

AP Physics C: Electricity and Magnetism

AP Physics

Every AP physics exam supplies a table of equations, so the useful question is not what exists but what the table leaves you to remember. Anything not marked GIVEN is one to carry in your head. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not a College Board document; the official syllabus is the authority.

8 Electric Charges, Fields, and Gauss's Law

CALCULUS Field of a continuous charge distribution

$$\mathbf{E} = \left(\frac{1}{4\pi\epsilon_0} \right) \int \frac{dq}{r^2}$$

dq — charge on an element of the distribution (C)
 r — distance from that element to the field point (m)
 $\mathbf{E}$ — field, summed as a vector (N/C)

Add the components, not the magnitudes: symmetry usually kills one component and is what makes the integral doable.

CALCULUS Charge densities

$$\lambda = dq / dl \quad \sigma = dq / dA \quad \rho = dq / dV$$

λ — charge per unit length (C/m)
 σ — charge per unit area (C/m²)
 ρ — charge per unit volume (C/m³)

Definitions rather than results, but choosing the right one is what turns dq into something you can integrate.

CALCULUS Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}} / \epsilon_0$$

$\oint \mathbf{E} \cdot d\mathbf{A}$ — flux of the field out through a closed surface (N m²/C)
 Q_{enc} — net charge enclosed by that surface (C)
 ϵ_0 — permittivity of free space (C²/(N m²))

Always true, but only useful where symmetry makes $\mathbf{E}$ constant and parallel to $d\mathbf{A}$ over the surface — spherical, cylindrical or planar.

CALCULUS DERIVED What Gauss's law gives for the standard symmetries

$$\text{sphere, outside: } \mathbf{E} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{r^2} \quad \text{infinite line: } \mathbf{E} = \lambda / (2\pi\epsilon_0 r) \quad \text{infinite plane: } \mathbf{E} = \sigma / (2\epsilon_0)$$

Q — total charge on the sphere (C)
 λ, σ — line and surface charge density (C/m, C/m²)
 r — distance from the centre or the axis (m)

Results of the derivation, not statements to quote in place of it. A uniformly charged sphere pulls from outside as if all its charge sat at the centre; inside a uniform shell the field is zero.

9 Electric Potential

CALCULUS Potential difference from the field

$$\Delta V = - \int \mathbf{E} \cdot d\mathbf{r}$$

ΔV — potential difference between the two points (V)
 $\mathbf{E} \cdot d\mathbf{r}$ — component of the field along the path, times the step (V)

The path does not matter, only the endpoints. Moving against the field raises the potential.

CALCULUS

Field from the potential

$$\mathbf{E} = - \partial V / \partial \mathbf{x}$$

E_x — component of the field along x (V/m)

$\partial V / \partial x$ — rate of change of potential in that direction (V/m)

The field points the way the potential falls fastest, and is perpendicular to the equipotentials.

CALCULUS

Potential of a continuous charge distribution

$$V = (1 / 4\pi\epsilon_0) \int dq / r$$

V — potential at the point, taken as zero at infinity (V)

dq — charge element (C)

r — its distance from the point (m)

A scalar integral with no components to resolve, which is why it is usually easier to find V first and differentiate for E .

CALCULUS

Potential energy of a group of charges

$$U = (1 / 4\pi\epsilon_0) \sum q_i q_j / r_{ij}$$

q_i, q_j — each pair of charges, signs included (C)

r_{ij} — separation of that pair (m)

Summed over each pair once — the work needed to assemble the configuration from infinity.

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Conductors and Capacitors

CALCULUS

DERIVED

Field just outside a charged conductor

$$\mathbf{E} = \sigma / \epsilon_0$$

σ — local surface charge density (C/m²)

E — field just outside, perpendicular to the surface (N/C)

From Gauss's law on a small pillbox straddling the surface. Twice the field of an isolated charged sheet, because all the flux leaves on one side.

CALCULUS

Energy density of an electric field

$$u = \frac{1}{2} \epsilon_0 E^2$$

u — energy stored per unit volume of the field (J/m³)

E — field strength there (V/m)

The energy in a capacitor located in the field between the plates rather than on them.

CALCULUS

Finding a capacitance from the geometry

$$Q \rightarrow E \text{ by Gauss's law} \rightarrow \Delta V = - \int \mathbf{E} \cdot d\mathbf{r} \rightarrow C = Q / \Delta V$$

Q — charge assumed on the conductors (C)

C — capacitance, which the Q cancels out of (F)

A method, not a printed formula, and the way this course gets the capacitance of a spherical or cylindrical capacitor rather than only the parallel-plate one.

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Electric Circuits

CALCULUS

Current as a derivative

$$\mathbf{I} = dQ / dt$$

I — instantaneous current (A)

Q — charge that has passed (C)

Needed because in an RC or LR circuit the current is not constant.

CALCULUS **Current density and field in a conductor**

$$\mathbf{J} = \mathbf{I} / A \cdot \mathbf{E} = \rho \mathbf{J}$$

J — current density (A/m^2)

ρ — resistivity ($\Omega \text{ m}$)

E — field driving the current inside the wire (V/m)

The local form of Ohm's law. $R = \rho L/A$ follows from it.

CALCULUS **Charging a capacitor through a resistor**

$$q(t) = C \varepsilon (1 - e^{-(t / RC)}) \cdot I(t) = (\varepsilon / R) e^{-(t / RC)}$$

q — charge on the capacitor at time t (C)

ε — emf of the source (V)

RC — time constant of the circuit (s)

The solution of the loop equation $\varepsilon = IR + q/C$. This course expects the differential equation to be set up and solved, so learn the shape of the curves as well as the algebra.

CALCULUS **Discharging a capacitor through a resistor**

$$q(t) = Q_0 e^{-(t / RC)} \cdot I(t) = (Q_0 / RC) e^{-(t / RC)}$$

Q_0 — charge on the capacitor at $t = 0$ (C)

R — resistance in the loop (Ω)

C — capacitance (F)

Same exponential, no source. After one time constant RC the charge has fallen to about 37% of its starting value.

CALCULUS **The two steady states of an RC circuit**

at $t = 0$ the capacitor behaves as a wire · after a long time it behaves as a break

$t = 0$ — the instant the switch closes, when the capacitor is uncharged

$t \rightarrow \infty$ — when the capacitor is fully charged and no current flows into it

Not an equation, but it answers most of what an AP C circuit question actually asks, without solving anything.

12 **Magnetic Fields and Electromagnetism**

CALCULUS **Magnetic force as a cross product**

$$\mathbf{F} = q \mathbf{v} \times \mathbf{B}$$

F — magnetic force on the moving charge (N)

$\mathbf{v}$ — velocity of the charge (m/s)

$\mathbf{B}$ — magnetic field (T)

The vector form of the magnitude $qvB \sin \theta$ the algebra-based course uses. It does no work: speed cannot change, only direction.

CALCULUS **Force on a current element**

$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$$

$d\mathbf{l}$ — element of the wire, in the direction of the current (m)

I — current (A)

Integrated along the wire, which is how a curved or looped conductor is handled.

CALCULUS **Biot-Savart law**

$$d\mathbf{B} = (\mu_0 / 4\pi) I (d\mathbf{l} \times \mathbf{r}) / r^3$$

$d\mathbf{B}$ — field contributed by one current element (T)

$\mathbf{r}$ — vector from the element to the field point (m)

μ_0 — permeability of free space (T m/A)

The magnetic counterpart of adding up dE from dq . The r^3 in the denominator is there because $\mathbf{r}$ on top is not a unit vector.

CALCULUS

Ampère's law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

$\oint \mathbf{B} \cdot d\mathbf{l}$ — field summed round a closed loop (T m)

I_{enc} — current passing through the loop (A)

Gauss's law's magnetic twin, and useful under the same condition: enough symmetry that B is constant along the chosen loop.

CALCULUS

Field inside a long solenoid

$$\mathbf{B} = \mu_0 n \mathbf{I}$$

n — turns per unit length (m^{-1})

I — current in the winding (A)

Uniform along the axis and independent of the radius, which is what makes a solenoid useful.

CALCULUS

DERIVED

Radius of a charged particle's circular path

$$r = m v / (q B)$$

r — radius of the circle (m)

m — mass of the particle (kg)

q — charge (C)

From $qvB = mv^2/r$, the magnetic force supplying the centripetal force. The period does not depend on the speed.

13

Electromagnetic Induction

CALCULUS

Magnetic flux as an integral

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

Φ_B — flux through the surface (Wb)

$\mathbf{B} \cdot d\mathbf{A}$ — component of the field normal to each area element, times that area (Wb)

Needed where the field varies across the loop, as it does near a straight wire.

CALCULUS

Faraday's law

$$\varepsilon = - d\Phi_B / dt$$

ε — induced emf (V)

$d\Phi_B / dt$ — rate of change of flux, from B , from the area, or from the angle (Wb/s)

The instantaneous form of the $\Delta\Phi/\Delta t$ statement the algebra-based course uses. The minus sign is Lenz's law, and this unit is built on it.

CALCULUS

Self-inductance

$$\varepsilon = - L dI / dt$$

L — inductance (H)

dI / dt — rate of change of current (A/s)

An inductor opposes changes in current, not current itself: with a steady current it is just a wire.

CALCULUS

Energy stored in an inductor

$$U_L = \frac{1}{2} L I^2$$

U_L — energy stored in the magnetic field (J)

I — current through the inductor (A)

CALCULUS

Current growth in an LR circuit

$$I(t) = (\varepsilon / R) (1 - e^{-(R t / L)})$$

ε — emf of the source (V)

L / R — time constant of the circuit (s)

The solution of $\varepsilon = IR + L dI/dt$. As with RC, the differential equation is the examinable part; at $t = 0$ the inductor blocks the current entirely and after a long time it does nothing.

M

Maths you must recall

ALGEBRA

Right-angled triangles

$$c^2 = a^2 + b^2 \cdot \sin \theta = a / c \cdot \cos \theta = b / c \cdot \tan \theta = a / b$$

- c — hypotenuse
a — side opposite θ
b — side adjacent to θ

How a vector is broken into components and how the components are put back together. Used from the first unit onwards.

ALGEBRA

Areas

$$\text{rectangle } A = b h \cdot \text{triangle } A = \frac{1}{2} b h \cdot \text{circle } A = \pi r^2, \text{ circumference } C = 2\pi r$$

- A — area (m^2)
b, h — base and height (m)
r — radius (m)

The triangle is what the area under a velocity-time graph usually comes to; the circle is what pressure and flow questions need.

ALGEBRA

Volumes and surface areas

$$\text{rectangular solid } V = l w h \cdot \text{cylinder } V = \pi r^2 l \cdot \text{sphere } V = (4/3) \pi r^3 \cdot \text{sphere surface } S = 4\pi r^2$$

- V — volume (m^3)
S — surface area (m^2)
r — radius (m)

Needed for density, buoyancy and for writing dm in terms of a density on the calculus-based route.

CALCULUS

Derivatives that come up

$$d(x^n)/dx = n x^{n-1} \cdot d(e^{ax})/dx = a e^{ax} \cdot d(\ln x)/dx = 1/x \cdot d(\sin ax)/dx = a \cos ax \cdot d(\cos ax)/dx = -a \sin ax$$

- a, n — constants
x — the variable being differentiated with respect to

Almost every derivative in AP Physics C is one of these five. The exponential is the one that makes RC and LR circuits work.

CALCULUS

Integrals that come up

$$\int x^n dx = x^{n+1} / (n+1), \quad n \neq -1 \cdot \int dx / x = \ln x \cdot \int e^{ax} dx = e^{ax} / a \cdot \int \sin ax dx = -(\cos ax) / a$$

- a, n — constants
+ C — the constant of integration, fixed by an initial condition

In a physics problem the constant of integration is a starting position, velocity or charge, and forgetting it loses the initial conditions.

CALCULUS

Dot and cross products

$$A \cdot B = A B \cos \theta \cdot |A \times B| = A B \sin \theta$$

- A, B — magnitudes of the two vectors
 θ — angle between them ($^\circ$)

The dot product is why work is a scalar; the cross product is why torque, angular momentum and the magnetic force point along an axis, with the direction from the right hand.

— **Values to know**

ALGEBRA	Acceleration due to gravity at the Earth's surface	9.8 m/s ²
ALGEBRA	Universal gravitational constant	6.67 × 10 ⁻¹¹ N m ² /kg ²
ALGEBRA	Speed of light in a vacuum	3.00 × 10 ⁸ m/s
ALGEBRA	Elementary charge	1.60 × 10 ⁻¹⁹ C
ALGEBRA	Coulomb's law constant	9.0 × 10 ⁹ N m ² /C ²
ALGEBRA	Permittivity of free space	8.85 × 10 ⁻¹² C ² /(N m ²)
ALGEBRA	Permeability of free space	4π × 10 ⁻⁷ T m/A
ALGEBRA	Boltzmann's constant	1.38 × 10 ⁻²³ J/K

ALGEBRA	Avogadro's number	$6.02 \times 10^{23} \text{ mol}^{-1}$
ALGEBRA	Universal gas constant	8.31 J/(mol K)
ALGEBRA	Planck's constant	$6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s}$
ALGEBRA	Mass of an electron	$9.11 \times 10^{-31} \text{ kg}$
ALGEBRA	Mass of a proton	$1.67 \times 10^{-27} \text{ kg}$
ALGEBRA	Mass of a neutron	$1.67 \times 10^{-27} \text{ kg}$
ALGEBRA	Unified atomic mass unit	$1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
ALGEBRA	One electronvolt	$1.60 \times 10^{-19} \text{ J}$
ALGEBRA	One atmosphere of pressure	$1.0 \times 10^5 \text{ Pa}$