it takes whatever value up to the maximum keeps the surfaces from sliding.

ALGEBRA

Force from an ideal spring

$$F_s = -kx$$

- F_s — force the spring exerts (N)
- k — spring constant (N/m)
- x — displacement from the natural length (m)

The minus sign says the force points back towards the natural length; in magnitude, $F_s = k|x|$.

ALGEBRA

Newton's law of universal gravitation

$$F_g = G\, m_1 m_2 / r^2$$

- F_g — attractive force on each mass (N)
- G — universal gravitational constant (N m²/kg²)
- m_1, m_2 — the two masses (kg)
- r — distance between their centres (m)

ALGEBRA

Gravitational field strength

$$g = F_g / m = G M / r^2$$

g — field strength at that point (N/kg)
M — mass producing the field (kg)
r — distance from its centre (m)

Why g at the surface has the value it does, and why it falls off with altitude.

ALGEBRA

Circular motion

$$a_c = v^2 / r \cdot F_{net} = m v^2 / r$$

a_c — centripetal acceleration, towards the centre (m/s²)
v — speed along the circle (m/s)
r — radius (m)

Centripetal force is not an extra force to add to the diagram: it is the name for whatever real forces already point towards the centre.

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Speed and period of a circular orbit

$$v = \sqrt{G M / r} \cdot T^2 = 4\pi^2 r^3 / (G M)$$

v — orbital speed (m/s)
T — orbital period (s)
M — mass of the central body (kg)
r — radius of the orbit, measured from the centre (m)

Gravity supplying the centripetal force: $G M m / r^2 = m v^2 / r$, and the orbiting mass cancels, so everything at that radius orbits at the same speed whatever its own mass. The period relation is Kepler's third law for a circular orbit. Neither is printed — rebuild them from the two equations above.

3 Work, Energy, and Power

ALGEBRA

Work done by a constant force

$$W = F d \cos \theta$$

W — work done (J)
F — force (N)
d — displacement (m)
θ — angle between the force and the displacement (°)

A force perpendicular to the motion does no work, which is why the centripetal force never appears in an energy equation.

ALGEBRA

Work-energy theorem

$$W_{net} = \Delta K$$

W_{net} — work done by the net force (J)
ΔK — change in kinetic energy (J)

The bridge between the force unit and this one.

ALGEBRA

Kinetic energy

$$K = \tfrac{1}{2}mv^2$$

K — kinetic energy (J)
m — mass (kg)
v — speed (m/s)

ALGEBRA

Gravitational potential energy near a surface

$$\Delta U_g = mg\Delta y$$

ΔU_g — change in gravitational potential energy (J)
Δy — change in height (m)

For a field that can be treated as uniform. Only the change has meaning; you choose where zero sits.

ALGEBRA

Gravitational potential energy of two masses

$$U_g = -G\, m_1 m_2 / r$$

- U_g — potential energy of the pair (J)
- m_1, m_2 — the two masses (kg)
- r — separation of their centres (m)

Negative because the zero is taken at infinite separation. This is the form orbit and escape-speed questions need.

ALGEBRA

Elastic potential energy

$$U_s = \tfrac{1}{2}kx^2$$

- U_s — energy stored in the spring (J)
- k — spring constant (N/m)
- x — displacement from the natural length (m)

The area under a force–displacement graph.

ALGEBRA

Conservation of energy

$$\Delta K + \Delta U + \Delta E_{thermal} = 0$$

- ΔK, ΔU — changes in kinetic and potential energy (J)
- $\Delta E_{thermal}$ — energy that has gone to heating the surfaces (J)

For an isolated system. Defining the system carefully is half of an AP energy question.

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Energy dissipated by friction

$$\Delta E_{thermal} = F_f\, d$$

- F_f — kinetic friction force (N)
- d — distance slid along the surface (m)

Not a printed statement: it is $W = Fd\cos\theta$ with $\theta = 180°$, counted as energy leaving the mechanical system. Distance slid, not displacement.

ALGEBRA

Power

$$P = \Delta E / \Delta t \cdot P = F\, v \cos\theta$$

- P — power (W)
- ΔE — energy transferred (J)
- v — speed (m/s)

4 Linear Momentum

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Linear momentum

$$p = mv$$

- p — momentum, a vector along the velocity (kg m/s)
- m — mass (kg)
- v — velocity (m/s)

ALGEBRA

Impulse

$$J = F\Delta t = \Delta p$$

- J — impulse (N s)
- F — average force during the interaction (N)
- Δt — how long it acts (s)

Also the area under a force–time graph, which is how AP usually asks for it.

ALGEBRA

Conservation of momentum

$$\Sigma p \text{ before} = \Sigma p \text{ after}$$

- Σp — vector sum over the whole system (kg m/s)

Holds when no external force acts on the system, and holds component by component.

ALGEBRA

Elastic and inelastic collisions

elastic: ΣK before = ΣK after · perfectly inelastic: the objects move off together

- K — kinetic energy (J)
- p — conserved in both cases (kg m/s)

Momentum is conserved in every collision here; kinetic energy only in the elastic one. The distinction is the point of the unit.

ALGEBRA

Centre of mass

$$x_{cm} = \Sigma(m\ x) / \Sigma m$$

- x_{cm} — position of the centre of mass (m)
- m — mass of each part (kg)
- x — position of each part (m)

The point that moves as if all the mass and all the external force acted there. AP Physics C: Mechanics turns this sum into an integral.

5 Torque and Rotational Dynamics

ALGEBRA

Torque

$$\tau = r F \sin\theta$$

- τ — torque about the chosen axis (N m)
- r — distance from the axis to where the force acts (m)
- F — force (N)
- θ — angle between r and F (°)

$r \sin\theta$ is the lever arm — the perpendicular distance from the axis to the line of the force.

ALGEBRA

Rotational form of Newton's second law

$$\alpha = \Sigma\tau / I$$

- α — angular acceleration (rad/s²)
- $\Sigma\tau$ — net torque about the axis (N m)
- I — rotational inertia about that axis (kg m²)

ALGEBRA

Rotational inertia of point masses

$$I = \Sigma m r^2$$

- I — rotational inertia (kg m²)
- m — each mass (kg)
- r — its distance from the axis (m)

It depends on the axis, so it is meaningless to quote one without saying about what.

ALGEBRA

Rotational kinematics

$$\omega = \omega_0 + \alpha t \cdot \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

- ω, ω_0 — final and initial angular velocity (rad/s)
- θ, θ_0 — final and initial angular position (rad)
- α — constant angular acceleration (rad/s²)
- t — time (s)

The straight-line equations with every symbol swapped for its angular twin.

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Angular velocity without time

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

- ω, ω_0 — final and initial angular velocity (rad/s)
- $\theta - \theta_0$ — angle turned through (rad)

The twin of $v^2 = v_0^2 + 2a\Delta x$. Worth deriving once rather than counting on it being printed.

ALGEBRA

Linear and angular quantities

$$s = r\theta \cdot v = r\omega \cdot a = r\alpha$$

- s — arc length (m)
- r — distance from the axis (m)
- θ — angle, in radians (rad)

Only true with the angle in radians, which is why AP works in radians throughout.

ALGEBRA

Rotational equilibrium

$$\Sigma F = 0 \cdot \Sigma\tau = 0$$

- $\Sigma\tau$ — about any axis you choose (N m)

Both conditions are needed. Choosing the axis through an unknown force removes it from the torque equation.

6 Energy and Momentum of Rotating Systems

ALGEBRA

Rotational kinetic energy

$$K = \tfrac{1}{2}I\omega^2$$

- K — kinetic energy of rotation (J)
- I — rotational inertia (kg m²)
- ω — angular velocity (rad/s)

ALGEBRA

Angular momentum of a rotating body

$$L = I\omega$$

- L — angular momentum about the axis (kg m²/s)
- I — rotational inertia (kg m²)
- ω — angular velocity (rad/s)

ALGEBRA

Angular impulse

$$\Delta L = \tau\,\Delta t$$

- ΔL — change in angular momentum (kg m²/s)
- τ — average external torque (N m)
- Δt — how long it acts (s)

ALGEBRA

Conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

- I_1, I_2 — rotational inertia before and after (kg m²)
- ω_1, ω_2 — angular velocity before and after (rad/s)

When no external torque acts. Pulling mass towards the axis lowers I, so ω rises — and K rises with it, because the pulling does work.

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Rolling without slipping

$$v_{cm} = r\omega \cdot K = \tfrac{1}{2}m\,v_{cm}^2 + \tfrac{1}{2}\,I_{cm}\,\omega^2$$

- v_{cm} — speed of the centre of mass (m/s)
- I_{cm} — rotational inertia about the centre of mass (kg m²)
- r — radius of the rolling object (m)

The condition and the energy split follow from the definitions rather than being printed. It is why a hoop loses a race to a solid cylinder.

7 Oscillations

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Period, frequency and angular frequency

$$T = 1 / f \cdot \omega = 2\pi f$$

- T — period (s)
- f — frequency (Hz)
- ω — angular frequency (rad/s)

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Displacement in simple harmonic motion

$$x = A\cos(2\pi f t)$$

- x — displacement from equilibrium (m)
- A — amplitude (m)
- f — frequency (Hz)
- t — time (s)

Written for an oscillator released from maximum displacement at $t = 0$; a sine describes one started from equilibrium.

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Period of a mass on a spring

$$T = 2\pi\sqrt{m / k}$$

- T — period (s)
- m — oscillating mass (kg)
- k — spring constant (N/m)

No amplitude in it: the period of a simple harmonic oscillator does not depend on how far you pull it.

ALGEBRA

Period of a simple pendulum

$$T = 2\pi\sqrt{L / g}$$

- L — length of the pendulum (m)
- g — gravitational field strength (m/s²)

For small angles only, and independent of the mass hung on the end.

ALGEBRA
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Energy of an oscillator

$$E = \tfrac{1}{2}kA^2$$

- E — total energy of the oscillation (J)
- A — amplitude (m)
- k — spring constant (N/m)

$U_s = \frac{1}{2}kx^2$ read at maximum displacement, where the kinetic energy is zero. Energy goes as amplitude squared.

8 Fluids

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Density

$$\rho = m / V$$

- ρ — density (kg/m³)
- m — mass (kg)
- V — volume (m³)

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Pressure

$$P = F / A$$

- P — pressure (Pa)
- F — force perpendicular to the surface (N)
- A — area (m²)

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Pressure with depth

$$P = P_0 + \rho g h$$

- P — absolute pressure at that depth (Pa)
- P_0 — pressure at the surface (Pa)
- ρ — density of the fluid (kg/m³)
- h — depth below the surface (m)

Depth, not the shape of the container. ρgh on its own is the gauge pressure.

ALGEBRA

Buoyant force

$$F_b = \rho_{fluid} V_{displaced}\, g$$

- F_b — upward force on the submerged object (N)
- ρ_{fluid} — density of the fluid, not the object (kg/m³)
- $V_{displaced}$ — volume of fluid pushed aside (m³)

Archimedes' principle. For a floating object the displaced weight equals the object's whole weight.

ALGEBRA

Continuity

$$A_1 v_1 = A_2 v_2$$

- A — cross-sectional area (m²)
- v — flow speed there (m/s)

Conservation of volume for an incompressible fluid: a narrower pipe means a faster flow.

ALGEBRA

Bernoulli's equation

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

- P — pressure at that point (Pa)
- y — height of that point (m)
- v — flow speed there (m/s)

Conservation of energy per unit volume, for steady non-viscous incompressible flow along one streamline.

AP 2

AP Physics 2: Algebra-Based

9

Thermodynamics

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Ideal gas law

$$PV = nRT = N k_B T$$

- P — pressure (Pa)
- V — volume (m³)
- n — amount of gas (mol)
- N — number of molecules
- T — absolute temperature (K)

The same law counted two ways, by moles or by molecules. T must be in kelvin.

ALGEBRA

Average kinetic energy of a molecule

$$K_{avg} = (3/2)\, k_B T$$

- K_{avg} — mean translational kinetic energy of one molecule (J)
- k_B — Boltzmann's constant (J/K)
- T — absolute temperature (K)

What temperature actually measures. It does not depend on which gas.

ALGEBRA

Root-mean-square speed

$$v_{rms} = \sqrt{3 k_B T / m} = \sqrt{3RT / M}$$

- v_{rms} — root-mean-square molecular speed (m/s)
- m — mass of one molecule (kg)
- M — molar mass (kg/mol)

Heavier molecules move more slowly at the same temperature.

ALGEBRA

First law of thermodynamics

$$\Delta U = Q + W$$

ΔU — change in the internal energy of the gas (J)
Q — energy added by heating (J)
W — work done on the gas (J)

AP takes W as the work done on the gas, so compressing it is positive. Signs are where marks are lost here.

ALGEBRA

Work done on a gas

$$W = -P\,\Delta V$$

W — work done on the gas (J)
P — pressure, held constant (Pa)
ΔV — change in volume (m^3)

At constant pressure. In general it is minus the area under the process on a PV diagram.

ALGEBRA

Heating a substance

$$Q = m\,c\,\Delta T$$

Q — energy transferred by heating (J)
m — mass (kg)
c — specific heat capacity (J/(kg K))
ΔT — temperature change (K)

10 Electric Force, Field, and Potential

ALGEBRA

Coulomb's law

$$F_E = (1 / 4\pi\varepsilon_0)\, q_1 q_2 / r^2$$

F_E — force on each charge, along the line joining them (N)
q_1, q_2 — the two charges (C)
r — separation (m)
ε_0 — permittivity of free space ($C^2/(N\,m^2)$)

$1 / 4\pi\varepsilon_0$ is the constant written $k = 9.0 \times 10^9\ N\,m^2/C^2$. Like charges repel, so signs tell you direction, not magnitude.

ALGEBRA

Electric field

$$E = F_E / q$$

E — electric field, a vector (N/C or V/m)
F_E — force on the test charge (N)
q — test charge (C)

The definition. Field points along the force on a positive charge.

ALGEBRA

Field of a point charge

$$E = (1 / 4\pi\varepsilon_0)\, q / r^2$$

E — field strength at distance r (N/C)
q — charge producing the field (C)
r — distance from it (m)

Away from a positive charge, towards a negative one. Fields from several charges add as vectors.

ALGEBRA

Uniform field between parallel plates

$$E = \Delta V / d$$

E — field strength, uniform between the plates (V/m)
ΔV — potential difference across the gap (V)
d — separation of the plates (m)

The field points from high potential to low.

ALGEBRA

Electric potential of a point charge

$$V = (1 / 4\pi\varepsilon_0)\ q / r$$

- V — potential at that point (V)
- q — charge producing it, sign included (C)
- r — distance from it (m)

A scalar: potentials from several charges add by ordinary arithmetic, with their signs.

ALGEBRA

Electric potential energy of two charges

$$UE = (1 / 4\pi\varepsilon_0)\ q_1 q_2 / r$$

- UE — potential energy of the pair (J)
- q_1, q_2 — the two charges, signs included (C)

Zero at infinite separation. Negative for opposite charges, which is why they are bound.

ALGEBRA

Energy change of a charge moved through a potential difference

$$\Delta UE = q\ \Delta V$$

- ΔUE — change in electric potential energy (J)
- q — charge moved (C)
- ΔV — potential difference it moves through (V)

The definition of the electronvolt, and the way accelerated-charge questions are set.

ALGEBRA

A conductor in electrostatic equilibrium

E = 0 inside · excess charge sits on the surface · the whole conductor is one equipotential

- E — field within the conducting material (N/C)
- V — the same everywhere on and in the conductor (V)

Not formulae, but the facts every conductor question turns on. Any field inside would move the free charges, so it cannot persist.

11 Electric Circuits

ALGEBRA

Electric current

$$I = \Delta Q / \Delta t$$

- I — current (A)
- ΔQ — charge passing a point (C)
- Δt — time taken (s)

Conventional current is the direction positive charge would move.

ALGEBRA

Resistance of a wire

$$R = \rho\ L / A$$

- R — resistance (Ω)
- ρ — resistivity of the material (Ω m)
- L — length (m)
- A — cross-sectional area (m^2)

ALGEBRA

Ohm's law

$$I = \Delta V / R$$

$\Delta V = IR$ · $R = \Delta V / I$

- ΔV — potential difference across the component (V)
- I — current through it (A)
- R — resistance (Ω)

A property of ohmic components only; a filament lamp or a diode does not obey it.

ALGEBRA

Electrical power

$$P = I\,\Delta V = I^2R = (\Delta V)^2 / R$$

- P — power delivered (W)
- I — current (A)
- ΔV — potential difference (V)

The last two forms hold only where $\Delta V = IR$ does.

ALGEBRA

Resistors in series

$$R_s = R_1 + R_2 + \dots$$

- R_s — equivalent resistance (Ω)

Same current through each; the potential differences add.

ALGEBRA

Resistors in parallel

$$1 / R_p = 1 / R_1 + 1 / R_2 + \dots$$

- R_p — equivalent resistance (Ω)

Same potential difference across each; the currents add. The result is always smaller than the smallest resistor.

ALGEBRA

Kirchhoff's rules

$$\Sigma\, I \text{ into a junction} = \Sigma\, I \text{ out of it} \cdot \Sigma\, \Delta V \text{ round any loop} = 0$$

- I — current at a junction (A)
- ΔV — potential change across each element in the loop (V)

Conservation of charge and conservation of energy. Stated as rules rather than printed as formulae, and needed for any circuit that will not reduce to series and parallel.

ALGEBRA

Capacitance

$$C = Q / \Delta V$$

- C — capacitance (F)
- Q — charge on either plate (C)
- ΔV — potential difference between the plates (V)

ALGEBRA

Parallel-plate capacitor

$$C = \kappa\, \varepsilon_0\, A / d$$

- κ — dielectric constant of the material between the plates
- A — plate area (m^2)
- d — plate separation (m)

$\kappa = 1$ for a vacuum and is greater than 1 for any dielectric, so a dielectric always raises the capacitance.

ALGEBRA

Energy stored in a capacitor

$$U_C = \tfrac{1}{2}\, Q\, \Delta V = \tfrac{1}{2}\, C\, (\Delta V)^2$$

- U_C — energy stored (J)
- Q — charge stored (C)
- C — capacitance (F)

The half is there because the potential difference grows as the charge arrives.

ALGEBRA

Capacitors in series and parallel

$$1 / C_s = 1 / C_1 + 1 / C_2 + \dots \cdot C_p = C_1 + C_2 + \dots$$

- C — equivalent capacitance (F)

The opposite way round from resistors, which is the mistake this pair exists to catch.

12 Magnetism and Electromagnetism

ALGEBRA

Force on a moving charge

$$F_M = q\ v\ B \sin\theta$$

- F_M — magnetic force (N)
- q — charge (C)
- v — speed (m/s)
- B — magnetic flux density (T)
- θ — angle between v and B (°)

The force is perpendicular to both v and B, found with the right hand, so it does no work and only changes direction.

ALGEBRA

Force on a current-carrying wire

$$F_M = B\ I\ L \sin\theta$$

- I — current (A)
- L — length of wire in the field (m)
- θ — angle between the wire and the field (°)

ALGEBRA

Field of a long straight wire

$$B = \mu_0\ I\ /\ (2\pi\ r)$$

- B — flux density at distance r (T)
- μ_0 — permeability of free space (T m/A)
- r — perpendicular distance from the wire (m)

The field circles the wire; grip it with the right hand, thumb along the current.

ALGEBRA

Magnetic flux

$$\Phi_B = B\ A \cos\theta$$

- Φ_B — magnetic flux through the loop (Wb)
- A — area of the loop (m^2)
- θ — angle between the field and the normal to the loop (°)

ALGEBRA

Faraday's law of induction

$$\varepsilon = -\ \Delta\Phi_B\ /\ \Delta t$$

- ε — induced emf (V)
- $\Delta\Phi_B$ — change in flux through the loop (Wb)
- Δt — time it takes (s)

The minus sign is Lenz's law: the induced current opposes the change that caused it. For N turns, multiply by N.

ALGEBRA
DERIVED

Motional emf

$$\varepsilon = B\ L\ v$$

- L — length of the moving conductor (m)
- v — speed across the field (m/s)

Faraday's law for a bar sliding on rails, where the area of the circuit changes at Lv. Worth being able to rebuild rather than recall.

13 Geometric Optics

ALGEBRA

Index of refraction

$$n = c\ /\ v$$

- n — index of refraction of the medium
- c — speed of light in a vacuum (m/s)
- v — speed of light in the medium (m/s)

n is never less than 1. The frequency does not change when light crosses a boundary; the wavelength does.

ALGEBRA

Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

n_1, n_2 — indices either side of the boundary
θ_1, θ_2 — angles to the normal (°)

Angles are measured from the normal, not the surface.

ALGEBRA
DERIVED

Critical angle

$$\sin \theta_c = n_2 / n_1$$

θ_c — critical angle (°)
n_1 — index of the denser medium the light is in
n_2 — index of the medium beyond

Snell's law with the refracted ray at 90°. Only possible when n_1 is greater than n_2; beyond θ_c the light is totally internally reflected.

ALGEBRA

Mirror and thin lens equation

$$1 / s_o + 1 / s_i = 1 / f$$

s_o — object distance (m)
s_i — image distance, negative for a virtual image (m)
f — focal length, negative for a diverging lens or convex mirror (m)

The sign convention carries the physics: get it wrong and a virtual image comes out real.

ALGEBRA

Magnification

$$M = h_i / h_o = - s_i / s_o$$

M — magnification
h_i, h_o — image and object height (m)

A negative M means the image is inverted.

ALGEBRA

Focal length of a spherical mirror

$$f = R / 2$$

f — focal length (m)
R — radius of curvature (m)

14 Waves, Sound, and Physical Optics

ALGEBRA

Wave speed

$$v = f \lambda$$

$f = v / \lambda$ · $\lambda = v / f$

v — wave speed, set by the medium (m/s)
f — frequency, set by the source (Hz)
λ — wavelength (m)

ALGEBRA
DERIVED

Standing waves with two matching ends

$$\lambda_n = 2L / n, \quad n = 1, 2, 3 \ldots$$

L — length of the string or pipe (m)
n — harmonic number

A string fixed at both ends, or a pipe open at both ends. Read off the boundary conditions rather than recalled — draw the pattern and count.

ALGEBRA
DERIVED

Standing waves in a pipe closed at one end

$$\lambda_n = 4L / n, \quad n = 1, 3, 5 \ldots$$

L — length of the pipe (m)
n — odd harmonics only

A node at the closed end and an antinode at the open one, which is why the even harmonics are missing.

ALGEBRA

Double-slit and grating maxima

$$d \sin \theta = m \lambda$$

- d — slit separation, or grating spacing (m)
- θ — angle of the bright fringe from the centre (°)
- m — order of the fringe, 0, 1, 2 …

Constructive interference, where the path difference is a whole number of wavelengths.

ALGEBRA

Fringe spacing on a distant screen

$$\Delta y = \lambda L / d$$

- Δy — distance between neighbouring bright fringes (m)
- L — distance from the slits to the screen (m)
- d — slit separation (m)

For small angles, with L much greater than d.

ALGEBRA

Single-slit minima

$$a \sin \theta = m \lambda, \quad m = 1, 2, 3 \ldots$$

- a — width of the single slit (m)
- θ — angle of the dark fringe (°)

The same-looking equation as the two-slit one but locating dark fringes, not bright. Note which it is before substituting.

15 Modern Physics

ALGEBRA

Photon energy

$$E = h f = h c / \lambda$$

- E — energy of one photon (J or eV)
- h — Planck's constant (J s)
- f — frequency of the light (Hz)

ALGEBRA

Photoelectric effect

$$K_{max} = h f - \varphi$$

- K_{max} — maximum kinetic energy of an emitted electron (J or eV)
- φ — work function of the surface (J or eV)

Below the threshold frequency nothing is emitted however bright the light — the observation that made the photon necessary.

ALGEBRA

de Broglie wavelength

$$\lambda = h / p$$

- λ — wavelength associated with the particle (m)
- p — momentum (kg m/s)

ALGEBRA

Photon emitted between energy levels

$$h f = |\Delta E|$$

- ΔE — difference between the two levels (J or eV)
- f — frequency of the photon (Hz)

Levels are discrete, so the emitted frequencies are too. That is what a line spectrum is.

ALGEBRA

Mass-energy equivalence

$$E = m c^2$$

- E — energy equivalent of the mass (J)
- m — mass (kg)
- c — speed of light in a vacuum (m/s)

Used as $\Delta E = \Delta m c^2$ for the mass defect of a nucleus and the energy released in a decay.

ALGEBRA

What a nuclear reaction conserves

$$\Sigma A \text{ unchanged} \cdot \Sigma Z \text{ unchanged}$$

A — mass number, the count of nucleons
Z — atomic number, the count of protons

Charge and nucleon number balance across the arrow; energy and momentum are conserved as well. Not a formula, but it is what the equations are checked against.

AP C:M

AP Physics C: Mechanics

1

Kinematics

CALCULUS

Velocity as a derivative

$$v = dx / dt$$

v — instantaneous velocity, a vector (m/s)
x — position (m)
t — time (s)

The slope of the position graph, said exactly. In two dimensions each component is differentiated separately.

CALCULUS

Acceleration as a derivative

$$a = dv / dt = d^2x / dt^2$$

a — instantaneous acceleration (m/s²)
v — velocity (m/s)

CALCULUS

Velocity and position by integration

$$v = v_0 + \int a\,dt \cdot x = x_0 + \int v\,dt$$

v_0, x_0 — the initial values, which the integration constants fix (m/s, m)
$\int \ldots dt$ — over the interval in question

The same two definitions read backwards, and the reason this course can set motion where the acceleration is not constant. The algebra-based equations of motion are what these give when a is constant.

2

Force and Translational Dynamics

CALCULUS

Force as the rate of change of momentum

$$F_{net} = dp / dt$$

Fnet — net force (N)
p — momentum (kg m/s)

Newton's second law in the form he wrote it. Fnet = ma is the special case of constant mass.

CALCULUS

Motion against a resistive force

$$m\,dv / dt = m g - b v$$

b — drag coefficient for a force proportional to speed (kg/s)
v — speed (m/s)
m g — the driving force, here the weight (N)

Written for a falling object with drag proportional to v; a drag proportional to v^2 is set the same way. This separable differential equation is the piece of calculus this unit exists to use.

CALCULUS
DERIVED

Terminal speed

$$v_{term} = m g / b$$

vterm — speed at which the acceleration reaches zero (m/s)
b — drag coefficient, for drag proportional to speed (kg/s)

Set dv/dt = 0 in the equation above. The speed is approached but never quite reached.

3

Work, Energy, and Power

CALCULUS

Work done by a varying force

$$W = \int F \cdot dr$$

W — work done (J)
F · dr — the component of the force along the path, times the step along it (J)

The area under a force–displacement graph, and the general statement $W = Fd \cos\theta$ is the constant-force case of.

CALCULUS

Instantaneous power

$$P = dW / dt = F \cdot v$$

P — power at that instant (W)
F · v — force times the component of velocity along it (W)

CALCULUS

Force from a potential energy function

$$F = - dU / dx$$

U — potential energy as a function of position (J)
F — conservative force, along x (N)

The force points down the slope of the U–x graph. Equilibrium is where the slope is zero, stable where the curve is a minimum.

CALCULUS

Potential energy from a conservative force

$$\Delta U = - \int F \cdot dr$$

ΔU — change in potential energy (J)
F — the conservative force (N)

Where $\frac{1}{2}kx^2$ and $-Gm_1m_2/r$ come from, and only defined for a force whose work does not depend on the path.

4 Linear Momentum

CALCULUS

Impulse as an integral

$$J = \int F\, dt = \Delta p$$

J — impulse (N s)
F — force, varying through the interaction (N)

The area under a force–time graph, without needing the force to be constant.

CALCULUS

Centre of mass of a continuous body

$$r_{cm} = (1 / M) \int r\, dm$$

rcm — position of the centre of mass (m)
M — total mass (kg)
dm — mass of an element at position r (kg)

The discrete sum turned into an integral. Writing dm in terms of a density and a length, area or volume element is the whole technique.

CALCULUS

Motion of the centre of mass

$$M a_{cm} = \Sigma F_{external}$$

acm — acceleration of the centre of mass (m/s^2)
Σ Fexternal — sum of the external forces only (N)

Internal forces cancel in pairs, so an exploding shell's centre of mass carries on along the same parabola.

5 Torque and Rotational Dynamics

CALCULUS

Torque as a cross product

$$\tau = r \times F$$

- τ — torque, a vector along the axis (N m)
- r — vector from the axis to the point of application (m)
- F — force (N)

Magnitude $rF \sin\theta$, direction from the right hand. The vector form is what lets torques about different axes be added properly.

CALCULUS

Rotational inertia of a continuous body

$$I = \int r^2 \, dm$$

- I — rotational inertia about the chosen axis (kg m²)
- r — perpendicular distance of the element from the axis (m)
- dm — mass element (kg)

Deriving I for a rod, a disc and a sphere from this is standard work in this course, not a lookup.

CALCULUS

Parallel-axis theorem

$$I = I_{cm} + M d^2$$

- I_{cm} — rotational inertia about a parallel axis through the centre of mass (kg m²)
- M — total mass (kg)
- d — distance between the two axes (m)

Only works between parallel axes, one of which passes through the centre of mass.

6 Energy and Momentum of Rotating Systems

CALCULUS

Angular momentum of a particle

$$L = r \times p$$

- L — angular momentum about the chosen point (kg m²/s)
- r — vector from that point to the particle (m)
- p — momentum (kg m/s)

Magnitude $m v r \sin\theta$. A particle travelling in a straight line still has angular momentum about a point off its line of motion, and that angular momentum is constant.

CALCULUS

Torque as the rate of change of angular momentum

$$\tau = dL / dt$$

- τ — net external torque about the point (N m)
- L — angular momentum about the same point (kg m²/s)

The rotational twin of $F_{net} = dp/dt$, and the cleanest statement of why L is conserved when no external torque acts.

7 Oscillations

CALCULUS

The equation of simple harmonic motion

$$d^2x / dt^2 = -\omega^2 x$$

- x — displacement from equilibrium (m)
- ω — angular frequency (rad/s)

The definition of simple harmonic motion in this course: show a restoring force proportional to displacement and this is what you have. ω^2 is read straight off it.

CALCULUS

General solution

$$x = A \cos(\omega t + \varphi)$$

- A — amplitude (m)
- φ — phase constant, fixed by the initial conditions (rad)
- ω — angular frequency (rad/s)

Differentiate it for $v = -A\omega \sin(\omega t + \varphi)$ and $a = -A\omega^2 \cos(\omega t + \varphi)$, which is where the maximum speed $A\omega$ and maximum acceleration $A\omega^2$ come from.

CALCULUS
DERIVED

Physical pendulum

$$T = 2\pi \sqrt{I / (m g d)}$$

- I — rotational inertia about the pivot (kg m²)
- d — distance from the pivot to the centre of mass (m)
- m — mass of the body (kg)

From $\Sigma\tau = I\alpha$ for small angles. The simple pendulum is the case $I = mL^2$ and $d = L$.

AP C:E&M

AP Physics C: Electricity and Magnetism

8

Electric Charges, Fields, and Gauss's Law

CALCULUS

Field of a continuous charge distribution

$$E = (1 / 4\pi\varepsilon_0) \int dq / r^2$$

- dq — charge on an element of the distribution (C)
- r — distance from that element to the field point (m)
- E — field, summed as a vector (N/C)

Add the components, not the magnitudes: symmetry usually kills one component and is what makes the integral doable.

CALCULUS

Charge densities

$$\lambda = dq / dl \cdot \sigma = dq / dA \cdot \rho = dq / dV$$

- λ — charge per unit length (C/m)
- σ — charge per unit area (C/m²)
- ρ — charge per unit volume (C/m³)

Definitions rather than results, but choosing the right one is what turns dq into something you can integrate.

CALCULUS

Gauss's law

$$\oint E \cdot dA = Q_{enc} / \varepsilon_0$$

- $\oint E \cdot dA$ — flux of the field out through a closed surface (N m²/C)
- Q_{enc} — net charge enclosed by that surface (C)
- ε_0 — permittivity of free space (C²/(N m²))

Always true, but only useful where symmetry makes E constant and parallel to dA over the surface — spherical, cylindrical or planar.

CALCULUS
DERIVED

What Gauss's law gives for the standard symmetries

sphere, outside: $E = (1 / 4\pi\varepsilon_0) Q / r^2$ · infinite line: $E = \lambda / (2\pi\varepsilon_0 r)$ · infinite plane: $E = \sigma / (2\varepsilon_0)$

- Q — total charge on the sphere (C)
- λ, σ — line and surface charge density (C/m, C/m²)
- r — distance from the centre or the axis (m)

Results of the derivation, not statements to quote in place of it. A uniformly charged sphere pulls from outside as if all its charge sat at the centre; inside a uniform shell the field is zero.

9

Electric Potential

CALCULUS

Potential difference from the field

$$\Delta V = -\int E \cdot dr$$

- ΔV — potential difference between the two points (V)
- $E \cdot dr$ — component of the field along the path, times the step (V)

The path does not matter, only the endpoints. Moving against the field raises the potential.

CALCULUS

Field from the potential

$$E_x = -\partial V / \partial x$$

- E_x — component of the field along x (V/m)
- $\partial V / \partial x$ — rate of change of potential in that direction (V/m)

The field points the way the potential falls fastest, and is perpendicular to the equipotentials.

CALCULUS

Potential of a continuous charge distribution

$$V = (1 / 4\pi\varepsilon_0) \int dq / r$$

- V — potential at the point, taken as zero at infinity (V)
- dq — charge element (C)
- r — its distance from the point (m)

A scalar integral with no components to resolve, which is why it is usually easier to find V first and differentiate for E.

CALCULUS

Potential energy of a group of charges

$$U_E = (1 / 4\pi\varepsilon_0) \Sigma q_i q_j / r_{ij}$$

- q_i, q_j — each pair of charges, signs included (C)
- r_{ij} — separation of that pair (m)

Summed over each pair once — the work needed to assemble the configuration from infinity.

10 Conductors and Capacitors

CALCULUS
DERIVED

Field just outside a charged conductor

$$E = \sigma / \varepsilon_0$$

- σ — local surface charge density (C/m²)
- E — field just outside, perpendicular to the surface (N/C)

From Gauss's law on a small pillbox straddling the surface. Twice the field of an isolated charged sheet, because all the flux leaves on one side.

CALCULUS

Energy density of an electric field

$$u = \tfrac{1}{2} \varepsilon_0 E^2$$

- u — energy stored per unit volume of the field (J/m³)
- E — field strength there (V/m)

The energy in a capacitor located in the field between the plates rather than on them.

CALCULUS

Finding a capacitance from the geometry

$$Q \rightarrow E \text{ by Gauss's law} \rightarrow \Delta V = -\int E \cdot dr \rightarrow C = Q / \Delta V$$

- Q — charge assumed on the conductors (C)
- C — capacitance, which the Q cancels out of (F)

A method, not a printed formula, and the way this course gets the capacitance of a spherical or cylindrical capacitor rather than only the parallel-plate one.

11 Electric Circuits

CALCULUS

Current as a derivative

$$I = dQ / dt$$

- I — instantaneous current (A)
- Q — charge that has passed (C)

Needed because in an RC or LR circuit the current is not constant.

CALCULUS

Current density and field in a conductor

$$J = I / A \cdot E = \rho J$$

- J — current density (A/m²)
- ρ — resistivity (Ω m)
- E — field driving the current inside the wire (V/m)

The local form of Ohm's law. R = ρL/A follows from it.

CALCULUS

Charging a capacitor through a resistor

$$q(t) = C\,\varepsilon\,(1 - e^{(-t / RC)}) \cdot I(t) = (\varepsilon / R)\,e^{(-t / RC)}$$

- q — charge on the capacitor at time t (C)
- ε — emf of the source (V)
- RC — time constant of the circuit (s)

The solution of the loop equation $\varepsilon = IR + q/C$. This course expects the differential equation to be set up and solved, so learn the shape of the curves as well as the algebra.

CALCULUS

Discharging a capacitor through a resistor

$$q(t) = Q_0\,e^{(-t / RC)} \cdot I(t) = (Q_0 / RC)\,e^{(-t / RC)}$$

- Q_0 — charge on the capacitor at $t = 0$ (C)
- R — resistance in the loop (Ω)
- C — capacitance (F)

Same exponential, no source. After one time constant RC the charge has fallen to about 37% of its starting value.

CALCULUS

The two steady states of an RC circuit

at t = 0 the capacitor behaves as a wire · after a long time it behaves as a break

- $t = 0$ — the instant the switch closes, when the capacitor is uncharged
- $t \to \infty$ — when the capacitor is fully charged and no current flows into it

Not an equation, but it answers most of what an AP C circuit question actually asks, without solving anything.

12 Magnetic Fields and Electromagnetism

CALCULUS

Magnetic force as a cross product

$$F = q\,v \times B$$

- F — magnetic force on the moving charge (N)
- v — velocity of the charge (m/s)
- B — magnetic field (T)

The vector form of the magnitude $qvB \sin\theta$ the algebra-based course uses. It does no work: speed cannot change, only direction.

CALCULUS

Force on a current element

$$dF = I\,dl \times B$$

- dl — element of the wire, in the direction of the current (m)
- I — current (A)

Integrated along the wire, which is how a curved or looped conductor is handled.

CALCULUS

Biot–Savart law

$$dB = (\mu_0 / 4\pi)\,I\,(dl \times r) / r^3$$

- dB — field contributed by one current element (T)
- r — vector from the element to the field point (m)
- μ_0 — permeability of free space (T m/A)

The magnetic counterpart of adding up dE from dq. The r^3 in the denominator is there because r on top is not a unit vector.

CALCULUS

Ampère's law

$$\oint B \cdot dl = \mu_0\,I_{enc}$$

- $\oint B \cdot dl$ — field summed round a closed loop (T m)
- I_{enc} — current passing through the loop (A)

Gauss's law's magnetic twin, and useful under the same condition: enough symmetry that B is constant along the chosen loop.

CALCULUS

Field inside a long solenoid

$$B = \mu_0 \, n \, I$$

- n — turns per unit length (m^{-1})
- I — current in the winding (A)

Uniform along the axis and independent of the radius, which is what makes a solenoid useful.

CALCULUS
DERIVED

Radius of a charged particle's circular path

$$r = m\,v / (q\,B)$$

- r — radius of the circle (m)
- m — mass of the particle (kg)
- q — charge (C)

From $qvB = mv^2/r$, the magnetic force supplying the centripetal force. The period does not depend on the speed.

13 Electromagnetic Induction

CALCULUS

Magnetic flux as an integral

$$\Phi B = \int B \cdot dA$$

- ΦB — flux through the surface (Wb)
- B · dA — component of the field normal to each area element, times that area (Wb)

Needed where the field varies across the loop, as it does near a straight wire.

CALCULUS

Faraday's law

$$\varepsilon = - d\Phi B / dt$$

- ε — induced emf (V)
- dΦB / dt — rate of change of flux, from B, from the area, or from the angle (Wb/s)

The instantaneous form of the ΔΦ/Δt statement the algebra-based course uses. The minus sign is Lenz's law, and this unit is built on it.

CALCULUS

Self-inductance

$$\varepsilon = - L\,dI / dt$$

- L — inductance (H)
- dI / dt — rate of change of current (A/s)

An inductor opposes changes in current, not current itself: with a steady current it is just a wire.

CALCULUS

Energy stored in an inductor

$$UL = \tfrac{1}{2}\, L\, I^2$$

- UL — energy stored in the magnetic field (J)
- I — current through the inductor (A)

CALCULUS

Current growth in an LR circuit

$$I(t) = (\varepsilon / R)\,(1 - e^{(-R\,t / L)})$$

- ε — emf of the source (V)
- L / R — time constant of the circuit (s)

The solution of ε = IR + L dI/dt. As with RC, the differential equation is the examinable part; at t = 0 the inductor blocks the current entirely and after a long time it does nothing.

M Maths you must recall

ALGEBRA

Right-angled triangles

$$c^2 = a^2 + b^2 \cdot \sin\theta = a / c \cdot \cos\theta = b / c \cdot \tan\theta = a / b$$

- c — hypotenuse
- a — side opposite θ
- b — side adjacent to θ

How a vector is broken into components and how the components are put back together. Used from the first unit onwards.

ALGEBRA

Areas

$$\text{rectangle } A = b\,h \cdot \text{triangle } A = \tfrac{1}{2}\,b\,h \cdot \text{circle } A = \pi\,r^2,\ \text{circumference } C = 2\pi\,r$$

- A — area (m²)
- b, h — base and height (m)
- r — radius (m)

The triangle is what the area under a velocity–time graph usually comes to; the circle is what pressure and flow questions need.

ALGEBRA

Volumes and surface areas

$$\text{rectangular solid } V = l\,w\,h \cdot \text{cylinder } V = \pi\,r^2\,l \cdot \text{sphere } V = (4/3)\,\pi\,r^3 \cdot \text{sphere surface } S = 4\pi\,r^2$$

- V — volume (m³)
- S — surface area (m²)
- r — radius (m)

Needed for density, buoyancy and for writing dm in terms of a density on the calculus-based route.

CALCULUS

Derivatives that come up

$$d(x^n)/dx = n\,x^{n-1} \cdot d(e^{ax})/dx = a\,e^{ax} \cdot d(\ln x)/dx = 1/x \cdot d(\sin ax)/dx = a\cos ax \cdot d(\cos ax)/dx = -\,a\sin ax$$

- a, n — constants
- x — the variable being differentiated with respect to

Almost every derivative in AP Physics C is one of these five. The exponential is the one that makes RC and LR circuits work.

CALCULUS

Integrals that come up

$$\int x^n\,dx = x^{n+1} / (n+1),\quad n \neq -1 \cdot \int dx / x = \ln x \cdot \int e^{ax}\,dx = e^{ax} / a \cdot \int \sin ax\,dx = -\,(\cos ax) / a$$

- a, n — constants
- + C — the constant of integration, fixed by an initial condition

In a physics problem the constant of integration is a starting position, velocity or charge, and forgetting it loses the initial conditions.

CALCULUS

Dot and cross products

$$A \cdot B = A\,B\cos\theta \cdot |A \times B| = A\,B\sin\theta$$

- A, B — magnitudes of the two vectors
- θ — angle between them (°)

The dot product is why work is a scalar; the cross product is why torque, angular momentum and the magnetic force point along an axis, with the direction from the right hand.

Values to know

ALGEBRA	Acceleration due to gravity at the Earth's surface	9.8 m/s²
ALGEBRA	Universal gravitational constant	6.67×10^{-11} N m²/kg²
ALGEBRA	Speed of light in a vacuum	3.00×10^{8} m/s
ALGEBRA	Elementary charge	1.60×10^{-19} C
ALGEBRA	Coulomb's law constant	9.0×10^{9} N m²/C²
ALGEBRA	Permittivity of free space	8.85×10^{-12} C²/(N m²)
ALGEBRA	Permeability of free space	$4\pi \times 10^{-7}$ T m/A
ALGEBRA	Boltzmann's constant	1.38×10^{-23} J/K

ALGEBRA	Avogadro's number	$6.02 \times 10^{23}\ mol^{-1}$
ALGEBRA	Universal gas constant	8.31 J/(mol K)
ALGEBRA	Planck's constant	6.63×10^{-34} J s = 4.14×10^{-15} eV s
ALGEBRA	Mass of an electron	9.11×10^{-31} kg
ALGEBRA	Mass of a proton	1.67×10^{-27} kg
ALGEBRA	Mass of a neutron	1.67×10^{-27} kg
ALGEBRA	Unified atomic mass unit	1.66×10^{-27} kg = 931 MeV/c^2
ALGEBRA	One electronvolt	1.60×10^{-19} J
ALGEBRA	One atmosphere of pressure	1.0×10^{5} Pa