Every formula

IB Diploma Physics

> *The data booklet is on the desk in every paper, so the useful question is what you have to supply yourself: the definitions, the decay bookkeeping, and the results you are expected to derive. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not an IB document; the official syllabus is the authority.*

A Space, time and motion

A.1 Kinematics

SL **Average velocity**

$$v = \Delta s / \Delta t$$

- v — average velocity (m s^{-1})
- Δs — displacement (m)
- Δt — time interval (s)

A definition, not a booklet equation. Average speed uses distance travelled and average velocity uses displacement; the two differ whenever the path is not straight.

SL **Average acceleration**

$$a = \Delta v / \Delta t$$

- a — average acceleration (m s^{-2})
- Δv — change in velocity (m s^{-1})
- Δt — time interval (s)

A definition, not a booklet equation.

SL **Velocity after uniform acceleration**

$$v = u + at$$

- v — final velocity (m s^{-1})
- u — initial velocity (m s^{-1})
- a — acceleration (m s^{-2})
- t — time (s)

SL **Displacement under uniform acceleration**

$$s = ut + \tfrac{1}{2}at^2$$

- s — displacement (m)
- u — initial velocity (m s^{-1})
- a — acceleration (m s^{-2})
- t — time (s)

SL **Uniform acceleration, without time**

$$v^2 = u^2 + 2as$$

- v — final velocity (m s^{-1})
- u — initial velocity (m s^{-1})
- a — acceleration (m s^{-2})
- s — displacement (m)

SL **Displacement from average velocity**

$$s = \tfrac{1}{2}(u + v)t$$

- s — displacement (m)
- u, v — initial and final velocity (m s^{-1})
- t — time (s)

All four of these hold only while the acceleration is constant.

SL

From the graphs

velocity = gradient of s–t · acceleration = gradient of v–t · displacement = area under v–t

gradient — of a tangent where the motion is not uniform

Not printed in the data booklet. Distance travelled is the area under a speed–time graph, which is not the same as displacement once the direction reverses.

SL
DERIVED

Projectile motion

$$\text{horizontal: } s_x = u_x t \cdot \text{vertical: } s_y = u_y t - \tfrac{1}{2}gt^2$$

u_x — horizontal component of the launch velocity (m s^{-1})
u_y — vertical component of the launch velocity (m s^{-1})
g — acceleration of free fall (m s^{-2})

Not a separate booklet equation: it is the four equations above applied to each axis, with zero acceleration horizontally. Fluid resistance is left out here, and its effect on a projectile — shorter range, a steeper descent — is wanted only qualitatively; the drag force itself is in A.2, and terminal speed is where it balances the weight.

A.2

Forces and momentum

SL

Newton's second law

$$F = ma$$

F — resultant force (N)
m — mass (kg)
a — acceleration (m s^{-2})

The special case of $F = \Delta p/\Delta t$ for constant mass. Force and acceleration are in the same direction.

SL

Force as rate of change of momentum

$$F = \Delta p / \Delta t$$

F — resultant force (N)
Δp — change in momentum (kg m s^{-1})
Δt — time taken (s)

SL

Linear momentum

$$p = mv$$

p — momentum (kg m s^{-1}, N s)
m — mass (kg)
v — velocity (m s^{-1})

SL

Impulse

$$J = F\Delta t = \Delta p$$

J — impulse (N s)
F — constant force (N)
Δt — time for which it acts (s)

For a varying force, impulse is the area under a force–time graph.

SL

Translational equilibrium

$$\Sigma F = 0$$

ΣF — vector sum of every force on the body (N)

Not printed in the data booklet. It is the condition for constant velocity, drawn from a free-body diagram.

SL

Conservation of linear momentum

$$\text{total momentum before} = \text{total momentum after}$$

p — summed over the system, with direction

Not printed in the data booklet. It holds for an isolated system, in collisions and in explosions; kinetic energy is conserved only in an elastic collision.

SL

Static friction

$$F_f \le \mu_s N$$

F_f — friction force (N)
μ_s — coefficient of static friction
N — normal reaction force (N)

An inequality: static friction takes whatever value balances the applied force, up to this maximum.

SL

Dynamic friction

$$F_f = \mu_d N$$

F_f — friction force while sliding (N)
μ_d — coefficient of dynamic friction
N — normal reaction force (N)

SL

Buoyancy

$$F_b = \rho V g$$

F_b — buoyancy force, the upthrust on the body (N)
ρ — density of the fluid (kg m^{-3})
V — volume of fluid displaced (m^3)
g — gravitational field strength (N kg^{-1})

Archimedes' principle: the upthrust equals the weight of the fluid displaced. V is the volume of fluid pushed aside, which is the volume of the whole body only when it is fully submerged.

SL

Viscous drag on a sphere (Stokes' law)

$$F_d = 6\pi\eta r v$$

F_d — viscous drag force (N)
η — viscosity of the fluid (Pa s)
r — radius of the sphere (m)
v — speed of the sphere through the fluid (m s^{-1})

For a small sphere in laminar flow only. The drag grows with speed, which is what fixes terminal speed: the body stops accelerating once the drag plus the buoyancy balance the weight.

SL

Angular speed in a circle

$$\omega = 2\pi / T \cdot v = \omega r$$

ω — angular speed (rad s^{-1})
T — period of the circular motion (s)
v — linear speed (m s^{-1})
r — radius of the circle (m)

SL

Centripetal acceleration

$$a = v^2/r = \omega^2 r = 4\pi^2 r / T^2$$

a — centripetal acceleration (m s^{-2})
v — linear speed (m s^{-1})
r — radius (m)

Directed towards the centre, so uniform circular motion is accelerated motion even though the speed does not change.

SL

Centripetal force

$$F = mv^2/r = m\omega^2 r = 4\pi^2 m r / T^2$$

F — resultant force towards the centre (N)
m — mass (kg)
r — radius (m)

Not a new force: it is the name for whatever resultant — tension, gravity, friction, the normal force — points at the centre.

A.3 Work, energy and power

SL

Work done by a constant force

$$W = Fs\cos\theta$$

- W — work done (J)
- F — force (N)
- s — displacement (m)
- θ — angle between force and displacement (°)

A force at right angles to the motion does no work, which is why a centripetal force never changes the speed.

SL

Work from a force–displacement graph

$$\text{work done} = \text{area under a force–displacement graph}$$

- area — counted negative where the force opposes the motion

Not printed in the data booklet. This is how work is found when the force varies, as it does for a spring.

SL

Kinetic energy

$$E_k = \tfrac{1}{2}mv^2 = p^2 / (2m)$$

- E_k — kinetic energy (J)
- m — mass (kg)
- v — speed (m s⁻¹)
- p — momentum (kg m s⁻¹)

The momentum form is worth having: it turns a momentum answer into an energy answer without going back through the speed.

SL

Change in gravitational potential energy

$$\Delta E_p = mg\Delta h$$

- ΔE_p — change in gravitational potential energy (J)
- m — mass (kg)
- g — gravitational field strength (N kg⁻¹)
- Δh — change in height (m)

For a uniform field, which is the near-Earth case. D.1 replaces it with $E_p = -GMm/r$ once the field is no longer uniform.

SL

Force in a stretched spring

$$F = k\Delta x$$

$k = F / \Delta x$ · $\Delta x = F / k$

- F — force applied to the spring (N)
- k — spring constant (N m⁻¹)
- Δx — extension or compression (m)

Within the limit of proportionality.

SL

Elastic potential energy

$$E_e = \tfrac{1}{2}k\Delta x^2$$

- E_e — energy stored in the spring (J)
- k — spring constant (N m⁻¹)
- Δx — extension or compression (m)

The area under the force–extension graph.

SL

Power

$$P = W / \Delta t = Fv$$

- P — power (W)
- W — work done (J)
- Δt — time taken (s)
- F — force (N)
- v — speed in the direction of the force (m s⁻¹)

SL

Efficiency

$$\eta = \text{useful work out} / \text{total work in} = \text{useful power out} / \text{total power in}$$

η — efficiency (a ratio, no unit)

Multiply by 100% for a percentage. Energy is conserved either way — the shortfall is degraded, not destroyed.

A.4

Rigid body mechanics

HL

Torque of a force

$$\tau = Fr \sin\theta$$

τ — torque about the axis (N m)
F — force (N)
r — distance from the axis to the point where the force is applied (m)
θ — angle between the force and the line joining the axis to that point (°)

Higher level only. A body is in rotational equilibrium when the resultant torque about every axis is zero.

HL

Moment of inertia

$$I = \Sigma mr^2$$

I — moment of inertia (kg m^2)
m — mass of each particle (kg)
r — its distance from the axis (m)

Higher level only. It depends on the axis chosen, not only on the body — the rotational counterpart of mass.

HL

Newton's second law for rotation

$$\tau = I\alpha$$

τ — resultant torque (N m)
I — moment of inertia (kg m^2)
α — angular acceleration (rad s^{-2})

Higher level only.

HL

Angular velocity after uniform angular acceleration

$$\omega = \omega_0 + \alpha t$$

ω — final angular velocity (rad s^{-1})
ω_0 — initial angular velocity (rad s^{-1})
α — angular acceleration (rad s^{-2})
t — time (s)

Higher level only.

HL

Angular displacement under uniform angular acceleration

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \cdot \theta = \tfrac{1}{2}(\omega_0 + \omega)t$$

θ — angular displacement (rad)
ω_0, ω — initial and final angular velocity (rad s^{-1})
α — angular acceleration (rad s^{-2})

Higher level only. The four rotational equations mirror the four in A.1 term for term.

HL

Angular velocity without time

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

ω, ω_0 — final and initial angular velocity (rad s^{-1})
α — angular acceleration (rad s^{-2})
θ — angular displacement (rad)

Higher level only.

HL

Angular momentum and angular impulse

$$L = I\omega \cdot \Delta L = \tau\Delta t$$

- L — angular momentum (kg m² s⁻¹)
- I — moment of inertia (kg m²)
- ω — angular velocity (rad s⁻¹)
- τ — resultant torque (N m)

Higher level only. Angular momentum is conserved when the resultant torque is zero, which is why a skater spins faster with their arms pulled in.

HL

Rotational kinetic energy

$$E_k = \tfrac{1}{2}I\omega^2 = L^2 / (2I)$$

- E_k — rotational kinetic energy (J)
- I — moment of inertia (kg m²)
- ω — angular velocity (rad s⁻¹)
- L — angular momentum (kg m² s⁻¹)

Higher level only. A rolling body has this on top of $\tfrac{1}{2}mv^2$ for its centre of mass.

A.5 Galilean and special relativity

HL

Galilean transformations

$$x' = x - vt \cdot u' = u - v$$

- x', x — position in the moving and the original frame (m)
- u', u — velocity of an object in each frame (m s⁻¹)
- v — speed of one frame relative to the other (m s⁻¹)

Higher level only. The low-speed limit, and the thing special relativity replaces.

HL

Lorentz factor

$$\gamma = 1 / \sqrt{(1 - v^2/c^2)}$$

- γ — Lorentz factor (a ratio, no unit)
- v — relative speed of the two frames (m s⁻¹)
- c — speed of light in a vacuum (m s⁻¹)

Higher level only. $\gamma \geq 1$ always, and every relativistic effect below is γ doing the work.

HL

Lorentz transformations

$$x' = \gamma(x - vt) \cdot t' = \gamma(t - vx/c^2)$$

- x', t' — coordinates of an event in the moving frame (m, s)
- x, t — its coordinates in the original frame (m, s)
- γ — Lorentz factor

Higher level only. The same form works for intervals: $\Delta x' = \gamma(\Delta x - v\Delta t)$ and $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$.

HL

Relativistic velocity addition

$$u' = (u - v) / (1 - uv/c^2)$$

- u' — velocity measured in the moving frame (m s⁻¹)
- u — velocity measured in the original frame (m s⁻¹)
- v — relative speed of the frames (m s⁻¹)

Higher level only. Feed in $u = c$ and you get $u' = c$, whatever v is — which is the postulate the whole theory is built on.

HL

Time dilation

$$\Delta t = \gamma\Delta t_0$$

- Δt — time interval measured in the frame the clock moves in (s)
- Δt_0 — proper time, measured where the two events happen at the same place (s)
- γ — Lorentz factor

Higher level only. Getting the answer right is mostly a matter of deciding which observer measures the proper time.

HL

Length contraction

$$L = L_0 / \gamma$$

- L — length measured in the frame the object moves in (m)
- L_0 — proper length, measured at rest with respect to the object (m)
- γ — Lorentz factor

Higher level only. Contraction happens along the direction of motion only.

HL

Spacetime interval

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

- Δs — spacetime interval between two events (m)
- Δt — time between them (s)
- Δx — distance between them (m)

Higher level only. Every inertial observer disagrees about Δt and Δx and agrees about Δs — the invariant the spacetime diagram is drawn around.

HL

Rest energy

$$E_0 = mc^2$$

- E_0 — rest energy (J, MeV)
- m — rest mass (kg, MeV c^{-2})
- c — speed of light in a vacuum (m s^{-1})

Higher level only. Working in MeV and MeV c^{-2} saves most of the arithmetic.

HL

Total energy

$$E = \gamma mc^2$$

- E — total energy (J, MeV)
- γ — Lorentz factor
- m — rest mass (kg)

Higher level only.

HL

Relativistic kinetic energy

$$E_k = (\gamma - 1)mc^2$$

- E_k — kinetic energy (J, MeV)
- γ — Lorentz factor
- m — rest mass (kg)

Higher level only. Total energy minus rest energy; $\frac{1}{2}mv^2$ is what this collapses to at low speed.

HL

Relativistic momentum

$$p = \gamma mv$$

- p — momentum (kg m s^{-1}, MeV c^{-1})
- γ — Lorentz factor
- m — rest mass (kg)
- v — speed (m s^{-1})

HL

Energy–momentum relation

$$E^2 = p^2c^2 + m^2c^4$$

- E — total energy (J, MeV)
- p — momentum (kg m s^{-1}, MeV c^{-1})
- m — rest mass (kg, MeV c^{-2})

Higher level only. It links E and p without needing the speed, and for a massless particle it reduces to $E = pc$.

B The particulate nature of matter

B.1 Thermal energy transfers

SL

Kelvin and Celsius

$$T / \mathrm{K} = \theta / {}^\circ\mathrm{C} + 273$$

$\theta / {}^\circ\mathrm{C} = T / \mathrm{K} - 273$

- T — absolute temperature (K)
- θ — temperature (°C)

Absolute zero is 0 K. Every gas and radiation equation below needs the temperature in kelvin.

SL

Specific heat capacity

$$Q = mc\Delta T$$

$c = Q / (m\Delta T)$ · $\Delta T = Q / (mc)$

- Q — energy transferred (J)
- m — mass (kg)
- c — specific heat capacity (J kg^{-1} K^{-1})
- ΔT — temperature change (K)

A temperature change in kelvin and in degrees Celsius is the same number, so either unit works here.

SL

Specific latent heat

$$Q = mL$$

- Q — energy transferred (J)
- m — mass changing phase (kg)
- L — specific latent heat of fusion or vaporisation (J kg^{-1})

The temperature does not change during a phase change: the energy goes into potential energy between the molecules, not kinetic.

SL

Conduction

$$\Delta Q / \Delta t = kA\, \Delta T / \Delta x$$

- $\Delta Q/\Delta t$ — rate of thermal energy transfer (W)
- k — thermal conductivity (W m^{-1} K^{-1})
- A — cross-sectional area (m^2)
- $\Delta T/\Delta x$ — temperature gradient (K m^{-1})

Convection is treated qualitatively, through density differences in a fluid.

SL

Stefan–Boltzmann law

$$L = \sigma A T^4$$

- L — power radiated by a black body (W)
- σ — Stefan–Boltzmann constant (W m^{-2} K^{-4})
- A — surface area (m^2)
- T — surface temperature (K)

The fourth power is the point: doubling the absolute temperature multiplies the power by sixteen.

SL

Emissivity

e = power radiated by the body / power radiated by a black body of the same area and temperature

- e — emissivity, between 0 and 1 (a ratio, no unit)

A black body has $e = 1$, so a real surface radiates $P = e\sigma AT^4$.

SL

Wien's displacement law

$$\lambda_{max} = 2.90 \times 10^{-3} / T$$

- λ_{max} — wavelength at which the emitted power peaks (m)
- T — surface temperature (K)

The constant is in m K. Hotter bodies peak at shorter wavelengths, which is how a star's colour gives its temperature.

SL

Apparent brightness

$$b = L / (4\pi d^2)$$

- b — apparent brightness, the intensity received (W m^{-2})
- L — luminosity, the power emitted (W)
- d — distance to the source (m)

An inverse square law, because the power spreads over a sphere of area $4\pi d^2$. E.5 uses the same equation for stars.

B.2 Greenhouse effect

SL

Albedo

$$\alpha = \text{total scattered power} / \text{total incident power}$$

- α — albedo, between 0 and 1 (a ratio, no unit)

The fraction of incoming radiation reflected away. Ice and cloud raise it; open ocean and forest lower it.

SL

Solar intensity at a planet

$$S = L / (4\pi d^2)$$

- S — intensity arriving on a surface facing the Sun (W m^{-2})
- L — luminosity of the Sun (W)
- d — distance from the Sun (m)

At the Earth's orbit this is the solar constant, about 1.36×10^3 W m^{-2}.

SL
DERIVED

Average intensity absorbed by a planet

$$\text{average absorbed intensity} = S(1 - \alpha) / 4$$

- S — solar intensity at that orbit (W m^{-2})
- α — albedo

Not printed in the data booklet. The factor of four is the disc the planet presents, πr^2, spread over the whole surface it radiates from, $4\pi r^2$.

SL

Power radiated by a planet

$$P = e\sigma AT^4$$

- P — power radiated (W)
- e — emissivity
- σ — Stefan–Boltzmann constant (W m^{-2} K^{-4})
- A — surface area (m^2)
- T — average surface temperature (K)

SL

Energy balance of a planet

$$\text{power absorbed} = \text{power radiated}$$

- T — the equilibrium temperature this fixes (K)

Not printed in the data booklet. Greenhouse gases — CO_2, H_2O, CH_4, N_2O — absorb outgoing infrared and re-emit it, so equilibrium is reached at a higher surface temperature.

B.3 Gas laws

SL

Amount of substance

$$n = N / NA$$

- n — amount of substance (mol)
- N — number of molecules
- NA — Avogadro constant (mol^{-1})

SL

Ideal gas equation, by moles

$$pV = nRT$$

- p — pressure (Pa)
- V — volume (m^3)
- n — amount of substance (mol)
- R — molar gas constant ($J\ K^{-1}\ mol^{-1}$)
- T — absolute temperature (K)

SL

Ideal gas equation, by molecules

$$pV = Nk_BT$$

- N — number of molecules
- k_B — Boltzmann constant ($J\ K^{-1}$)
- T — absolute temperature (K)

The same statement counted in molecules rather than moles; $k_B = R / N_A$.

SL
DERIVED

A fixed mass of gas, changing state

$$p_1V_1 / T_1 = p_2V_2 / T_2$$

- p, V, T — pressure, volume and absolute temperature before and after (Pa, m^3, K)

Not printed in the data booklet: it is $pV = nRT$ with n fixed. Hold one quantity constant and it becomes Boyle's, Charles's or Gay-Lussac's law.

SL

Average kinetic energy of a molecule

$$\bar{E}_k = (3/2)k_BT$$

- $\bar{E}_k$ — average translational kinetic energy of one molecule (J)
- k_B — Boltzmann constant ($J\ K^{-1}$)
- T — absolute temperature (K)

This is what absolute temperature measures — and why 0 K is the floor.

SL

Internal energy of an ideal monatomic gas

$$U = (3/2)nRT = (3/2)Nk_BT = (3/2)pV$$

- U — internal energy (J)
- n — amount of substance (mol)
- T — absolute temperature (K)

N times the average molecular kinetic energy. An ideal gas has no intermolecular potential energy, so this is all of it. B.4 puts it into the first law.

SL
DERIVED

Root-mean-square speed

$$v_{rms} = \sqrt{3k_BT / m} = \sqrt{3RT / M}$$

- v_{rms} — root-mean-square molecular speed ($m\ s^{-1}$)
- m — mass of one molecule (kg)
- M — molar mass ($kg\ mol^{-1}$)

Not printed in the data booklet: it is $\bar{E}_k = \frac{1}{2}m\,v_{rms}^2$ set equal to $(3/2)k_BT$. Lighter molecules move faster at the same temperature.

B.4 Thermodynamics

HL

First law of thermodynamics

$$Q = \Delta U + W$$

- Q — energy transferred to the gas by heating (J)
- ΔU — increase in internal energy (J)
- W — work done BY the gas (J)

Higher level only. Note the sign convention: W here is work done by the gas, so an expansion makes it positive.

HL

Work done by an expanding gas

$$W = p\Delta V$$

- W — work done by the gas (J)
- p — pressure (Pa)
- ΔV — change in volume (m^3)

Higher level only. At constant pressure. In general the work is the area under the p–V curve.

HL

Change in internal energy of a monatomic ideal gas

$$\Delta U = (3/2)nR\Delta T$$

- ΔU — change in internal energy (J)
- n — amount of substance (mol)
- ΔT — change in absolute temperature (K)

Higher level only. No temperature change means no change in internal energy, which is what makes an isothermal process tractable.

HL

Isothermal process

$$pV = \text{constant} \cdot W = nRT \ln(V_2 / V_1)$$

- W — work done by the gas (J)
- V_1, V_2 — initial and final volume (m^3)
- T — the constant absolute temperature (K)

Higher level only. $\Delta U = 0$, so $Q = W$: everything supplied by heating leaves as work.

HL

Adiabatic process for a monatomic ideal gas

$$pV^{(5/3)} = \text{constant}$$

- p — pressure (Pa)
- V — volume (m^3)

Higher level only. $Q = 0$, so $\Delta U = -W$: the gas cools as it expands because the work comes out of its internal energy.

HL

Entropy change

$$\Delta S = \Delta Q / T$$

- ΔS — change in entropy ($J\,K^{-1}$)
- ΔQ — energy transferred by heating (J)
- T — absolute temperature at which it is transferred (K)

Higher level only. The second law: the entropy of an isolated system never decreases.

HL

Efficiency of a heat engine

$$\eta = W / Q_{in} = 1 - Q_{out} / Q_{in}$$

- η — efficiency (a ratio, no unit)
- W — useful work per cycle (J)
- Q_{in}, Q_{out} — energy taken from the hot reservoir and dumped to the cold one (J)

Higher level only.

HL

Carnot efficiency

$$\eta_{Carnot} = 1 - T_{cold} / T_{hot}$$

- η_{Carnot} — the highest efficiency any engine between the two reservoirs can reach
- T_{cold}, T_{hot} — absolute temperatures of the two reservoirs (K)

Higher level only. A ceiling set by the temperatures alone — no engineering gets past it.

B.5 Current and circuits

SL

Electric current

$$I = \Delta q / \Delta t$$

$\Delta q = I\Delta t$

- I — current (A)
- Δq — charge passing a point (C)
- Δt — time taken (s)

Conventional current runs from positive to negative; electrons drift the other way.

SL

Potential difference

$$V = W / q$$

- V — potential difference (V)
- W — work done on or by the charge (J)
- q — charge moved (C)

SL

Resistance

$$R = V / I$$

$V = IR$ · $I = V / R$

- R — resistance (Ω)
- V — potential difference (V)
- I — current (A)

This defines resistance for any component. It is Ohm's law only where R stays constant, which a filament lamp does not.

SL

Resistivity

$$R = \rho L / A$$

- ρ — resistivity (Ω m)
- L — length (m)
- A — cross-sectional area (m^2)

SL

Current and drift speed

$$I = nAvq$$

- n — number density of charge carriers (m^{-3})
- A — cross-sectional area (m^2)
- v — drift speed ($m\ s^{-1}$)
- q — charge on each carrier (C)

SL

Electrical power

$$P = IV = I^2R = V^2 / R$$

- P — power dissipated (W)
- I — current (A)
- V — potential difference (V)
- R — resistance (Ω)

Pick the form that matches what is held constant — I^2R for a fixed current, V^2/R for a fixed supply.

SL

Resistors in series

$$R_s = R_1 + R_2 + \ldots$$

- R_s — combined resistance (Ω)

SL

Resistors in parallel

$$1 / R_p = 1/R_1 + 1/R_2 + \ldots$$

- R_p — combined resistance (Ω)

The combined resistance is always smaller than the smallest branch.

SL

Kirchhoff's circuit laws

$$\Sigma I = 0 \text{ at a junction} \cdot \Sigma V = 0 \text{ around a loop}$$

- I — currents in and out of the junction, with sign (A)
- V — e.m.f.s and potential drops around the loop, with sign (V)

Not printed in the data booklet as equations. The first is conservation of charge, the second conservation of energy.

SL

e.m.f. and internal resistance

$$\varepsilon = I(R + r)$$

$V = \varepsilon - Ir$

- ε — electromotive force of the cell (V)
- I — current drawn (A)
- R — external resistance (Ω)
- r — internal resistance of the cell (Ω)

The terminal p.d. V is what a voltmeter across the cell reads, and it falls below ε as soon as current flows.

C

Wave behaviour

C.1

Simple harmonic motion

SL

Defining condition for s.h.m.

$$a = -\omega^2 x$$

- a — acceleration ($\mathrm{m\ s^{-2}}$)
- ω — angular frequency ($\mathrm{rad\ s^{-1}}$)
- x — displacement from equilibrium (m)

The minus sign is the whole idea: the acceleration is proportional to the displacement and always points back towards equilibrium.

SL

Period and frequency

$$T = 1 / f$$

- T — period (s)
- f — frequency (Hz)

SL

Angular frequency

$$\omega = 2\pi / T = 2\pi f$$

- ω — angular frequency ($\mathrm{rad\ s^{-1}}$)
- T — period (s)
- f — frequency (Hz)

SL

Period of a mass on a spring

$$T = 2\pi\sqrt{(m / k)}$$

- T — period (s)
- m — mass (kg)
- k — spring constant ($\mathrm{N\ m^{-1}}$)

Independent of the amplitude — which is what makes the motion isochronous.

SL

Period of a simple pendulum

$$T = 2\pi\sqrt{(l / g)}$$

- T — period (s)
- l — length of the pendulum (m)
- g — gravitational field strength ($\mathrm{m\ s^{-2}}$)

For small amplitudes only, where sin θ ≈ θ. The mass of the bob does not appear.

HL

Displacement in s.h.m.

$$x = x_0 \sin(\omega t + \varphi) \cdot x = x_0 \cos(\omega t + \varphi)$$

x_0 — amplitude (m)
φ — phase angle (rad)
t — time (s)

Higher level only. Which of the two you use is a choice of where t = 0 sits — the sine form starts at the equilibrium point, the cosine form at maximum displacement.

HL

Velocity in s.h.m.

$$v = \omega x_0 \cos(\omega t + \varphi)$$

v — velocity (m s^{-1})
ω — angular frequency (rad s^{-1})
x_0 — amplitude (m)

Higher level only. The maximum speed is ωx_0, reached at the equilibrium point.

HL

Velocity from displacement

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$

v — velocity (m s^{-1})
x — displacement (m)
x_0 — amplitude (m)

Higher level only. The form to use when the question gives a position rather than a time.

HL

Kinetic energy in s.h.m.

$$E_k = \tfrac{1}{2}m\omega^2(x_0^2 - x^2)$$

E_k — kinetic energy (J)
m — mass (kg)
x_0 — amplitude (m)
x — displacement (m)

Higher level only.

HL

Potential energy in s.h.m.

$$E_p = \tfrac{1}{2}m\omega^2x^2$$

E_p — potential energy (J)
m — mass (kg)
x — displacement (m)

Higher level only.

HL

Total energy of an oscillator

$$E_T = \tfrac{1}{2}m\omega^2x_0^2$$

E_T — total energy (J)
m — mass (kg)
ω — angular frequency (rad s^{-1})
x_0 — amplitude (m)

Higher level only. Constant while the oscillator is undamped, and proportional to the square of the amplitude.

C.2

Wave model

SL

The wave equation

$$v = f\lambda$$

$f = v / \lambda \quad \cdot \quad \lambda = v / f$

v — wave speed (m s^{-1})
f — frequency (Hz)
λ — wavelength (m)

When a wave crosses into a new medium the frequency is fixed by the source, so the speed and the wavelength change together.

SL

Period and frequency of a wave

$$T = 1 / f$$

T — period (s)
f — frequency (Hz)

One wavelength passes a fixed point in one period, which is where $v = f\lambda$ comes from.

SL

Intensity and amplitude

$$I \propto A^2$$

I — intensity ($W\ m^{-2}$)
A — amplitude (m)

Intensity also falls as $1/d^2$ from a point source — the same inverse square law as B.1.

C.3 Wave phenomena

SL

Refractive index

$$n = c / v$$

n — refractive index of the medium (a ratio, no unit)
c — speed of light in a vacuum ($m\ s^{-1}$)
v — speed of light in the medium ($m\ s^{-1}$)

SL

Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \cdot \sin \theta_1 / \sin \theta_2 = v_1 / v_2 = n_2 / n_1$$

n_1, n_2 — refractive indices of the two media
θ_1, θ_2 — angles to the normal in each medium (°)
v_1, v_2 — wave speeds in each medium ($m\ s^{-1}$)

Angles are measured from the normal, not the surface.

SL

Critical angle

$$\sin \theta_c = n_2 / n_1$$

θ_c — critical angle (°)
n_1 — refractive index of the denser medium
n_2 — refractive index of the less dense medium

Total internal reflection needs $n_1 > n_2$ and an angle of incidence beyond θ_c.

SL

Condition for interference

$$\text{path difference} = n\lambda \text{ (constructive)} \cdot (n + \tfrac{1}{2})\lambda \text{ (destructive)}$$

n — any whole number
λ — wavelength (m)

For sources in phase. Two sources must be coherent — same frequency, constant phase difference — for a steady pattern.

SL

Double-slit fringe spacing

$$s = \lambda D / d$$

s — fringe separation (m)
λ — wavelength (m)
D — slit-to-screen distance (m)
d — slit separation (m)

Valid where D is far greater than d.

HL

First minimum of single-slit diffraction

$$\theta = \lambda / b$$

- θ — angle to the first minimum (rad)
- λ — wavelength (m)
- b — slit width (m)

Higher level only. A narrower slit spreads the pattern wider. This envelope also modulates the double-slit fringes.

HL

Diffraction grating

$$n\lambda = d \sin \theta$$

- n — order of the maximum
- λ — wavelength (m)
- d — spacing between adjacent slits (m)
- θ — angle of the maximum from the straight-through direction (°)

Higher level only. More slits make the maxima sharper, which is why a grating separates spectral lines better than a double slit.

HL

Rayleigh criterion

$$\theta = 1.22\, \lambda / b$$

- θ — smallest resolvable angular separation (rad)
- λ — wavelength (m)
- b — diameter of the circular aperture (m)

Higher level only. Two point sources are just resolved when the central maximum of one falls on the first minimum of the other.

C.4 Standing waves and resonance

SL

Standing wave with two fixed ends, or two open ends

$$\lambda_n = 2L / n$$

- λ_n — wavelength of the nth harmonic (m)
- L — length of the string or pipe (m)
- n — harmonic number, 1, 2, 3, …

A string fixed at both ends has a node at each end; a pipe open at both ends has an antinode at each end. The wavelengths come out the same.

SL

Standing wave in a pipe closed at one end

$$\lambda_n = 4L / n, \text{ with } n \text{ odd}$$

- λ_n — wavelength of the nth harmonic (m)
- L — length of the pipe (m)
- n — 1, 3, 5, … only

Node at the closed end, antinode at the open end. Only the odd harmonics exist.

SL
DERIVED

Frequency of a harmonic

$$f_n = v / \lambda_n$$

- f_n — frequency of the nth harmonic (Hz)
- v — wave speed in the string or the air ($m\ s^{-1}$)

Not a separate booklet equation: it is $v = f\lambda$ applied to the standing wave. For two fixed ends this gives $f_n = nv/2L$.

SL

Resonance and damping

$$\text{driving frequency} = \text{natural frequency at resonance}$$

- f_0 — natural frequency of the system (Hz)

No equation in the course: resonance and the three degrees of damping — light, critical, heavy — are examined qualitatively and from resonance curves. Heavier damping lowers and broadens the peak.

C.5 Doppler effect

SL

Doppler shift at low speed

$$\Delta f / f \approx \Delta\lambda / \lambda \approx v / c$$

- Δf — change in observed frequency (Hz)
- $\Delta\lambda$ — change in observed wavelength (m)
- v — relative speed of source and observer ($m\ s^{-1}$)
- c — wave speed — the speed of light for light ($m\ s^{-1}$)

An approximation, good only for v much less than c. It is the form used for the redshift of light from a receding source.

HL

Moving source, stationary observer

$$f' = f\ v / (v \mp u_s)$$

- f' — observed frequency (Hz)
- f — frequency emitted by the source (Hz)
- v — speed of the wave in the medium ($m\ s^{-1}$)
- u_s — speed of the source ($m\ s^{-1}$)

Higher level only. Minus when the source approaches — a smaller denominator, so a higher pitch — and plus when it recedes.

HL

Moving observer, stationary source

$$f' = f\ (v \pm u_o) / v$$

- f' — observed frequency (Hz)
- f — frequency emitted by the source (Hz)
- v — speed of the wave in the medium ($m\ s^{-1}$)
- u_o — speed of the observer ($m\ s^{-1}$)

Higher level only. Plus when the observer moves towards the source. The two cases are not symmetric, because the medium fixes the wave speed.

D Fields

D.1 Gravitational fields

SL

Newton's law of gravitation

$$F = GMm / r^2$$

- F — gravitational force between the two masses (N)
- G — gravitational constant ($N\ m^2\ kg^{-2}$)
- M, m — the two masses (kg)
- r — separation of their centres (m)

For point masses, and for uniform spheres measured centre to centre.

SL

Gravitational field strength

$$g = F / m = GM / r^2$$

- g — gravitational field strength ($N\ kg^{-1}$, $m\ s^{-2}$)
- M — mass producing the field (kg)
- r — distance from its centre (m)

The force per unit mass, and numerically the same as the acceleration of free fall at that point.

SL
DERIVED

Speed in a circular orbit

$$v = \sqrt{GM / r}$$

- v — orbital speed ($m\ s^{-1}$)
- M — mass being orbited (kg)
- r — orbital radius (m)

Not printed in the data booklet: it comes from setting the gravitational force equal to the centripetal force, $GMm/r^2 = mv^2/r$. The orbiting mass cancels.

SL DERIVED

Kepler's third law

$$T^2 = 4\pi^2 r^3 / (GM)$$

- T — orbital period (s)
- r — orbital radius (m)
- M — mass being orbited (kg)

Not marked given here because the booklet's coverage of it has not been confirmed. It follows from $GMm/r^2 = 4\pi^2 mr/T^2$, and the useful form is that T^2 is proportional to r^3 for everything orbiting the same central mass.

HL

Gravitational potential

$$V_g = -GM / r$$

- V_g — gravitational potential (J kg⁻¹)
- M — mass producing the field (kg)
- r — distance from its centre (m)

Higher level only. Negative because the zero is taken at infinity and gravity only attracts.

HL

Gravitational potential energy

$$E_p = -GMm / r$$

- E_p — potential energy of the pair (J)
- M, m — the two masses (kg)
- r — their separation (m)

Higher level only. This replaces $\Delta E_p = mg\Delta h$ once the field can no longer be treated as uniform.

HL

Field as potential gradient

$$g = -\Delta V_g / \Delta r$$

- g — gravitational field strength ($N\ kg^{-1}$)
- $\Delta V_g/\Delta r$ — potential gradient ($J\ kg^{-1}\ m^{-1}$)

Higher level only. The field is the gradient of the potential graph, and equipotential surfaces are everywhere perpendicular to the field lines.

HL

Work done moving a mass in a field

$$W = m\Delta V_g$$

- W — work done (J)
- m — mass moved (kg)
- ΔV_g — change in gravitational potential ($J\ kg^{-1}$)

Higher level only. No work is done moving along an equipotential.

HL

Escape speed

$$v_{esc} = \sqrt{2GM / r}$$

- v_{esc} — escape speed ($m\ s^{-1}$)
- M — mass of the planet or star (kg)
- r — distance from its centre (m)

Higher level only. The speed at which kinetic energy just cancels the negative potential energy, so the total energy reaches zero.

HL

Total energy of an orbiting body

$$E_T = -GMm / (2r)$$

- E_T — total energy of the orbiting body (J)
- M — mass being orbited (kg)
- m — mass in orbit (kg)
- r — orbital radius (m)

Higher level only. Kinetic plus potential. It is negative for a bound orbit, and a satellite losing energy to drag falls to a smaller r and speeds up.

D.2 Electric and magnetic fields

SL

Coulomb's law

$$F = kQ_1Q_2 / r^2, \text{ with } k = 1 / (4\pi\varepsilon_0)$$

F — force between the charges (N)
k — Coulomb constant ($N\ m^2\ C^{-2}$)
Q_1, Q_2 — the two point charges (C)
ε_0 — permittivity of free space ($C^2\ N^{-1}\ m^{-2}$)
r — separation (m)

Same inverse square shape as gravitation, but this one can repel as well as attract.

SL

Electric field strength

$$E = F / q = kQ / r^2$$

E — electric field strength ($N\ C^{-1}$, $V\ m^{-1}$)
F — force on the test charge (N)
q — test charge (C)
Q — charge producing the field (C)

Force per unit positive charge, so the field direction is the direction of the force on a positive charge.

SL

Uniform field between parallel plates

$$E = V / d$$

E — field strength ($V\ m^{-1}$)
V — potential difference between the plates (V)
d — plate separation (m)

HL

Electric potential

$$V_e = kQ / r$$

V_e — electric potential (V, $J\ C^{-1}$)
Q — charge producing the field (C)
r — distance from it (m)

Higher level only. Unlike gravitational potential this can be positive or negative, following the sign of Q.

HL

Electric potential energy

$$E_p = kQq / r$$

E_p — potential energy of the pair of charges (J)
Q, q — the two charges (C)
r — their separation (m)

Higher level only. Positive for two like charges, which is why they fly apart when released.

HL

Field as potential gradient

$$E = -\Delta V_e / \Delta r$$

E — electric field strength ($V\ m^{-1}$)
$\Delta V_e/\Delta r$ — potential gradient ($V\ m^{-1}$)

Higher level only. In a uniform field this is exactly E = V/d.

HL

Work done moving a charge

$$W = q\Delta V_e$$

W — work done (J)
q — charge moved (C)
ΔV_e — change in electric potential (V)

Higher level only.

D.3 Motion in electromagnetic fields

SL

Energy gained by an accelerated charge

$$\Delta E_k = qV$$

- ΔE_k — kinetic energy gained (J, eV)
- q — charge (C)
- V — accelerating potential difference (V)

This is what the electronvolt is: the energy an electron gains across one volt.

SL

Force on a moving charge in a magnetic field

$$F = qvB \sin \theta$$

- F — magnetic force (N)
- q — charge (C)
- v — speed ($m\ s^{-1}$)
- B — magnetic flux density (T)
- θ — angle between the velocity and the field (°)

The force is perpendicular to both v and B, so it does no work and cannot change the speed — only the direction.

SL

Force on a current-carrying conductor

$$F = BIL \sin \theta$$

- F — force on the conductor (N)
- B — magnetic flux density (T)
- I — current (A)
- L — length of conductor in the field (m)
- θ — angle between the conductor and the field (°)

Direction from the left-hand rule for conventional current.

SL
DERIVED

Radius of a charged particle's circular path

$$r = mv / (qB)$$

- r — radius of the path (m)
- m — mass of the particle (kg)
- v — speed ($m\ s^{-1}$)
- q — charge (C)
- B — magnetic flux density (T)

Not printed in the data booklet: it is $qvB = mv^2/r$ rearranged, for a particle moving at right angles to the field.

SL

Force between parallel current-carrying wires

$$F / L = \mu_0 I_1 I_2 / (2\pi r)$$

- F/L — force per unit length ($N\ m^{-1}$)
- μ_0 — permeability of free space ($T\ m\ A^{-1}$)
- I_1, I_2 — the two currents (A)
- r — separation of the wires (m)

Currents in the same direction attract; opposite directions repel.

D.4 Induction

HL

Magnetic flux

$$\Phi = BA \cos \theta$$

- Φ — magnetic flux (Wb)
- B — magnetic flux density (T)
- A — area of the loop (m^2)
- θ — angle between the field and the normal to the area (°)

Higher level only. Note that θ is measured from the normal, so the flux is greatest when the field is perpendicular to the plane of the loop.

HL

Flux linkage

$$\text{flux linkage} = N\Phi$$

- N — number of turns in the coil
- Φ — flux through one turn (Wb)

Higher level only.

HL

Faraday's law

$$\varepsilon = -N\,\Delta\Phi / \Delta t$$

- ε — induced electromotive force (V)
- N — number of turns
- $\Delta\Phi/\Delta t$ — rate of change of flux (Wb s^{-1})

Higher level only. The magnitude is the rate of change of flux linkage; the minus sign is Lenz's law, which is conservation of energy — the induced current opposes the change that made it.

HL

e.m.f. induced in a moving conductor

$$\varepsilon = BvL$$

- ε — induced e.m.f. (V)
- B — magnetic flux density (T)
- v — speed of the conductor (m s^{-1})
- L — length of the conductor in the field (m)

Higher level only. For a rod moving at right angles to a uniform field, sweeping out area as it goes.

E

Nuclear and quantum physics

E.1

Structure of the atom

SL

Nuclide notation

$${}^{A}ZX \cdot A = Z + N$$

- A — nucleon number
- Z — proton number
- N — number of neutrons
- X — chemical symbol of the element

Not printed in the data booklet — it is notation rather than an equation. Isotopes share Z and differ in N.

SL

Photon emitted or absorbed in a transition

$$E = hf = hc / \lambda$$

- E — difference between the two energy levels (J, eV)
- h — Planck constant (J s)
- f — frequency of the photon (Hz)
- λ — wavelength of the photon (m)

Discrete energy levels are why emission and absorption spectra are lines rather than a continuum, and why the two are complementary.

HL

Bohr model for hydrogen

$$E_n = -13.6 / n^2 \text{ eV}$$

- E_n — energy of the nth level (eV)
- n — principal quantum number, 1, 2, 3, …

Higher level only. Hydrogen only — a hydrogen-like ion of proton number Z has $E_n = -13.6Z^2 / n^2$ eV. The levels crowd together as n grows and reach zero at ionisation.

HL

Nuclear radius

$$R = R_0 A^{(1/3)}$$

R — radius of the nucleus (m)
R_0 — Fermi radius (m)
A — nucleon number

Higher level only. R^3 is proportional to A, so nuclear density is roughly the same for every nuclide.

HL
DERIVED

Distance of closest approach

$$qV = E_k, \text{ giving } d = kQq / E_k$$

d — closest approach of the alpha particle to the nucleus (m)
E_k — initial kinetic energy of the alpha particle (J)
Q, q — charges of the nucleus and the alpha particle (C)

Higher level only, and not printed in the data booklet: it is the kinetic energy converted entirely to electric potential energy. This is how Rutherford scattering puts an upper bound on nuclear size.

E.2

Quantum physics

HL

Photon energy and momentum

$$E = hf \cdot p = h / \lambda$$

E — photon energy (J, eV)
h — Planck constant (J s)
f — frequency (Hz)
p — photon momentum (kg m s^{-1})

Higher level only. A photon has momentum without having mass.

HL

Photoelectric equation

$$E_{max} = hf - \Phi$$

E_{max} — maximum kinetic energy of an emitted electron (J, eV)
hf — energy of the incident photon (J, eV)
Φ — work function of the metal (J, eV)

Higher level only. One photon, one electron — which is why intensity changes the number emitted and never their maximum energy.

HL

Threshold frequency

$$\Phi = hf_0$$

Φ — work function (J, eV)
f_0 — threshold frequency (Hz)

Higher level only. Below f_0 nothing is emitted however bright the light is.

HL

Stopping voltage

$$eV_s = E_{max}$$

e — elementary charge (C)
V_s — stopping voltage (V)
E_{max} — maximum kinetic energy of the photoelectrons (J)

Higher level only. The p.d. that just stops the fastest electron, and the way E_{max} is measured.

HL

de Broglie wavelength

$$\lambda = h / p$$

λ — de Broglie wavelength (m)
h — Planck constant (J s)
p — momentum of the particle (kg m s^{-1})

Higher level only. Electron diffraction is the evidence: matter shows interference where the wavelength is comparable to the spacing it meets.

HL

Compton scattering

$$\lambda_f - \lambda_i = (h / (m_e c))(1 - \cos\theta)$$

λ_i, λ_f — wavelength of the photon before and after scattering (m)
m_e — rest mass of the electron (kg)
θ — angle through which the photon is scattered (°)

Higher level only. The photon loses energy to the electron, so the scattered wavelength is always the longer one.

E.3 Radioactive decay

SL

Alpha decay

$$A \text{ decreases by } 4 \cdot Z \text{ decreases by } 2$$

α — an alpha particle is a helium nucleus, ${}^4_2He$

Not printed in the data booklet: it is bookkeeping you write out as a nuclear equation, with nucleon number and proton number balancing on both sides.

SL

Beta decay

$$\beta^-: n \rightarrow p + e^- + \bar{\nu}_e \cdot \beta^+: p \rightarrow n + e^+ + \nu_e$$

$\bar{\nu}_e$ — electron antineutrino
ν_e — electron neutrino

Not printed in the data booklet. The neutrino was proposed to account for the continuous beta energy spectrum — without it energy and momentum do not balance.

SL

Gamma emission

$$A \text{ unchanged} \cdot Z \text{ unchanged}$$

γ — a photon emitted as the nucleus drops to a lower energy state

Not printed in the data booklet. It usually follows an alpha or beta decay that leaves the nucleus excited.

SL
DERIVED

Fraction remaining after n half-lives

$$\text{fraction remaining} = (\tfrac{1}{2})^n$$

n — number of half-lives elapsed

Not printed in the data booklet: it is the definition of half-life applied repeatedly, and it answers most SL half-life questions without any logarithms.

SL

Mass defect and binding energy

$$\Delta m = Zm_p + Nm_n - m_{nucleus} \cdot E = \Delta mc^2$$

Δm — mass defect (kg, u)
m_p, m_n — rest masses of a free proton and a free neutron (kg, u)
E — binding energy released (J, MeV)

A nucleus weighs less than its parts; the shortfall is the energy that had to be supplied to pull it apart. Working in u and MeV c^{-2} is faster — 1 u is 931.5 MeV c^{-2}.

SL

Binding energy per nucleon

$$\text{binding energy per nucleon} = \text{binding energy} / A$$

A — nucleon number
binding energy per nucleon — how tightly bound the nuclide is (MeV)

Not printed in the data booklet. The curve peaks around iron-56, which is why fusion releases energy below the peak and fission above it.

HL

Activity and decay constant

$$A = \lambda N$$

A — activity (Bq)
λ — decay constant (s^{-1})
N — number of nuclei not yet decayed

Higher level only. λ is the probability per unit time that any one nucleus decays.

HL

Exponential decay law

$$N = N_0\, e^{(-\lambda t)} \cdot A = A_0\, e^{(-\lambda t)}$$

N — nuclei remaining after time t
N_0 — number present at $t = 0$
A_0 — initial activity (Bq)
t — time (s)

Higher level only. Activity and count rate fall on the same exponential, since both are proportional to N.

HL

Decay constant and half-life

$$\lambda = \ln 2 / t_{1/2}$$

λ — decay constant (s^{-1})
$t_{1/2}$ — half-life (s)

Higher level only. ln 2 is 0.693.

E.4 Fission

SL

Energy released in a fission reaction

$$E = \Delta mc^2 = c^2 \times (\text{total mass before} - \text{total mass after})$$

E — energy released (J, MeV)
Δm — loss of mass in the reaction (kg, u)
c — speed of light in a vacuum ($m\ s^{-1}$)

The same mass–energy relation as E.3, read off the binding-energy-per-nucleon curve: splitting a heavy nucleus moves the fragments up towards the peak, and the difference comes out as energy. The rest of E.4 — chain reactions, moderators, control rods, enrichment — is qualitative.

E.5 Fusion and stars

SL

Energy released in fusion

$$E = \Delta mc^2$$

E — energy released (J, MeV)
Δm — loss of mass in the reaction (kg, u)

Light nuclei fusing move up the binding-energy curve. Fusion needs enormous temperature and pressure because the nuclei have to overcome their mutual electrostatic repulsion.

SL

Stellar equilibrium

outward radiation pressure = inward gravitational pressure

equilibrium — what keeps a main sequence star at a stable radius

Not an equation in the course. A star leaves the main sequence when the fuel for the outward pressure runs out.

SL

Luminosity of a star

$$L = \sigma A T^4$$

L — luminosity, the total power radiated (W)
σ — Stefan–Boltzmann constant ($W\ m^{-2}\ K^{-4}$)
A — surface area, $4\pi R^2$ for a star of radius R (m^2)
T — surface temperature (K)

The same law as B.1, applied to a star treated as a black body.

SL

Apparent brightness of a star

$$b = L / (4\pi d^2)$$

- b — apparent brightness (W m^{-2})
- L — luminosity (W)
- d — distance to the star (m)

Measure b, know L, and you have the distance — the whole basis of the standard candle method.

SL

Surface temperature from colour

$$\lambda_{max} = 2.90 \times 10^{-3} / T$$

- λ_{max} — wavelength of peak emission (m)
- T — surface temperature (K)

Wien's law again, and the axis a Hertzsprung–Russell diagram is plotted against.

SL

Mass–luminosity relation

$$L \propto M^{3.5}$$

- L — luminosity (W)
- M — mass of the star (kg)

For main sequence stars only. The steep power is why massive stars burn out fastest despite having more fuel.

SL

Stellar parallax

$$d / \text{parsec} = 1 / (p / \text{arcsecond})$$

- d — distance to the star (pc)
- p — parallax angle (arcsecond)

Measured against the Earth's orbit six months apart. The angles are tiny, so the method runs out for distant stars.

M

Maths you must recall

SL

Uncertainty in a sum or a difference

$$\text{if } y = a \pm b \text{ then } \Delta y = \Delta a + \Delta b$$

- Δa, Δb — absolute uncertainties in the measured quantities
- Δy — absolute uncertainty in the result

Absolute uncertainties add even when the quantities subtract — which is why a small difference between two large measurements is so poorly known.

SL

Uncertainty in a product or a quotient

$$\text{if } y = ab / c \text{ then } \Delta y/y = \Delta a/a + \Delta b/b + \Delta c/c$$

- $\Delta a/a$ — fractional uncertainty in each measured quantity
- $\Delta y/y$ — fractional uncertainty in the result

Fractional uncertainties add here, not absolute ones. Multiply by 100% for a percentage uncertainty.

SL

Uncertainty in a power

$$\text{if } y = a^n \text{ then } \Delta y/y = |n| \, \Delta a/a$$

- n — the power, positive or negative
- $\Delta a/a$ — fractional uncertainty in a

A cube triples the fractional uncertainty, so a radius measured to 2% gives a volume known only to 6%.

SL

Gradient and intercept of a straight line

$$y = mx + c$$

- m — gradient
- c — intercept on the y axis

Most of the data analysis in this course is rearranging a relationship into this form so that the gradient carries the quantity you want.

SL **Circle**

$$\text{circumference} = 2\pi r \cdot A = \pi r^2$$

r — radius (m)
A — area (m^2)

SL **Cylinder**

$$\text{surface area} = 2\pi rh + 2\pi r^2 \cdot V = \pi r^2 h$$

r — radius (m)
h — height (m)
V — volume (m^3)

SL **Sphere**

$$\text{surface area} = 4\pi r^2 \cdot V = (4/3)\pi r^3$$

r — radius (m)
V — volume (m^3)

The $4\pi r^2$ is also the area a point source spreads its power over, which is where every inverse square law on this sheet comes from.

Values to know

SL	Acceleration of free fall at the Earth's surface, g	$9.81\ m\ s^{-2}$
SL	Gravitational constant, G	$6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$
SL	Coulomb constant, k	$8.99 \times 10^{9}\ N\ m^2\ C^{-2}$
SL	Permittivity of free space, ε_0	$8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$
SL	Permeability of free space, μ_0	$4\pi \times 10^{-7}\ T\ m\ A^{-1}$
SL	Speed of light in a vacuum, c	$3.00 \times 10^{8}\ m\ s^{-1}$
SL	Planck constant, h	$6.63 \times 10^{-34}\ J\ s$
SL	Elementary charge, e	$1.60 \times 10^{-19}\ C$
SL	Electronvolt, 1 eV	$1.60 \times 10^{-19}\ J$
SL	Avogadro constant, NA	$6.02 \times 10^{23}\ mol^{-1}$
SL	Molar gas constant, R	$8.31\ J\ K^{-1}\ mol^{-1}$
SL	Boltzmann constant, kB	$1.38 \times 10^{-23}\ J\ K^{-1}$
SL	Stefan–Boltzmann constant, σ	$5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$
SL	Wien displacement law constant, b	$2.90 \times 10^{-3}\ m\ K$
SL	Unified atomic mass unit, u	$1.661 \times 10^{-27}\ kg = 931.5\ MeV\ c^{-2}$
SL	Rest mass of the electron, me	$9.11 \times 10^{-31}\ kg = 0.000549\ u = 0.511\ MeV\ c^{-2}$
SL	Rest mass of the proton, mp	$1.673 \times 10^{-27}\ kg = 1.007276\ u = 938.3\ MeV\ c^{-2}$
SL	Rest mass of the neutron, mn	$1.675 \times 10^{-27}\ kg = 1.008665\ u = 939.6\ MeV\ c^{-2}$
HL	Fermi radius, R_0	$1.20 \times 10^{-15}\ m$
SL	Solar constant, S	$1.36 \times 10^{3}\ W\ m^{-2}$
SL	Mass of the Sun	$1.99 \times 10^{30}\ kg$
SL	Mass of the Earth	$5.97 \times 10^{24}\ kg$
SL	Radius of the Earth	$6.37 \times 10^{6}\ m$

SL	Astronomical unit, AU	1.50×10^{11} m
SL	Light year, ly	9.46×10^{15} m
SL	Parsec, pc	3.09×10^{16} m = 3.26 ly