Wave behaviour

IB Diploma Physics

The data booklet is on the desk in every paper, so the useful question is what you have to supply yourself: the definitions, the decay bookkeeping, and the results you are expected to derive. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not an IB document; the official syllabus is the authority.

C.1 Simple harmonic motion

SL

Defining condition for s.h.m.

$$a = -\omega^2 x$$

- a — acceleration (m s⁻²)
- ω — angular frequency (rad s⁻¹)
- x — displacement from equilibrium (m)

The minus sign is the whole idea: the acceleration is proportional to the displacement and always points back towards equilibrium.

SL

Period and frequency

$$T = 1 / f$$

- T — period (s)
- f — frequency (Hz)

SL

Angular frequency

$$\omega = 2\pi / T = 2\pi f$$

- ω — angular frequency (rad s⁻¹)
- T — period (s)
- f — frequency (Hz)

SL

Period of a mass on a spring

$$T = 2\pi\sqrt{(m / k)}$$

- T — period (s)
- m — mass (kg)
- k — spring constant (N m⁻¹)

Independent of the amplitude — which is what makes the motion isochronous.

SL

Period of a simple pendulum

$$T = 2\pi\sqrt{(l / g)}$$

- T — period (s)
- l — length of the pendulum (m)
- g — gravitational field strength (m s⁻²)

For small amplitudes only, where $\sin\theta \approx \theta$. The mass of the bob does not appear.

HL

Displacement in s.h.m.

$$x = x_0 \sin(\omega t + \varphi) \cdot x = x_0 \cos(\omega t + \varphi)$$

- x_0 — amplitude (m)
- φ — phase angle (rad)
- t — time (s)

Higher level only. Which of the two you use is a choice of where $t = 0$ sits — the sine form starts at the equilibrium point, the cosine form at maximum displacement.

HL

Velocity in s.h.m.

$$v = \omega x_0 \cos(\omega t + \varphi)$$

- v — velocity (m s⁻¹)
- ω — angular frequency (rad s⁻¹)
- x_0 — amplitude (m)

Higher level only. The maximum speed is ωx_0, reached at the equilibrium point.

HL

Velocity from displacement

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$

- v — velocity (m s⁻¹)
- x — displacement (m)
- x_0 — amplitude (m)

Higher level only. The form to use when the question gives a position rather than a time.

HL

Kinetic energy in s.h.m.

$$E_k = \tfrac{1}{2}m\omega^2(x_0^2 - x^2)$$

- E_k — kinetic energy (J)
- m — mass (kg)
- x_0 — amplitude (m)
- x — displacement (m)

Higher level only.

HL

Potential energy in s.h.m.

$$E_p = \tfrac{1}{2}m\omega^2 x^2$$

- E_p — potential energy (J)
- m — mass (kg)
- x — displacement (m)

Higher level only.

HL

Total energy of an oscillator

$$E_T = \tfrac{1}{2}m\omega^2 x_0^2$$

- E_T — total energy (J)
- m — mass (kg)
- ω — angular frequency (rad s⁻¹)
- x_0 — amplitude (m)

Higher level only. Constant while the oscillator is undamped, and proportional to the square of the amplitude.

C.2 Wave model

SL

The wave equation

$$v = f\lambda$$

$f = v / \lambda$ · $\lambda = v / f$

- v — wave speed (m s⁻¹)
- f — frequency (Hz)
- λ — wavelength (m)

When a wave crosses into a new medium the frequency is fixed by the source, so the speed and the wavelength change together.

SL

Period and frequency of a wave

$$T = 1 / f$$

- T — period (s)
- f — frequency (Hz)

One wavelength passes a fixed point in one period, which is where $v = f\lambda$ comes from.

SL **Intensity and amplitude**

$$I \propto A^2$$

- I — intensity (W m⁻²)
- A — amplitude (m)

Intensity also falls as $1/d^2$ from a point source — the same inverse square law as B.1.

C.3 Wave phenomena

SL **Refractive index**

$$n = c / v$$

- n — refractive index of the medium (a ratio, no unit)
- c — speed of light in a vacuum (m s⁻¹)
- v — speed of light in the medium (m s⁻¹)

SL **Snell's law**

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \cdot \sin\theta_1 / \sin\theta_2 = v_1 / v_2 = n_2 / n_1$$

- n_1, n_2 — refractive indices of the two media
- θ_1, θ_2 — angles to the normal in each medium (°)
- v_1, v_2 — wave speeds in each medium (m s⁻¹)

Angles are measured from the normal, not the surface.

SL **Critical angle**

$$\sin\theta_c = n_2 / n_1$$

- θ_c — critical angle (°)
- n_1 — refractive index of the denser medium
- n_2 — refractive index of the less dense medium

Total internal reflection needs $n_1 > n_2$ and an angle of incidence beyond θ_c.

SL **Condition for interference**

$$\text{path difference} = n\lambda \text{ (constructive)} \cdot (n + \tfrac{1}{2})\lambda \text{ (destructive)}$$

- n — any whole number
- λ — wavelength (m)

For sources in phase. Two sources must be coherent — same frequency, constant phase difference — for a steady pattern.

SL **Double-slit fringe spacing**

$$s = \lambda D / d$$

- s — fringe separation (m)
- λ — wavelength (m)
- D — slit-to-screen distance (m)
- d — slit separation (m)

Valid where D is far greater than d.

HL **First minimum of single-slit diffraction**

$$\theta = \lambda / b$$

- θ — angle to the first minimum (rad)
- λ — wavelength (m)
- b — slit width (m)

Higher level only. A narrower slit spreads the pattern wider. This envelope also modulates the double-slit fringes.

HL

Diffraction grating

$$n\lambda = d \sin \theta$$

- n — order of the maximum
- λ — wavelength (m)
- d — spacing between adjacent slits (m)
- θ — angle of the maximum from the straight-through direction (°)

Higher level only. More slits make the maxima sharper, which is why a grating separates spectral lines better than a double slit.

HL

Rayleigh criterion

$$\theta = 1.22\ \lambda / b$$

- θ — smallest resolvable angular separation (rad)
- λ — wavelength (m)
- b — diameter of the circular aperture (m)

Higher level only. Two point sources are just resolved when the central maximum of one falls on the first minimum of the other.

C.4 Standing waves and resonance

SL

Standing wave with two fixed ends, or two open ends

$$\lambda_n = 2L / n$$

- λ_n — wavelength of the nth harmonic (m)
- L — length of the string or pipe (m)
- n — harmonic number, 1, 2, 3, …

A string fixed at both ends has a node at each end; a pipe open at both ends has an antinode at each end. The wavelengths come out the same.

SL

Standing wave in a pipe closed at one end

$$\lambda_n = 4L / n, \text{ with } n \text{ odd}$$

- λ_n — wavelength of the nth harmonic (m)
- L — length of the pipe (m)
- n — 1, 3, 5, … only

Node at the closed end, antinode at the open end. Only the odd harmonics exist.

SL
DERIVED

Frequency of a harmonic

$$f_n = v / \lambda_n$$

- f_n — frequency of the nth harmonic (Hz)
- v — wave speed in the string or the air (m s^{-1})

Not a separate booklet equation: it is $v = f\lambda$ applied to the standing wave. For two fixed ends this gives $f_n = nv/2L$.

SL

Resonance and damping

$$\text{driving frequency} = \text{natural frequency at resonance}$$

- f_0 — natural frequency of the system (Hz)

No equation in the course: resonance and the three degrees of damping — light, critical, heavy — are examined qualitatively and from resonance curves. Heavier damping lowers and broadens the peak.

C.5 Doppler effect

SL

Doppler shift at low speed

$$\Delta f / f \approx \Delta\lambda / \lambda \approx v / c$$

Δf — change in observed frequency (Hz)
$\Delta\lambda$ — change in observed wavelength (m)
v — relative speed of source and observer (m s^{-1})
c — wave speed — the speed of light for light (m s^{-1})

An approximation, good only for v much less than c. It is the form used for the redshift of light from a receding source.

HL

Moving source, stationary observer

$$f' = f v / (v \mp u_s)$$

f' — observed frequency (Hz)
f — frequency emitted by the source (Hz)
v — speed of the wave in the medium (m s^{-1})
u_s — speed of the source (m s^{-1})

Higher level only. Minus when the source approaches — a smaller denominator, so a higher pitch — and plus when it recedes.

HL

Moving observer, stationary source

$$f' = f (v \pm u_o) / v$$

f' — observed frequency (Hz)
f — frequency emitted by the source (Hz)
v — speed of the wave in the medium (m s^{-1})
u_o — speed of the observer (m s^{-1})

Higher level only. Plus when the observer moves towards the source. The two cases are not symmetric, because the medium fixes the wave speed.