AP Physics 1: Algebra-Based

AP Physics

> *Every AP physics exam supplies a table of equations, so the useful question is not what exists but what the table leaves you to remember. Anything not marked GIVEN is one to carry in your head. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not a College Board document; the official syllabus is the authority.*

1 Kinematics

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Average velocity

$$v_{avg} = \Delta x / \Delta t$$

- v_{avg} — average velocity (m/s)
- Δx — displacement, not distance travelled (m)
- Δt — time interval (s)

A definition rather than a result. Displacement on top is the whole difference between velocity and speed.

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Average acceleration

$$a_{avg} = \Delta v / \Delta t$$

- a_{avg} — average acceleration (m/s²)
- Δv — change in velocity (m/s)
- Δt — time interval (s)

Also a definition. Acceleration is a change in velocity, so a turn at constant speed is still an acceleration.

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Velocity under constant acceleration

$$v = v_0 + at$$

- v — velocity at time t (m/s)
- v_0 — velocity at $t = 0$ (m/s)
- a — constant acceleration (m/s²)
- t — time (s)

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Position under constant acceleration

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

- x — position at time t (m)
- x_0 — position at $t = 0$ (m)
- v_0 — velocity at $t = 0$ (m/s)
- a — constant acceleration (m/s²)
- t — time (s)

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Velocity without time

$$v^2 = v_0^2 + 2a(x - x_0)$$

- v, v_0 — final and initial velocity (m/s)
- a — constant acceleration (m/s²)
- $x - x_0$ — displacement (m)

The one to reach for when the question never mentions time.

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Projectile motion by components

$$a_x = 0 \cdot a_y = -g$$

- a_x — horizontal acceleration, with drag neglected (m/s²)
- a_y — vertical acceleration, downwards (m/s²)
- t — the same time for both components (s)

Not an equation the table needs to print: it is the method. Horizontal and vertical motion are solved separately and share only the time.

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Reading the motion graphs

velocity = slope of x–t · acceleration = slope of v–t · Δx = area under v–t

- slope — of a tangent where the graph is curved
- area — counted negative below the time axis

AP asks for these as often as it asks for the equations, and a graph question rarely gives you numbers to substitute.

2 Force and Translational Dynamics

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Newton's second law

$$a = F_{net} / m$$

$F_{net} = ma$

- F_{net} — vector sum of the forces on the object (N)
- m — mass (kg)
- a — acceleration, along F_{net} (m/s²)

Applied one axis at a time, off a free-body diagram.

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Weight

$$F_g = mg$$

- F_g — gravitational force on the object (N)
- m — mass (kg)
- g — gravitational field strength (N/kg or m/s²)

ALGEBRA

Friction

$$F_f \le \mu_s F_N \quad \text{(static)} \cdot F_f = \mu_k F_N \quad \text{(kinetic)}$$

- F_f — friction force, opposing relative sliding (N)
- μ_s, μ_k — coefficients of static and kinetic friction
- F_N — normal force (N)

Static friction is an inequality: it takes whatever value up to the maximum keeps the surfaces from sliding.

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Force from an ideal spring

$$F_s = -kx$$

- F_s — force the spring exerts (N)
- k — spring constant (N/m)
- x — displacement from the natural length (m)

The minus sign says the force points back towards the natural length; in magnitude, $F_s = k|x|$.

ALGEBRA

Newton's law of universal gravitation

$$F_g = G\, m_1 m_2 / r^2$$

- F_g — attractive force on each mass (N)
- G — universal gravitational constant (N m²/kg²)
- m_1, m_2 — the two masses (kg)
- r — distance between their centres (m)

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Gravitational field strength

$$g = F_g / m = G M / r^2$$

- g — field strength at that point (N/kg)
- M — mass producing the field (kg)
- r — distance from its centre (m)

Why g at the surface has the value it does, and why it falls off with altitude.

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Circular motion

$$a_c = v^2 / r \cdot F_{net} = m v^2 / r$$

- a_c — centripetal acceleration, towards the centre (m/s²)
- v — speed along the circle (m/s)
- r — radius (m)

Centripetal force is not an extra force to add to the diagram: it is the name for whatever real forces already point towards the centre.

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Speed and period of a circular orbit

$$v = \sqrt{G M / r} \cdot T^2 = 4\pi^2 r^3 / (G M)$$

- v — orbital speed (m/s)
- T — orbital period (s)
- M — mass of the central body (kg)
- r — radius of the orbit, measured from the centre (m)

Gravity supplying the centripetal force: $G M m / r^2 = m v^2 / r$, and the orbiting mass cancels, so everything at that radius orbits at the same speed whatever its own mass. The period relation is Kepler's third law for a circular orbit. Neither is printed — rebuild them from the two equations above.

3 Work, Energy, and Power

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Work done by a constant force

$$W = F d \cos \theta$$

- W — work done (J)
- F — force (N)
- d — displacement (m)
- θ — angle between the force and the displacement (°)

A force perpendicular to the motion does no work, which is why the centripetal force never appears in an energy equation.

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Work–energy theorem

$$W_{net} = \Delta K$$

- W_{net} — work done by the net force (J)
- ΔK — change in kinetic energy (J)

The bridge between the force unit and this one.

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Kinetic energy

$$K = \tfrac{1}{2} m v^2$$

- K — kinetic energy (J)
- m — mass (kg)
- v — speed (m/s)

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Gravitational potential energy near a surface

$$\Delta U_g = m g \Delta y$$

- ΔU_g — change in gravitational potential energy (J)
- Δy — change in height (m)

For a field that can be treated as uniform. Only the change has meaning; you choose where zero sits.

ALGEBRA

Gravitational potential energy of two masses

$$U_g = - G m_1 m_2 / r$$

- U_g — potential energy of the pair (J)
- m_1, m_2 — the two masses (kg)
- r — separation of their centres (m)

Negative because the zero is taken at infinite separation. This is the form orbit and escape-speed questions need.

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Elastic potential energy

$$U_s = \tfrac{1}{2}kx^2$$

- U_s — energy stored in the spring (J)
- k — spring constant (N/m)
- x — displacement from the natural length (m)

The area under a force–displacement graph.

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Conservation of energy

$$\Delta K + \Delta U + \Delta E_{thermal} = 0$$

- ΔK, ΔU — changes in kinetic and potential energy (J)
- $\Delta E_{thermal}$ — energy that has gone to heating the surfaces (J)

For an isolated system. Defining the system carefully is half of an AP energy question.

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Energy dissipated by friction

$$\Delta E_{thermal} = F_f\, d$$

- F_f — kinetic friction force (N)
- d — distance slid along the surface (m)

Not a printed statement: it is $W = Fd\cos\theta$ with $\theta = 180°$, counted as energy leaving the mechanical system. Distance slid, not displacement.

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Power

$$P = \Delta E / \Delta t \cdot P = F\, v \cos\theta$$

- P — power (W)
- ΔE — energy transferred (J)
- v — speed (m/s)

4 Linear Momentum

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Linear momentum

$$p = mv$$

- p — momentum, a vector along the velocity (kg m/s)
- m — mass (kg)
- v — velocity (m/s)

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Impulse

$$J = F\Delta t = \Delta p$$

- J — impulse (N s)
- F — average force during the interaction (N)
- Δt — how long it acts (s)

Also the area under a force–time graph, which is how AP usually asks for it.

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Conservation of momentum

$$\Sigma p \text{ before} = \Sigma p \text{ after}$$

- Σp — vector sum over the whole system (kg m/s)

Holds when no external force acts on the system, and holds component by component.

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Elastic and inelastic collisions

elastic: ΣK before = ΣK after · perfectly inelastic: the objects move off together

- K — kinetic energy (J)
- p — conserved in both cases (kg m/s)

Momentum is conserved in every collision here; kinetic energy only in the elastic one. The distinction is the point of the unit.

ALGEBRA

Centre of mass

$$x_{cm} = \Sigma(m\ x) / \Sigma m$$

- x_{cm} — position of the centre of mass (m)
- m — mass of each part (kg)
- x — position of each part (m)

The point that moves as if all the mass and all the external force acted there. AP Physics C: Mechanics turns this sum into an integral.

5 Torque and Rotational Dynamics

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Torque

$$\tau = r F \sin\theta$$

- τ — torque about the chosen axis (N m)
- r — distance from the axis to where the force acts (m)
- F — force (N)
- θ — angle between r and F (°)

$r \sin\theta$ is the lever arm — the perpendicular distance from the axis to the line of the force.

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Rotational form of Newton's second law

$$\alpha = \Sigma\tau / I$$

- α — angular acceleration (rad/s²)
- $\Sigma\tau$ — net torque about the axis (N m)
- I — rotational inertia about that axis (kg m²)

ALGEBRA

Rotational inertia of point masses

$$I = \Sigma\ m\ r^2$$

- I — rotational inertia (kg m²)
- m — each mass (kg)
- r — its distance from the axis (m)

It depends on the axis, so it is meaningless to quote one without saying about what.

ALGEBRA

Rotational kinematics

$$\omega = \omega_0 + \alpha t \cdot \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

- ω, ω_0 — final and initial angular velocity (rad/s)
- θ, θ_0 — final and initial angular position (rad)
- α — constant angular acceleration (rad/s²)
- t — time (s)

The straight-line equations with every symbol swapped for its angular twin.

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Angular velocity without time

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

- ω, ω_0 — final and initial angular velocity (rad/s)
- $\theta - \theta_0$ — angle turned through (rad)

The twin of $v^2 = v_0^2 + 2a\Delta x$. Worth deriving once rather than counting on it being printed.

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Linear and angular quantities

$$s = r\theta \cdot v = r\omega \cdot a = r\alpha$$

- s — arc length (m)
- r — distance from the axis (m)
- θ — angle, in radians (rad)

Only true with the angle in radians, which is why AP works in radians throughout.

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Rotational equilibrium

$$\Sigma F = 0 \cdot \Sigma\tau = 0$$

$\Sigma\tau$ — about any axis you choose (N m)

Both conditions are needed. Choosing the axis through an unknown force removes it from the torque equation.

6 Energy and Momentum of Rotating Systems

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Rotational kinetic energy

$$K = \tfrac{1}{2}I\omega^2$$

K — kinetic energy of rotation (J)
I — rotational inertia (kg m²)
ω — angular velocity (rad/s)

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Angular momentum of a rotating body

$$L = I\omega$$

L — angular momentum about the axis (kg m²/s)
I — rotational inertia (kg m²)
ω — angular velocity (rad/s)

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Angular impulse

$$\Delta L = \tau\ \Delta t$$

ΔL — change in angular momentum (kg m²/s)
τ — average external torque (N m)
Δt — how long it acts (s)

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Conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

I_1, I_2 — rotational inertia before and after (kg m²)
ω_1, ω_2 — angular velocity before and after (rad/s)

When no external torque acts. Pulling mass towards the axis lowers I, so ω rises — and K rises with it, because the pulling does work.

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Rolling without slipping

$$v_{cm} = r\omega \cdot K = \tfrac{1}{2}m\ v_{cm}^2 + \tfrac{1}{2}\ I_{cm}\ \omega^2$$

v_{cm} — speed of the centre of mass (m/s)
I_{cm} — rotational inertia about the centre of mass (kg m²)
r — radius of the rolling object (m)

The condition and the energy split follow from the definitions rather than being printed. It is why a hoop loses a race to a solid cylinder.

7 Oscillations

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Period, frequency and angular frequency

$$T = 1 / f \cdot \omega = 2\pi f$$

T — period (s)
f — frequency (Hz)
ω — angular frequency (rad/s)

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Displacement in simple harmonic motion

$$x = A\cos(2\pi ft)$$

- x — displacement from equilibrium (m)
- A — amplitude (m)
- f — frequency (Hz)
- t — time (s)

Written for an oscillator released from maximum displacement at t = 0; a sine describes one started from equilibrium.

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Period of a mass on a spring

$$T = 2\pi\sqrt{m / k}$$

- T — period (s)
- m — oscillating mass (kg)
- k — spring constant (N/m)

No amplitude in it: the period of a simple harmonic oscillator does not depend on how far you pull it.

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Period of a simple pendulum

$$T = 2\pi\sqrt{L / g}$$

- L — length of the pendulum (m)
- g — gravitational field strength (m/s²)

For small angles only, and independent of the mass hung on the end.

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Energy of an oscillator

$$E = \tfrac{1}{2}kA^2$$

- E — total energy of the oscillation (J)
- A — amplitude (m)
- k — spring constant (N/m)

$U_s = \frac{1}{2}kx^2$ read at maximum displacement, where the kinetic energy is zero. Energy goes as amplitude squared.

8 Fluids

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Density

$$\rho = m / V$$

- ρ — density (kg/m³)
- m — mass (kg)
- V — volume (m³)

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Pressure

$$P = F / A$$

- P — pressure (Pa)
- F — force perpendicular to the surface (N)
- A — area (m²)

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Pressure with depth

$$P = P_0 + \rho gh$$

- P — absolute pressure at that depth (Pa)
- P_0 — pressure at the surface (Pa)
- ρ — density of the fluid (kg/m³)
- h — depth below the surface (m)

Depth, not the shape of the container. ρgh on its own is the gauge pressure.

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Buoyant force

$$F_b = \rho_{fluid} V_{displaced}\, g$$

F_b — upward force on the submerged object (N)
ρ_{fluid} — density of the fluid, not the object (kg/m^3)
$V_{displaced}$ — volume of fluid pushed aside (m^3)

Archimedes' principle. For a floating object the displaced weight equals the object's whole weight.

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Continuity

$$A_1 v_1 = A_2 v_2$$

A — cross-sectional area (m^2)
v — flow speed there (m/s)

Conservation of volume for an incompressible fluid: a narrower pipe means a faster flow.

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Bernoulli's equation

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

P — pressure at that point (Pa)
y — height of that point (m)
v — flow speed there (m/s)

Conservation of energy per unit volume, for steady non-viscous incompressible flow along one streamline.