AP Physics C: Mechanics

AP Physics

> *Every AP physics exam supplies a table of equations, so the useful question is not what exists but what the table leaves you to remember. Anything not marked GIVEN is one to carry in your head. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not a College Board document; the official syllabus is the authority.*

1 Kinematics

CALCULUS

Velocity as a derivative

$$v = dx / dt$$

- v — instantaneous velocity, a vector (m/s)
- x — position (m)
- t — time (s)

The slope of the position graph, said exactly. In two dimensions each component is differentiated separately.

CALCULUS

Acceleration as a derivative

$$a = dv / dt = d^2x / dt^2$$

- a — instantaneous acceleration (m/s^2)
- v — velocity (m/s)

CALCULUS

Velocity and position by integration

$$v = v_0 + \int a\, dt \cdot x = x_0 + \int v\, dt$$

- v_0, x_0 — the initial values, which the integration constants fix (m/s, m)
- ∫ … dt — over the interval in question

The same two definitions read backwards, and the reason this course can set motion where the acceleration is not constant. The algebra-based equations of motion are what these give when a is constant.

2 Force and Translational Dynamics

CALCULUS

Force as the rate of change of momentum

$$F_{net} = dp / dt$$

- Fnet — net force (N)
- p — momentum (kg m/s)

Newton's second law in the form he wrote it. Fnet = ma is the special case of constant mass.

CALCULUS

Motion against a resistive force

$$m\, dv / dt = m g - b v$$

- b — drag coefficient for a force proportional to speed (kg/s)
- v — speed (m/s)
- m g — the driving force, here the weight (N)

Written for a falling object with drag proportional to v; a drag proportional to v^2 is set the same way. This separable differential equation is the piece of calculus this unit exists to use.

CALCULUS
DERIVED

Terminal speed

$$v_{term} = m g / b$$

- vterm — speed at which the acceleration reaches zero (m/s)
- b — drag coefficient, for drag proportional to speed (kg/s)

Set dv/dt = 0 in the equation above. The speed is approached but never quite reached.

3 Work, Energy, and Power

CALCULUS

Work done by a varying force

$$W = \int F \cdot dr$$

- W — work done (J)
- $F \cdot dr$ — the component of the force along the path, times the step along it (J)

The area under a force–displacement graph, and the general statement $W = Fd \cos\theta$ is the constant-force case of.

CALCULUS

Instantaneous power

$$P = dW / dt = F \cdot v$$

- P — power at that instant (W)
- $F \cdot v$ — force times the component of velocity along it (W)

CALCULUS

Force from a potential energy function

$$F = -\,dU / dx$$

- U — potential energy as a function of position (J)
- F — conservative force, along x (N)

The force points down the slope of the U–x graph. Equilibrium is where the slope is zero, stable where the curve is a minimum.

CALCULUS

Potential energy from a conservative force

$$\Delta U = -\int F \cdot dr$$

- ΔU — change in potential energy (J)
- F — the conservative force (N)

Where $\frac{1}{2}kx^2$ and $-Gm_1m_2/r$ come from, and only defined for a force whose work does not depend on the path.

4 Linear Momentum

CALCULUS

Impulse as an integral

$$J = \int F\, dt = \Delta p$$

- J — impulse (N s)
- F — force, varying through the interaction (N)

The area under a force–time graph, without needing the force to be constant.

CALCULUS

Centre of mass of a continuous body

$$r_{cm} = (1 / M) \int r\, dm$$

- r_{cm} — position of the centre of mass (m)
- M — total mass (kg)
- dm — mass of an element at position r (kg)

The discrete sum turned into an integral. Writing dm in terms of a density and a length, area or volume element is the whole technique.

CALCULUS

Motion of the centre of mass

$$M\, a_{cm} = \Sigma F_{external}$$

- a_{cm} — acceleration of the centre of mass (m/s²)
- $\Sigma F_{external}$ — sum of the external forces only (N)

Internal forces cancel in pairs, so an exploding shell's centre of mass carries on along the same parabola.

5 Torque and Rotational Dynamics

CALCULUS

Torque as a cross product

$$\tau = r \times F$$

- τ — torque, a vector along the axis (N m)
- r — vector from the axis to the point of application (m)
- F — force (N)

Magnitude $rF \sin\theta$, direction from the right hand. The vector form is what lets torques about different axes be added properly.

CALCULUS

Rotational inertia of a continuous body

$$I = \int r^2 \, dm$$

- I — rotational inertia about the chosen axis (kg m²)
- r — perpendicular distance of the element from the axis (m)
- dm — mass element (kg)

Deriving I for a rod, a disc and a sphere from this is standard work in this course, not a lookup.

CALCULUS

Parallel-axis theorem

$$I = I_{cm} + M d^2$$

- I_{cm} — rotational inertia about a parallel axis through the centre of mass (kg m²)
- M — total mass (kg)
- d — distance between the two axes (m)

Only works between parallel axes, one of which passes through the centre of mass.

6 Energy and Momentum of Rotating Systems

CALCULUS

Angular momentum of a particle

$$L = r \times p$$

- L — angular momentum about the chosen point (kg m²/s)
- r — vector from that point to the particle (m)
- p — momentum (kg m/s)

Magnitude $m v r \sin\theta$. A particle travelling in a straight line still has angular momentum about a point off its line of motion, and that angular momentum is constant.

CALCULUS

Torque as the rate of change of angular momentum

$$\tau = dL / dt$$

- τ — net external torque about the point (N m)
- L — angular momentum about the same point (kg m²/s)

The rotational twin of $F_{net} = dp/dt$, and the cleanest statement of why L is conserved when no external torque acts.

7 Oscillations

CALCULUS

The equation of simple harmonic motion

$$d^2x / dt^2 = -\omega^2 x$$

- x — displacement from equilibrium (m)
- ω — angular frequency (rad/s)

The definition of simple harmonic motion in this course: show a restoring force proportional to displacement and this is what you have. ω^2 is read straight off it.

CALCULUS

General solution

$$x = A \cos(\omega t + \varphi)$$

- A — amplitude (m)
- φ — phase constant, fixed by the initial conditions (rad)
- ω — angular frequency (rad/s)

Differentiate it for $v = -A\omega \sin(\omega t + \varphi)$ and $a = -A\omega^2 \cos(\omega t + \varphi)$, which is where the maximum speed $A\omega$ and maximum acceleration $A\omega^2$ come from.

CALCULUS
DERIVED

Physical pendulum

$$T = 2\pi \sqrt{I / (m\, g\, d)}$$

- I — rotational inertia about the pivot (kg m²)
- d — distance from the pivot to the centre of mass (m)
- m — mass of the body (kg)

From $\Sigma\tau = I\alpha$ for small angles. The simple pendulum is the case $I = mL^2$ and $d = L$.