Space, time and motion

IB Diploma Physics

> *The data booklet is on the desk in every paper, so the useful question is what you have to supply yourself: the definitions, the decay bookkeeping, and the results you are expected to derive. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not an IB document; the official syllabus is the authority.*

A.1 Kinematics

SL

Average velocity

$$v = \Delta s / \Delta t$$

- v — average velocity (m s^{-1})
- Δs — displacement (m)
- Δt — time interval (s)

A definition, not a booklet equation. Average speed uses distance travelled and average velocity uses displacement; the two differ whenever the path is not straight.

SL

Average acceleration

$$a = \Delta v / \Delta t$$

- a — average acceleration (m s^{-2})
- Δv — change in velocity (m s^{-1})
- Δt — time interval (s)

A definition, not a booklet equation.

SL

Velocity after uniform acceleration

$$v = u + at$$

- v — final velocity (m s^{-1})
- u — initial velocity (m s^{-1})
- a — acceleration (m s^{-2})
- t — time (s)

SL

Displacement under uniform acceleration

$$s = ut + \tfrac{1}{2}at^2$$

- s — displacement (m)
- u — initial velocity (m s^{-1})
- a — acceleration (m s^{-2})
- t — time (s)

SL

Uniform acceleration, without time

$$v^2 = u^2 + 2as$$

- v — final velocity (m s^{-1})
- u — initial velocity (m s^{-1})
- a — acceleration (m s^{-2})
- s — displacement (m)

SL

Displacement from average velocity

$$s = \tfrac{1}{2}(u + v)t$$

- s — displacement (m)
- u, v — initial and final velocity (m s^{-1})
- t — time (s)

All four of these hold only while the acceleration is constant.

SL

From the graphs

velocity = gradient of s–t · acceleration = gradient of v–t · displacement = area under v–t

gradient — of a tangent where the motion is not uniform

Not printed in the data booklet. Distance travelled is the area under a speed–time graph, which is not the same as displacement once the direction reverses.

SL
DERIVED

Projectile motion

$$\text{horizontal: } s_x = u_x t \cdot \text{vertical: } s_y = u_y t - \tfrac{1}{2} g t^2$$

- u_x — horizontal component of the launch velocity (m s^{-1})
- u_y — vertical component of the launch velocity (m s^{-1})
- g — acceleration of free fall (m s^{-2})

Not a separate booklet equation: it is the four equations above applied to each axis, with zero acceleration horizontally. Fluid resistance is left out here, and its effect on a projectile — shorter range, a steeper descent — is wanted only qualitatively; the drag force itself is in A.2, and terminal speed is where it balances the weight.

A.2

Forces and momentum

SL

Newton's second law

$$F = ma$$

- F — resultant force (N)
- m — mass (kg)
- a — acceleration (m s^{-2})

The special case of $F = \Delta p/\Delta t$ for constant mass. Force and acceleration are in the same direction.

SL

Force as rate of change of momentum

$$F = \Delta p / \Delta t$$

- F — resultant force (N)
- Δp — change in momentum (kg m s^{-1})
- Δt — time taken (s)

SL

Linear momentum

$$p = mv$$

- p — momentum (kg m s^{-1}, N s)
- m — mass (kg)
- v — velocity (m s^{-1})

SL

Impulse

$$J = F\Delta t = \Delta p$$

- J — impulse (N s)
- F — constant force (N)
- Δt — time for which it acts (s)

For a varying force, impulse is the area under a force–time graph.

SL

Translational equilibrium

$$\Sigma F = 0$$

- ΣF — vector sum of every force on the body (N)

Not printed in the data booklet. It is the condition for constant velocity, drawn from a free-body diagram.

SL

Conservation of linear momentum

$$\text{total momentum before} = \text{total momentum after}$$

- p — summed over the system, with direction

Not printed in the data booklet. It holds for an isolated system, in collisions and in explosions; kinetic energy is conserved only in an elastic collision.

SL

Static friction

$$F_f \le \mu_s N$$

F_f — friction force (N)
μ_s — coefficient of static friction
N — normal reaction force (N)

An inequality: static friction takes whatever value balances the applied force, up to this maximum.

SL

Dynamic friction

$$F_f = \mu_d N$$

F_f — friction force while sliding (N)
μ_d — coefficient of dynamic friction
N — normal reaction force (N)

SL

Buoyancy

$$F_b = \rho V g$$

F_b — buoyancy force, the upthrust on the body (N)
ρ — density of the fluid (kg m^{-3})
V — volume of fluid displaced (m^3)
g — gravitational field strength (N kg^{-1})

Archimedes' principle: the upthrust equals the weight of the fluid displaced. V is the volume of fluid pushed aside, which is the volume of the whole body only when it is fully submerged.

SL

Viscous drag on a sphere (Stokes' law)

$$F_d = 6\pi\eta r v$$

F_d — viscous drag force (N)
η — viscosity of the fluid (Pa s)
r — radius of the sphere (m)
v — speed of the sphere through the fluid (m s^{-1})

For a small sphere in laminar flow only. The drag grows with speed, which is what fixes terminal speed: the body stops accelerating once the drag plus the buoyancy balance the weight.

SL

Angular speed in a circle

$$\omega = 2\pi / T \cdot v = \omega r$$

ω — angular speed (rad s^{-1})
T — period of the circular motion (s)
v — linear speed (m s^{-1})
r — radius of the circle (m)

SL

Centripetal acceleration

$$a = v^2/r = \omega^2 r = 4\pi^2 r / T^2$$

a — centripetal acceleration (m s^{-2})
v — linear speed (m s^{-1})
r — radius (m)

Directed towards the centre, so uniform circular motion is accelerated motion even though the speed does not change.

SL

Centripetal force

$$F = mv^2/r = m\omega^2 r = 4\pi^2 m r / T^2$$

F — resultant force towards the centre (N)
m — mass (kg)
r — radius (m)

Not a new force: it is the name for whatever resultant — tension, gravity, friction, the normal force — points at the centre.

A.3 Work, energy and power

SL

Work done by a constant force

$$W = Fs\cos\theta$$

- W — work done (J)
- F — force (N)
- s — displacement (m)
- θ — angle between force and displacement (°)

A force at right angles to the motion does no work, which is why a centripetal force never changes the speed.

SL

Work from a force–displacement graph

$$\text{work done} = \text{area under a force–displacement graph}$$

- area — counted negative where the force opposes the motion

Not printed in the data booklet. This is how work is found when the force varies, as it does for a spring.

SL

Kinetic energy

$$E_k = \tfrac{1}{2}mv^2 = p^2 / (2m)$$

- E_k — kinetic energy (J)
- m — mass (kg)
- v — speed ($m\ s^{-1}$)
- p — momentum ($kg\ m\ s^{-1}$)

The momentum form is worth having: it turns a momentum answer into an energy answer without going back through the speed.

SL

Change in gravitational potential energy

$$\Delta E_p = mg\Delta h$$

- ΔE_p — change in gravitational potential energy (J)
- m — mass (kg)
- g — gravitational field strength ($N\ kg^{-1}$)
- Δh — change in height (m)

For a uniform field, which is the near-Earth case. D.1 replaces it with $E_p = -GMm/r$ once the field is no longer uniform.

SL

Force in a stretched spring

$$F = k\Delta x$$

$k = F / \Delta x$ · $\Delta x = F / k$

- F — force applied to the spring (N)
- k — spring constant ($N\ m^{-1}$)
- Δx — extension or compression (m)

Within the limit of proportionality.

SL

Elastic potential energy

$$E_e = \tfrac{1}{2}k\Delta x^2$$

- E_e — energy stored in the spring (J)
- k — spring constant ($N\ m^{-1}$)
- Δx — extension or compression (m)

The area under the force–extension graph.

SL

Power

$$P = W / \Delta t = Fv$$

- P — power (W)
- W — work done (J)
- Δt — time taken (s)
- F — force (N)
- v — speed in the direction of the force ($m\ s^{-1}$)

SL

Efficiency

$$\eta = \text{useful work out} / \text{total work in} = \text{useful power out} / \text{total power in}$$

η — efficiency (a ratio, no unit)

Multiply by 100% for a percentage. Energy is conserved either way — the shortfall is degraded, not destroyed.

A.4

Rigid body mechanics

HL

Torque of a force

$$\tau = Fr \sin \theta$$

τ — torque about the axis (N m)
F — force (N)
r — distance from the axis to the point where the force is applied (m)
θ — angle between the force and the line joining the axis to that point (°)

Higher level only. A body is in rotational equilibrium when the resultant torque about every axis is zero.

HL

Moment of inertia

$$I = \Sigma mr^2$$

I — moment of inertia (kg m²)
m — mass of each particle (kg)
r — its distance from the axis (m)

Higher level only. It depends on the axis chosen, not only on the body — the rotational counterpart of mass.

HL

Newton's second law for rotation

$$\tau = I\alpha$$

τ — resultant torque (N m)
I — moment of inertia (kg m²)
α — angular acceleration (rad s^{-2})

Higher level only.

HL

Angular velocity after uniform angular acceleration

$$\omega = \omega_0 + \alpha t$$

ω — final angular velocity (rad s^{-1})
ω_0 — initial angular velocity (rad s^{-1})
α — angular acceleration (rad s^{-2})
t — time (s)

Higher level only.

HL

Angular displacement under uniform angular acceleration

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \cdot \theta = \tfrac{1}{2}(\omega_0 + \omega)t$$

θ — angular displacement (rad)
ω_0, ω — initial and final angular velocity (rad s^{-1})
α — angular acceleration (rad s^{-2})

Higher level only. The four rotational equations mirror the four in A.1 term for term.

HL

Angular velocity without time

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

ω, ω_0 — final and initial angular velocity (rad s^{-1})
α — angular acceleration (rad s^{-2})
θ — angular displacement (rad)

Higher level only.

HL

Angular momentum and angular impulse

$$L = I\omega \cdot \Delta L = \tau\Delta t$$

- L — angular momentum (kg m² s⁻¹)
- I — moment of inertia (kg m²)
- ω — angular velocity (rad s⁻¹)
- τ — resultant torque (N m)

Higher level only. Angular momentum is conserved when the resultant torque is zero, which is why a skater spins faster with their arms pulled in.

HL

Rotational kinetic energy

$$E_k = \tfrac{1}{2}I\omega^2 = L^2 / (2I)$$

- E_k — rotational kinetic energy (J)
- I — moment of inertia (kg m²)
- ω — angular velocity (rad s⁻¹)
- L — angular momentum (kg m² s⁻¹)

Higher level only. A rolling body has this on top of $\tfrac{1}{2}mv^2$ for its centre of mass.

A.5 Galilean and special relativity

HL

Galilean transformations

$$x' = x - vt \cdot u' = u - v$$

- x', x — position in the moving and the original frame (m)
- u', u — velocity of an object in each frame (m s⁻¹)
- v — speed of one frame relative to the other (m s⁻¹)

Higher level only. The low-speed limit, and the thing special relativity replaces.

HL

Lorentz factor

$$\gamma = 1 / \sqrt{(1 - v^2/c^2)}$$

- γ — Lorentz factor (a ratio, no unit)
- v — relative speed of the two frames (m s⁻¹)
- c — speed of light in a vacuum (m s⁻¹)

Higher level only. $\gamma \geq 1$ always, and every relativistic effect below is γ doing the work.

HL

Lorentz transformations

$$x' = \gamma(x - vt) \cdot t' = \gamma(t - vx/c^2)$$

- x', t' — coordinates of an event in the moving frame (m, s)
- x, t — its coordinates in the original frame (m, s)
- γ — Lorentz factor

Higher level only. The same form works for intervals: $\Delta x' = \gamma(\Delta x - v\Delta t)$ and $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$.

HL

Relativistic velocity addition

$$u' = (u - v) / (1 - uv/c^2)$$

- u' — velocity measured in the moving frame (m s⁻¹)
- u — velocity measured in the original frame (m s⁻¹)
- v — relative speed of the frames (m s⁻¹)

Higher level only. Feed in $u = c$ and you get $u' = c$, whatever v is — which is the postulate the whole theory is built on.

HL

Time dilation

$$\Delta t = \gamma\Delta t_0$$

- Δt — time interval measured in the frame the clock moves in (s)
- Δt_0 — proper time, measured where the two events happen at the same place (s)
- γ — Lorentz factor

Higher level only. Getting the answer right is mostly a matter of deciding which observer measures the proper time.

HL

Length contraction

$$L = L_0 / \gamma$$

L — length measured in the frame the object moves in (m)
L_0 — proper length, measured at rest with respect to the object (m)
γ — Lorentz factor

Higher level only. Contraction happens along the direction of motion only.

HL

Spacetime interval

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

Δs — spacetime interval between two events (m)
Δt — time between them (s)
Δx — distance between them (m)

Higher level only. Every inertial observer disagrees about Δt and Δx and agrees about Δs — the invariant the spacetime diagram is drawn around.

HL

Rest energy

$$E_0 = mc^2$$

E_0 — rest energy (J, MeV)
m — rest mass (kg, MeV c^{-2})
c — speed of light in a vacuum (m s^{-1})

Higher level only. Working in MeV and MeV c^{-2} saves most of the arithmetic.

HL

Total energy

$$E = \gamma mc^2$$

E — total energy (J, MeV)
γ — Lorentz factor
m — rest mass (kg)

Higher level only.

HL

Relativistic kinetic energy

$$E_k = (\gamma - 1)mc^2$$

E_k — kinetic energy (J, MeV)
γ — Lorentz factor
m — rest mass (kg)

Higher level only. Total energy minus rest energy; $\frac{1}{2}mv^2$ is what this collapses to at low speed.

HL

Relativistic momentum

$$p = \gamma mv$$

p — momentum (kg m s^{-1}, MeV c^{-1})
γ — Lorentz factor
m — rest mass (kg)
v — speed (m s^{-1})

HL

Energy–momentum relation

$$E^2 = p^2c^2 + m^2c^4$$

E — total energy (J, MeV)
p — momentum (kg m s^{-1}, MeV c^{-1})
m — rest mass (kg, MeV c^{-2})

Higher level only. It links E and p without needing the speed, and for a massless particle it reduces to $E = pc$.