Nuclear and quantum physics

IB Diploma Physics

The data booklet is on the desk in every paper, so the useful question is what you have to supply yourself: the definitions, the decay bookkeeping, and the results you are expected to derive. Compiled by GioPhysics from the published syllabus. This is an independent study aid, not an IB document; the official syllabus is the authority.

E.1 Structure of the atom

SL

Nuclide notation

$^{A}ZX \cdot A = Z + N$

- A — nucleon number
- Z — proton number
- N — number of neutrons
- X — chemical symbol of the element

Not printed in the data booklet — it is notation rather than an equation. Isotopes share Z and differ in N.

SL

Photon emitted or absorbed in a transition

$E = hf = hc / \lambda$

- E — difference between the two energy levels (J, eV)
- h — Planck constant (J s)
- f — frequency of the photon (Hz)
- λ — wavelength of the photon (m)

Discrete energy levels are why emission and absorption spectra are lines rather than a continuum, and why the two are complementary.

HL

Bohr model for hydrogen

$E_n = -13.6 / n^2$ eV

- En — energy of the nth level (eV)
- n — principal quantum number, 1, 2, 3, …

Higher level only. Hydrogen only — a hydrogen-like ion of proton number Z has $E_n = -13.6Z^2 / n^2$ eV. The levels crowd together as n grows and reach zero at ionisation.

HL

Nuclear radius

$R = R_0 A^{(1/3)}$

- R — radius of the nucleus (m)
- R_0 — Fermi radius (m)
- A — nucleon number

Higher level only. R^3 is proportional to A, so nuclear density is roughly the same for every nuclide.

HL
DERIVED

Distance of closest approach

$qV = E_k$, giving $d = kQq / E_k$

- d — closest approach of the alpha particle to the nucleus (m)
- Ek — initial kinetic energy of the alpha particle (J)
- Q, q — charges of the nucleus and the alpha particle (C)

Higher level only, and not printed in the data booklet: it is the kinetic energy converted entirely to electric potential energy. This is how Rutherford scattering puts an upper bound on nuclear size.

E.2 Quantum physics

HL

Photon energy and momentum

$$E = hf \cdot p = h / \lambda$$

- E — photon energy (J, eV)
- h — Planck constant (J s)
- f — frequency (Hz)
- p — photon momentum (kg m s^{-1})

Higher level only. A photon has momentum without having mass.

HL

Photoelectric equation

$$E_{max} = hf - \Phi$$

- E_{max} — maximum kinetic energy of an emitted electron (J, eV)
- hf — energy of the incident photon (J, eV)
- Φ — work function of the metal (J, eV)

Higher level only. One photon, one electron — which is why intensity changes the number emitted and never their maximum energy.

HL

Threshold frequency

$$\Phi = hf_0$$

- Φ — work function (J, eV)
- f_0 — threshold frequency (Hz)

Higher level only. Below f_0 nothing is emitted however bright the light is.

HL

Stopping voltage

$$eV_s = E_{max}$$

- e — elementary charge (C)
- V_s — stopping voltage (V)
- E_{max} — maximum kinetic energy of the photoelectrons (J)

Higher level only. The p.d. that just stops the fastest electron, and the way E_{max} is measured.

HL

de Broglie wavelength

$$\lambda = h / p$$

- λ — de Broglie wavelength (m)
- h — Planck constant (J s)
- p — momentum of the particle (kg m s^{-1})

Higher level only. Electron diffraction is the evidence: matter shows interference where the wavelength is comparable to the spacing it meets.

HL

Compton scattering

$$\lambda_f - \lambda_i = (h / (m_e c))(1 - \cos\theta)$$

- λ_i, λ_f — wavelength of the photon before and after scattering (m)
- m_e — rest mass of the electron (kg)
- θ — angle through which the photon is scattered (°)

Higher level only. The photon loses energy to the electron, so the scattered wavelength is always the longer one.

E.3 Radioactive decay

SL

Alpha decay

A decreases by 4 · Z decreases by 2

- α — an alpha particle is a helium nucleus, ${}^{4}_{2}He$

Not printed in the data booklet: it is bookkeeping you write out as a nuclear equation, with nucleon number and proton number balancing on both sides.

SL

Beta decay

$$\beta^-: n \rightarrow p + e^- + \bar{\nu}_e \cdot \beta^+: p \rightarrow n + e^+ + \nu_e$$

- $\bar{\nu}_e$ — electron antineutrino
- ν_e — electron neutrino

Not printed in the data booklet. The neutrino was proposed to account for the continuous beta energy spectrum — without it energy and momentum do not balance.

SL

Gamma emission

$$A \text{ unchanged} \cdot Z \text{ unchanged}$$

- γ — a photon emitted as the nucleus drops to a lower energy state

Not printed in the data booklet. It usually follows an alpha or beta decay that leaves the nucleus excited.

SL
DERIVED

Fraction remaining after n half-lives

$$\text{fraction remaining} = \left(\tfrac{1}{2}\right)^n$$

- n — number of half-lives elapsed

Not printed in the data booklet: it is the definition of half-life applied repeatedly, and it answers most SL half-life questions without any logarithms.

SL

Mass defect and binding energy

$$\Delta m = Zm_p + Nm_n - m_{nucleus} \cdot E = \Delta mc^2$$

- Δm — mass defect (kg, u)
- m_p, m_n — rest masses of a free proton and a free neutron (kg, u)
- E — binding energy released (J, MeV)

A nucleus weighs less than its parts; the shortfall is the energy that had to be supplied to pull it apart. Working in u and MeV c^{-2} is faster — 1 u is 931.5 MeV c^{-2}.

SL

Binding energy per nucleon

$$\text{binding energy per nucleon} = \text{binding energy} / A$$

- A — nucleon number
- binding energy per nucleon — how tightly bound the nuclide is (MeV)

Not printed in the data booklet. The curve peaks around iron-56, which is why fusion releases energy below the peak and fission above it.

HL

Activity and decay constant

$$A = \lambda N$$

- A — activity (Bq)
- λ — decay constant (s^{-1})
- N — number of nuclei not yet decayed

Higher level only. λ is the probability per unit time that any one nucleus decays.

HL

Exponential decay law

$$N = N_0 e^{(-\lambda t)} \cdot A = A_0 e^{(-\lambda t)}$$

- N — nuclei remaining after time t
- N_0 — number present at $t = 0$
- A_0 — initial activity (Bq)
- t — time (s)

Higher level only. Activity and count rate fall on the same exponential, since both are proportional to N.

HL

Decay constant and half-life

$$\lambda = \ln 2 / t_{\frac{1}{2}}$$

- λ — decay constant (s^{-1})
- $t_{1/2}$ — half-life (s)

Higher level only. ln 2 is 0.693.

E.4

Fission

SL

Energy released in a fission reaction

$$E = \Delta mc^2 = c^2 \times (\text{total mass before} - \text{total mass after})$$

- E — energy released (J, MeV)
- Δm — loss of mass in the reaction (kg, u)
- c — speed of light in a vacuum ($m\ s^{-1}$)

The same mass–energy relation as E.3, read off the binding-energy-per-nucleon curve: splitting a heavy nucleus moves the fragments up towards the peak, and the difference comes out as energy. The rest of E.4 — chain reactions, moderators, control rods, enrichment — is qualitative.

E.5 Fusion and stars

SL

Energy released in fusion

$$E = \Delta mc^2$$

- E — energy released (J, MeV)
- Δm — loss of mass in the reaction (kg, u)

Light nuclei fusing move up the binding-energy curve. Fusion needs enormous temperature and pressure because the nuclei have to overcome their mutual electrostatic repulsion.

SL

Stellar equilibrium

$$\text{outward radiation pressure} = \text{inward gravitational pressure}$$

- equilibrium — what keeps a main sequence star at a stable radius

Not an equation in the course. A star leaves the main sequence when the fuel for the outward pressure runs out.

SL

Luminosity of a star

$$L = \sigma A T^4$$

- L — luminosity, the total power radiated (W)
- σ — Stefan–Boltzmann constant ($W\ m^{-2}\ K^{-4}$)
- A — surface area, $4\pi R^2$ for a star of radius R (m^2)
- T — surface temperature (K)

The same law as B.1, applied to a star treated as a black body.

SL

Apparent brightness of a star

$$b = L / (4\pi d^2)$$

- b — apparent brightness ($W\ m^{-2}$)
- L — luminosity (W)
- d — distance to the star (m)

Measure b, know L, and you have the distance — the whole basis of the standard candle method.

SL

Surface temperature from colour

$$\lambda_{max} = 2.90 \times 10^{-3} / T$$

- λ_{max} — wavelength of peak emission (m)
- T — surface temperature (K)

Wien's law again, and the axis a Hertzsprung–Russell diagram is plotted against.

SL

Mass–luminosity relation

$$L \propto M^{3.5}$$

- L — luminosity (W)
- M — mass of the star (kg)

For main sequence stars only. The steep power is why massive stars burn out fastest despite having more fuel.

SL

Stellar parallax

$$d / \text{parsec} = 1 / (p / \text{arcsecond})$$

- d — distance to the star (pc)
- p — parallax angle (arcsecond)

Measured against the Earth's orbit six months apart. The angles are tiny, so the method runs out for distant stars.

M

Maths you must recall

SL

Uncertainty in a sum or a difference

$$\text{if } y = a \pm b \text{ then } \Delta y = \Delta a + \Delta b$$

- Δa, Δb — absolute uncertainties in the measured quantities
- Δy — absolute uncertainty in the result

Absolute uncertainties add even when the quantities subtract — which is why a small difference between two large measurements is so poorly known.

SL

Uncertainty in a product or a quotient

$$\text{if } y = ab / c \text{ then } \Delta y/y = \Delta a/a + \Delta b/b + \Delta c/c$$

- $\Delta a/a$ — fractional uncertainty in each measured quantity
- $\Delta y/y$ — fractional uncertainty in the result

Fractional uncertainties add here, not absolute ones. Multiply by 100% for a percentage uncertainty.

SL

Uncertainty in a power

$$\text{if } y = a^n \text{ then } \Delta y/y = |n|\ \Delta a/a$$

- n — the power, positive or negative
- $\Delta a/a$ — fractional uncertainty in a

A cube triples the fractional uncertainty, so a radius measured to 2% gives a volume known only to 6%.

SL

Gradient and intercept of a straight line

$$y = mx + c$$

- m — gradient
- c — intercept on the y axis

Most of the data analysis in this course is rearranging a relationship into this form so that the gradient carries the quantity you want.

SL

Circle

$$\text{circumference} = 2\pi r \cdot A = \pi r^2$$

- r — radius (m)
- A — area (m^2)

SL

Cylinder

$$\text{surface area} = 2\pi rh + 2\pi r^2 \cdot V = \pi r^2 h$$

- r — radius (m)
- h — height (m)
- V — volume (m^3)

SL

Sphere

$$\text{surface area} = 4\pi r^2 \cdot V = (4/3)\pi r^3$$

- r — radius (m)
- V — volume (m^3)

The $4\pi r^2$ is also the area a point source spreads its power over, which is where every inverse square law on this sheet comes from.

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Values to know

SL	Acceleration of free fall at the Earth's surface, g	$9.81\ \mathrm{m\ s^{-2}}$
SL	Gravitational constant, G	$6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$
SL	Coulomb constant, k	$8.99 \times 10^{9}\ \mathrm{N\ m^2\ C^{-2}}$
SL	Permittivity of free space, ε_0	$8.85 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$
SL	Permeability of free space, μ_0	$4\pi \times 10^{-7}\ \mathrm{T\ m\ A^{-1}}$
SL	Speed of light in a vacuum, c	$3.00 \times 10^{8}\ \mathrm{m\ s^{-1}}$
SL	Planck constant, h	$6.63 \times 10^{-34}\ \mathrm{J\ s}$
SL	Elementary charge, e	$1.60 \times 10^{-19}\ \mathrm{C}$
SL	Electronvolt, 1 eV	$1.60 \times 10^{-19}\ \mathrm{J}$
SL	Avogadro constant, NA	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
SL	Molar gas constant, R	$8.31\ \mathrm{J\ K^{-1}\ mol^{-1}}$
SL	Boltzmann constant, kB	$1.38 \times 10^{-23}\ \mathrm{J\ K^{-1}}$
SL	Stefan–Boltzmann constant, σ	$5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
SL	Wien displacement law constant, b	$2.90 \times 10^{-3}\ \mathrm{m\ K}$
SL	Unified atomic mass unit, u	$1.661 \times 10^{-27}\ \mathrm{kg} = 931.5\ \mathrm{MeV\ c^{-2}}$
SL	Rest mass of the electron, me	$9.11 \times 10^{-31}\ \mathrm{kg} = 0.000549\ \mathrm{u} = 0.511\ \mathrm{MeV\ c^{-2}}$
SL	Rest mass of the proton, mp	$1.673 \times 10^{-27}\ \mathrm{kg} = 1.007276\ \mathrm{u} = 938.3\ \mathrm{MeV\ c^{-2}}$
SL	Rest mass of the neutron, mn	$1.675 \times 10^{-27}\ \mathrm{kg} = 1.008665\ \mathrm{u} = 939.6\ \mathrm{MeV\ c^{-2}}$
HL	Fermi radius, R_0	$1.20 \times 10^{-15}\ \mathrm{m}$
SL	Solar constant, S	$1.36 \times 10^{3}\ \mathrm{W\ m^{-2}}$
SL	Mass of the Sun	$1.99 \times 10^{30}\ \mathrm{kg}$
SL	Mass of the Earth	$5.97 \times 10^{24}\ \mathrm{kg}$
SL	Radius of the Earth	$6.37 \times 10^{6}\ \mathrm{m}$
SL	Astronomical unit, AU	$1.50 \times 10^{11}\ \mathrm{m}$
SL	Light year, ly	$9.46 \times 10^{15}\ \mathrm{m}$
SL	Parsec, pc	$3.09 \times 10^{16}\ \mathrm{m} = 3.26\ \mathrm{ly}$